сообщения
объединенного
института
ядерных
исследований
дубна

E17-82-733

K.Lukierska-Walasek

EFFECT OF QUANTUM PROPERTIES ON THE CRITICAL BEHAVIOUR OF THE X-Y MODEL IN A LONGITUDINAL MAGNETIC FIELD

1982

Let us consider the quantum-mechanical X-Y model in a longitudinal field Γ described by the following Hamiltonian:

$$H = -\Gamma \sum_i S_i^z - \sum_{i,j} I_{ij} S_i^+ S_j^- \ , \tag{1}$$

where $S_i^{\pm} = S_i^x \pm i S_i^y$, S_i^{a} $(a = x, y, z)$ is the a-th component of the spin operator referred to the i-th site of the crystal lattice, and I_{ij} denotes the exchange integral.

In this system one can induce change of Γ at the constant temperature the second order phase transition. A line of critical points $\Gamma_c(T)$ ends at the multicritical point $[\Gamma_c(0), T=0]$, characterised by new, specifically quantum-mechanical exponents/1/. The critical behaviour at finite temperature corresponds to that of classical X-Y model/2,3/. At sufficiently low temperature, however, the region in which classical asymptotic behaviour should be observed becomes small, and one may expect to see the quantum-to-classical crossover behaviour similar to that for the transverse Ising model /4-7/.

In this report we show that the field theoretic renormalization group (R.G.) method can be easily adopted to the description of the crossover behaviour of the X-Y model in the field Γ. Instead of the spin Hamiltonian/1/ we use the functional Hamiltonian $S[\phi]$ which satisfies the following relation:

$$-\beta F = \ln \int d(\phi)\, e^{-S[\phi]} \ , \tag{2}$$

where F is the free energy, $\beta = 1/T$ and $\int d(\phi) \ldots$ denotes the integration over the complex scalar field ϕ (for the definition of $d(\phi)$ cf. Ref. /7/). The relevant part of $S[\phi]$ can be written as follows:

$$S[\phi] = \frac{1}{2} \sum_{m=-\infty}^{\infty} \int_{|\vec{p}|<\Lambda} (p^2 - imt + m_0^2)\, \phi^*_{\vec{p},m}\, \phi_{\vec{p},m} +$$

$$+ \frac{u_0 t}{4} \sum_{m_1} \ldots \sum_{m_4} \int_{|\vec{p}_1|<\Lambda} \ldots \int_{|\vec{p}_4|<\Lambda} \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)\, \delta_{m_1+m_2,\, m_3+m_4} \times \tag{3}$$

$$\times\, \phi^*_{\vec{p}_1,m_1}\, \phi^*_{\vec{p}_2,m_2}\, \phi_{\vec{p}_3,m_3}\, \phi_{\vec{p}_4,m_4} \ ,$$

where

$$\int\limits_{|\vec{p}|<\Lambda} \ldots = \frac{1}{(2\pi)^d} \int\limits_{|\vec{p}|<\Lambda} d^d\vec{p}, \quad \delta(\vec{p}) = (2\pi)^d\,\delta^{(d)}(\vec{p})$$

$\Lambda \sim \pi/a$ is the radius of the Brillouin zone for d-dimensional hypercubic crystal lattice, a denotes the lattice constant, m_0^2 is linear in the longitudinal field deviation $h = (\Gamma - \Gamma_c)/\Gamma_c$, $t \sim T\Lambda^2$, and $u_0 \sim \Lambda^{2-d}$ is a constant for the small temperature.

The vertex functions for the field $\phi_{\vec{p},m}$, calculated with the Hamiltonian /3/ diverge when $\Lambda \to \infty$ and, in addition, contain the singular terms in the limit $t \to 0$. In order to overcome these difficulties we introduce, similarly as in Ref. /7/, the renormalized field $\phi^R_{\vec{p},m}$, dimensionless coupling constant g, and renormalization momentum point μ. The equivalent procedure of removing the singularities can be performed with various values of g and μ. The set of transfromations of (g, μ) to the other possible pairs $(\bar{g}, \bar{\mu})$ forms the R.G. The definitions of $\phi^R_{\vec{p},m}$, g, μ, and a R.G. transformation are the following:

$$\phi^R_{\vec{p},m} = f^{-1/2}(t/\mu^2)\, Z_3^{-1/2}\, \phi_{\vec{p},m}, \tag{4}$$

$$u_0 t = \mu^{\epsilon} g f(t/\mu^2) Z_1 Z_3^{-2} = \bar{\mu}^{\epsilon} \bar{g} f(t/\mu^2) \bar{Z}_1 \bar{Z}_3^{-2} \tag{5}$$

and

$$m_0^2 - m_{0c}^2 = \mu^2 Z_2 h = \bar{\mu}^2 \bar{Z}_2 h\, e^{\ell}, \tag{6}$$

where $Z_i(g, t/\mu^2, \Lambda/\mu)$ is the renormalization constant, $\bar{Z}_i = Z_i(\bar{g}, t/\bar{\mu}^2, \Lambda/\bar{\mu})$, $f(x)$ is an arbitrary function with the following asymptotic properties:

$$\lim_{x\to\infty} f(x) = 1 \tag{7}$$

and

$$\lim_{x\to 0} f(x) \approx x, \tag{8}$$

m_{0c}^2 is the critical value of m_0^2 for $h=0$, $\epsilon = 4-d$ and $-\infty < \ell < \infty$ is the group parameter. The function $f(t/\mu^2)$ serves to remove the singularities when $t \to 0$. The final results of the theory do not depend on the explicit form of $f(x)$ (cf. Ref. /7/). From eqs. (5) and (6) it follows that $\bar{\mu} = \bar{\mu}(\ell, g, \mu)$ and $\bar{g} = \bar{g}(\ell, g, \mu)$ with the initial conditions $\bar{\mu}(0, g, \mu) = \mu$ and $\bar{g}(0, g, \mu) = g$. The renormalization constants Z_1 and Z_3 are determined by the equations:

$$\left.\frac{\partial \Gamma_R(\vec{p}, m=0)}{\partial p^2}\right|_{\substack{h=0\\ p^2=\mu^2}} = f(t/\mu^2) \tag{9}$$

and

$$\Gamma_R(\vec{p}_1, \ldots, \vec{p}_4; \{m_i = 0\})|_{h=0} = -\mu^{\epsilon} g f^3(t/\mu^2), \tag{10}$$

where $\vec{p}_i\vec{p}_j = \mu^2(\delta_{ij} - \frac{1}{4})$ and $\Gamma_R^{(n)}(\vec{p}_1, \ldots, \vec{p}_n, m_1, \ldots, m_n)$ denotes the renormalized n-point vertex function. In order to obtain the third condition for Z_2, we separate in $\Gamma_R^{(2)}(\vec{p}, 0)|_{p^2=\mu^2}$ the terms proportional to $\mu^2 h$. The singularities in these terms, when $\Lambda \to \infty$, are compensated by the suitable choice of Z_2.

The differential R.G. equations can be obtained from eqs. (5) and (6) and are the following:

$$\bar{\mu}\frac{d\bar{g}}{d\bar{\mu}} = \beta(\bar{g}, t/\bar{\mu}^2) \tag{11}$$

and

$$h\frac{d\bar{\mu}}{dh} = \nu(\bar{g}, t/\bar{\mu}^2)\bar{\mu}, \tag{12}$$

where $h = e^{-\ell}$ and the limit $\Lambda \to \infty$ has been taken. Note, that the inverse correlation length ξ^{-1} is asymptotically proportional to $\bar{\mu}(h)$ /6,7/.

The critical and multicritical behaviours can be obtained by the following transitions: $\lim_{\bar{\mu}\to 0}\lim_{t\equiv\infty}$ (since $t \sim T\Lambda^2$, $T \neq 0, \Lambda \to \infty$) and $\lim_{\bar{\mu}\to 0}\lim_{t\to 0}$, respectively. In order to get the crossover behaviour of ξ^{-1} to the first order in $\epsilon = 4-d$, it is necessary to calculate $\beta(\bar{g}, t/\bar{\mu}^2)$ and $\nu(\bar{g}, t/\bar{\mu}^2)$ to the second and first order with respect to $\bar{g}$. Thus, we get

$$\beta(\bar{g}, t/\bar{\mu}^2) = -\bar{g}\epsilon(t/\bar{\mu}^2)[1 - g/g^*(t/\bar{\mu}^2)] \tag{13}$$

and

$$\nu(\bar{g}, t/\bar{\mu}^2) = \frac{1}{2}\Big\{1 + \frac{\bar{g}}{2}\frac{\Omega_d}{(2\pi)^d} B\big(\frac{d}{2}, 3 - \frac{d}{2}\big) \times \tag{14}$$
$$\times \big(\sum_m \int_0^1 dx\, x(1-x)[x(1-x) - i\, m(t/\bar{\mu}^2)]^{-3+d/2},$$

where

$$\epsilon(t/\bar{\mu}^2) = \epsilon - 2(t/\bar{\mu}^2)\frac{d\ln f(t/\bar{\mu}^2)}{d(t/\bar{\mu}^2)}, \tag{15}$$

$$g^*(t/\bar{\mu}^2) = 2\frac{(2\pi)^d}{\Omega_d} B^{-1}\big(\frac{d}{2}, 3 - \frac{d}{2}\big) f^{-1}(t/\bar{\mu}^2) \times$$
$$\times \Big\{\sum_m \int_0^1 dx(1-x)([x(1-x) - i(1-2x)m(t/\bar{\mu}^2)]^{-3+d/2} + \tag{16}$$
$$+ 8[2x(1-x) - i\, m(t/\bar{\mu}^2)]^{-3+d/2}\,)\Big\}^{-1},$$

Ω_d is the solid angle in d-dimensions and $B(x, y)$ is the Euler beta function. The solution of eq. (11) with β (13) is the following:

$$\bar{g} = g(\bar{\mu}/\mu)^{-\epsilon}(f/\bar{f})[1 + \frac{1}{2}g(t/\mu^2)^{-\epsilon/2} f \times \times \int_{t/\mu^2}^{t/\bar{\mu}^2} g^{*-1}(y) f^{-1}(y)\epsilon(y) y^{(\epsilon-2)/2} dy]^{-1}, \quad (17)$$

where $\bar{f} = f(t/\bar{\mu}^2)$ and $f = f(t/\mu^2)$. From eq. (17) and with the help of eq. (16) we obtain the infrared stable fixed point coupling constant $g^*_{c\ell}$ as follows:

$$g^*_{c\ell} = \lim_{\bar{\mu}\to 0} \lim_{t\to\infty} \bar{g} = \frac{4}{5}\frac{(2\pi)^4}{\Omega_4}\epsilon + O(\epsilon^2), \quad (18)$$

where $d<4$. The multicritical behaviour is governed by the quantum fixed point with the following values of the coupling constant g^*_q:

$$g^*_q = \lim_{\bar{\mu}\to 0} \lim_{t\to 0} \bar{g} = \begin{cases} 0 & \text{for } d \geq 2 \\ \frac{4(2\pi)^2}{\Omega_2} \int_{-\infty}^{\infty} d\omega \int_0^1 dx\, x(1-x)[x(1-x) - i\omega(1-2x)]^{-2}(\epsilon-2) & \text{for } d < 2. \end{cases} \quad (19)$$

Similarly we can calculate from eqs. (14) and (17) the correlation length exponents. For the critical line we have

$$\nu_d = \lim_{\bar{\mu}\to 0} \lim_{t\to\infty} \nu(\bar{g}, t/\bar{\mu}^2) = \frac{1}{2} + \frac{1}{10}\epsilon + O(\epsilon^2) \quad d<4 \quad (20)$$

and for the multicritical point we obtain

$$\nu_{\bar{q}} = \lim_{\bar{\mu}\to 0} \lim_{t\to 0} \nu(\bar{g}, t/\bar{\mu}^2) = \begin{cases} \frac{1}{2} & \text{for } d \geq 2 \\ \frac{1}{2} + O[(\epsilon-2)^2] & \text{for } d < 2. \end{cases} \quad (21)$$

Eq. (21) agrees with the results of Ref. /1/.

In order to describe the crossover behaviour of $\bar{\mu}$ for $2 < d < 4$ we write $\bar{\mu}(h)$ in the following form:

$$\bar{\mu} = \mu\lambda_h^{\nu_q} X(t, \lambda_h), \quad (22)$$

where

$$\lambda_h = h e^{\nu_q F(t/\mu^2, t/\mu^2, g)} \quad (23)$$

is the non-linear scaling field,

$$F(t/y^2, t/\mu^2, g) = \int \frac{dy}{y}\{\nu^{-1}[g(t/y^2, t/\mu^2, g), t/y^2] - \nu_q^{-1}\} \quad (24)$$

$\int dy \ldots$ – denotes an indefinite integral and $\nu_q = \frac{1}{2}$. With the help of eq. (12) we obtain the following equation for X:

$$X \exp\{\nu_q F(t/\mu^2 \lambda_h^{-2\nu_q} X^{-2}, t/\mu^2, g)\} = 1. \quad (25)$$

The eq. (24) we cinsider in the scaling limit, e.g., $t \to 0$, $\lambda_h \to 0$ with X remaining unchanged. This leads to the separation of the variable $z = (t/\mu^2)\lambda_h^{-\phi}$ being finite in the scaling limit, where ϕ is the crossover exponent. Thus, X can be calculated as the functions of z. Taking into account eqs. (14), (17), (24) and (25) we obtain in the scaling limit for $\epsilon \ll 1$ the following result:

$$X(z) = X(0)[1 + 2\nu_{c\ell}(g/g^*_{c\ell})z]^{-1/5}, \quad (26)$$

where $z = (t/\mu^2)\lambda_h^{-\phi}$ with $\phi = \epsilon/2$. In the limit $z \to \infty$ eq. (26) reproduces the correct critical line behaviour:

$$\xi^{-1} \sim \bar{\mu} \sim \lambda_h^{\nu_{c\ell}}$$

REFERENCES

1. Gerber P.R., Beck H. J.Phys.C: Solid St.Phys., 1977, 10, p. 4013.
2. Betts P.D. In: Phase Transitions and Critical Phenomena vol.3, eds C.Domb and M.S.Green (Academic Press, New York) p. 569, 1974.
3. Dekeyser R., Rodgiers J. Physica, 1979, A81, p. 72.
4. Lawrie I.D. J.Phys.C: Solid St.Phys., 1978, 11, p. 1123.
5. Lawrie I.D. J.Phys.C: Solid St.Phys., 1978, 11, p. 3857.
6. Lukierska-Walasek K., Walasek K. Phys.Lett., 1981, 81A, p. 527.
7. Lukierska-Walasek K., Walasek K. J.Phys.C: Solid St.Phys. (in press).

Reseived by Publishing Department
on October 18 1982.

Люкерска-Валясек К. E17-82-733

Влияние квантовых свойств на критическое поведение модели X-Y в продольном магнитном поле

Теоретико-полевой метод ренормализационной группы, который недавно был предложен для описания квантового кроссовера для поперечной модели Изинга, приложен для исследования подобного эффекта происходящего в квантовой модели X-Y в продольном магнитном поле. Вычислены индекс кроссовера и масштабная функция до первого порядка в $\epsilon = 4 - d$ для пространственной размерности $2 < d < 4$.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1982

Lukerska-Walasek K. E17-82-733

Effect of Quantum Properties on the Critical Behaviour of the X-Y Model in a Longitudinal Magnetic Field

The field theoretical renormalization group method proposed recently to the description of the quantum crossover behaviour of the transverse Ising model is now adopted to the investigation of the similar effect in the quantum X-Y model in a longitudinal magnetic field. The crossover exponent and scaling function for the inverse correlation length are calculated to the first order in $\epsilon = 4 - d$, for the spatial dimension $2 < d < 4$.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1982