



# PATH INTEGRAL APPROACH TO POLARON CONDUCTIVITY

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### 1. INTRODUCTION

In a very well-known paper  $^{/1/}$  Feynman, Hellwarth, Iddings and Platzman (thereafter called FHIP) developed a powerful method to calculate the polaron conductivity free of the assumptions commonly made when the Boltzmann kinetic equation is applied to this problem  $^{/2-5/}$ . FHIP calculated the expectation value of the electron coordinate by means of the non-equilibrium density matrix of the electron phonon system in an external harmonic electric field  $\vec{E} = \vec{E}_0 e^{-i\omega t}$  and expressed the corresponding linear response function through a double path integral over electron trajectories. After approximating the path integral with the aid of Feynman's one-oscillator model of the polaron /6/ FHIP obtained a general expression for the polaron impedance, expected to be a very good approximation in the overall interval of the coupling strength a, at all temperatures T and arbitrary frequency  $\omega$ .General expressions for the polaron effective mass (at zero temperature) and the polaron drift mobility were obtained from the FHIP impedance. In papers /7,8/ the results of ref. /1/ were used to obtain the optical absorption coefficient of free polarons at T=0. It was shown that the resulting expression describes, at least qualitatively, all the expected features of the absorption spectrum such as one-phonon peaks and transitions to Franck-Condon and relaxed excited states. In papers /9,10/ the approach of FHIP was generalized to include the effect of a static magnetic field and the interaction with other phonon modes and applied to the analysis of galvanomagnetic phenomena and cyclotron resonance in polar crystals.

On the other hand there are some difficulties in the FHIP approach that have not been overcome up today. The first and perhaps less important one is that it is mathematically very complicated, because a general non-equilibrium density matrix is used to calculate the linear response function, a quantity which could be obtained in the framework of equilibrium theory (by using Kubo's formula). This leads to the appearance of a double path integral over trajectories defined in an infinite time interval where a much simpler integral over trajectories defined in a finite imaginary time interval could be used. The second difficulty is that the physically correct expression for the impedance is obtained by FHIP with a mathematically nonjustified expansion of a less accurate expression obtained for the admitance. In fact there is some uncertainty in this approach concerning which expression for the impedance corresponds to the Feynman approximation. The third difficulty is that the expression for the drift mobility at low temperatures differs by a factor  $3/2\beta$  ( $\beta = 1/kT$ , k is the Boltzmann factor) from the generally accepted result obtained from the Boltzman equation/2-5/ which must be correct at least at weak coupling and low temperatures. This incorrect expression for the mobility has been obtained in two quite independent ways<sup>(11)</sup>, and the source of the discrepancy or the way to overcome it while preserving the good features of the approach are not clear at all.

The present paper is a first attempt to solve the above-mentioned difficulties. By using Kubo's formula the polaron conductivity is expressed in terms of a double time retarded commutator Green function which can be obtained by analytical prolongation to the upper half-plane of the current-current Matsubara Green function. The last is exactly expressed by a simple path integral over cyclic electron trajectories defined in  $[0,\beta]$ Calculation of this integral in the framework of the variational principle for the polaron free energy leads in zeroth order to the FHIP results. The source of the discrepancy in the low temperature expression for the drift mobility and the way to overcome it are discussed. The expressions for the low temperature polaron effective mass and lifetime in this approximation are given and compared with the previous results.

### 2. KUBO'S FORMULA AND POLARON GREEN FUNCTIONS

We start with the well-known Frohlich Hamiltonian for the polaron  $^{\prime 12/}$ 

$$\hat{H} = \frac{\dot{p}^2}{2} + \sum_{\vec{k}} \hat{a}_{\vec{k}}^+ \hat{a}_{\vec{k}}^- + \sum_{\vec{k}} Q(k) (\hat{a}_{\vec{k}}^- + \hat{a}_{\vec{k}}^+) e^{i\vec{k}\cdot\vec{r}}; Q(k) = (\frac{2\sqrt{2}\pi a}{\Omega})^{1/2} \frac{1}{k}, (1)$$

where  $\vec{r}$  and  $\vec{p} = -iV$  are respectively the electron radius vector and quasimomentum operators,  $\hat{a}_{\vec{k}}$ ,  $\hat{a}_{\vec{k}}$  are the creation and annihilation operators for longitudinal optical phonons with quasimomentum  $\vec{k}$  and  $\Omega$  is the volume of the system. In (1) the phonon frequency, the conduction electron mass and the Planck constant are taken equal to unity.

Let us consider the double-time, retarded commutator Green function  $^{\prime\,13\prime}$ 

$$g_{R}(t-s) = i\Theta(t-s) < [\vec{p}_{H}(t), \vec{p}_{H}(s)] > ,$$

where:

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$$\hat{\vec{p}}(t) = e^{i\hat{H}t} \hat{\vec{p}} e^{-i\hat{H}t}; <...> = \frac{Sp[e^{-\beta\hat{H}}..]}{Sp e^{-\beta H}}; [\hat{A}(t), \hat{B}(s)] = \hat{A}(t) \hat{B}(s) - \hat{B}(s) \hat{A}(t)$$

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and

$$\Theta(t) = \begin{cases} 1, t > 0 \\ 0, t < 0 \end{cases}$$

By using Kubo's formula the conductivity  $\sigma(\omega)$  for the electronphonon system (1) can be expressed in terms of the Fourier transform of  $g_R(t)$  /13/

$$\sigma(\omega) = \frac{e^2}{3\pi i} \int_{-\infty}^{+\infty} \frac{dE}{E} \frac{\text{Im}\tilde{g}_{R}(E)}{E + \omega + i\epsilon},$$

$$\tilde{g}_{R}(\omega) = \int_{-\infty}^{+\infty} dt e^{i\omega t} g_{R}(t).$$
(2)

The function  $\vec{g}_{R}(\omega)$  is analytic in the upper half-plane of the complex variable  $\omega$  and hence can be obtained by analytical prolongation from the points  $i\omega_{n} = \frac{2\pi i n}{\beta} (n=1,2,...)$ . To obtain  $\vec{g}_{R}(i\omega_{n})$  we define the Matsubara Green function  $^{/14/}$ 

$$G(r-\sigma) = \langle T\{\vec{p}(r)\vec{p}(\sigma)\} \rangle; r, \sigma \in [0, \beta],$$

$$\tilde{G}(\omega_{n}) = \int_{0}^{\beta} dr e^{i\omega_{n}r} G(r); \omega_{n} = \frac{2\pi n}{\beta}; n = 0, \pm 1, \pm 2, \dots,$$
(3)

where  $\hat{\vec{p}}(r) = e^{\hat{H}r} \hat{\vec{p}} e^{-\hat{H}r}$  and  $T\{\hat{A}(r)\hat{B}(\sigma)\} = \Theta(r-\sigma)\hat{A}(r)\hat{B}(\sigma) + \Theta(\sigma-r)\hat{B}(\sigma)\hat{A}(r)$ . As is well-known /14/

$$\vec{g}_{R}(i\omega_{n}) = \vec{G}(\omega_{n}); \quad n = 1, 2, ...$$

Then to obtain  $\tilde{g}_{R}(\omega)$  we must calculate  $G(\omega_{n})$  and prolong it analytically to the upper half plane.

### 3. PATH INTEGRAL REPRESENTATION FOR THE MATSUBARA GREEN FUNCTION

Now we pass to obtain an exact path integral representation for  $G(r-\sigma)$  using the general method described in ref.<sup>15/</sup>. To do this we first apply to  $\hat{H}$  the Bogolubov canonical transformation<sup>16/</sup> and write it in the following way:

$$\hat{H} = \hat{H}_{0} + \frac{1}{2}\vec{p}^{2} + \hat{H}_{i}; \quad H_{0} = \sum_{\vec{k}} \hat{b}_{\vec{k}}^{+} \hat{b}_{\vec{k}}; \quad H_{i} = \sum_{\vec{k}} Q(k) (\hat{b}_{\vec{k}} + \hat{b}_{\vec{k}}^{+}), \quad (4)$$

where

$$\hat{\mathbf{b}}_{\vec{k}} = \hat{\mathbf{a}}_{\vec{k}} e^{\vec{i}\vec{k}\vec{r}}; \quad \hat{\mathbf{b}}_{\vec{k}}^+ = \hat{\mathbf{a}}_{\vec{k}}^+ e^{-\vec{i}\vec{k}\vec{r}}; \quad \hat{\vec{p}} = \vec{P} - \sum_{\vec{k}} \vec{k} \hat{\mathbf{b}}_{\vec{k}}^+ \hat{\mathbf{b}}_{\vec{k}}.$$

The Green function  $Q(r-\sigma)$  can be written in terms of the operators in the interaction picture  $^{/14/}$ 

$$G(r-\sigma) = \frac{1}{Z_{H}} \operatorname{Sp} \{ e^{-\beta H_{0}} T[\vec{p}_{I}(r)\vec{p}_{I}(\sigma)\hat{\sigma}(\beta)] \},$$
  

$$\hat{A}_{I}(r) = e^{\hat{H}_{0}r} \hat{A} e^{-\hat{H}_{0}r}; \quad Z_{H} = \operatorname{Sp} e^{-\beta \hat{H}},$$
  

$$\hat{\sigma}(\beta) = T \exp\{-\int_{0}^{\beta} dr [\hat{H}_{I}(r) + \frac{1}{2}\hat{p}_{I}^{2}(r)] \}.$$
(5)

Let us define the generating functional

$$Z(\vec{y}) = \text{Sp} \, e^{-\beta H_0} \text{Texp} \left\{ - \int_0^\beta dr \left[ \hat{H}_I(r) + \frac{1}{2} \hat{\vec{p}}_I^2(r) + i \vec{y}(r) \hat{\vec{p}}_I(r) \right] \right\}.$$

It is not difficult to verify that

$$Z_{H} = Z(0); G(r - \sigma) = -\frac{1}{Z(0)} \frac{\delta^{2} Z(\vec{y})}{\delta \vec{y}(r) \delta \vec{y}(\sigma)} |_{\vec{y} = 0}$$
(6)

On the other hand  $Z(\vec{y})\,$  can be expressed as an integral over Wiener's measure  $^{\prime\,17\prime}$ 

$$Z(\vec{\mathbf{y}}) = \int_{\vec{\mathbf{x}}(0)=0} D\vec{\mathbf{x}} e^{-\frac{1}{2} \int_{0}^{\beta} \vec{\mathbf{x}}^{2}(r) dr} W_{0}(\vec{\mathbf{x}} + \vec{\mathbf{y}}) ,$$

$$W_{0}(\vec{\mathbf{x}}) = \operatorname{Sp} e^{-\beta H_{0}} \operatorname{Texp} \left[ -\int_{0}^{\beta} dr \{\hat{H}_{i}(r) + i\vec{\mathbf{x}}(r)\hat{\vec{p}}_{i}(r)\} \right].$$
(7)

Then from (6) and (/) it follows that

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$$Z_{H} = \int_{\vec{x}(0)=0} D\vec{x} e^{-\frac{1}{2} \int_{0}^{\beta} d\vec{r} \cdot \vec{x}^{2}(r)} W_{0}(\vec{x}) ,$$

$$G(r-\sigma) = -\frac{1}{Z_{H}} \int_{\vec{x}(0)=0} D\vec{x} e^{-\frac{1}{2} \int_{0}^{\beta} dr \cdot \vec{x}^{2}(r)} \frac{\delta^{2} W_{0}(\vec{x})}{\delta \cdot \vec{x}(r) \delta \cdot \vec{x}(\sigma)} .$$
(8)

Integrating by parts the last expression we obtain

$$G(r-\sigma) = 3\delta(r-\sigma) - \frac{1}{Z_{\rm H}} \int_{\mathbf{x}(0)=0} \mathbf{D} \cdot \mathbf{x} e^{-\frac{1}{2} \int_{0}^{\beta} dr \cdot \mathbf{x}^{2}(r)} \mathbf{W} \cdot (\mathbf{x}) \cdot \mathbf{x}(r) \cdot \mathbf{x}(\sigma).$$
(9)

To obtain the explicit expression for  $W_0(\vec{x})$  we must calculate the trace in formula (7) for  $W_0(\vec{x})$ . The procedure to do this is described in ref.<sup>/18/</sup> and the result is:

$$W_{0}(\vec{x}) = \Omega \delta[\vec{x}(\beta) - \vec{x}(0)] Z_{H_{0}} e^{V[\vec{x}]} , \qquad (10)$$

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where

$$V[\mathbf{x}] = \sum_{\mathbf{k}} Q^{2}(\mathbf{k}) \int d\mathbf{r} \int d\sigma C(\mathbf{r} - \sigma) e^{i \mathbf{k} \left[ \mathbf{x}(\mathbf{r}) - \mathbf{x}(\sigma) \right]},$$
  

$$C(\mathbf{r}) = \frac{e^{\mathbf{r}}}{e^{\beta} - 1} + \frac{e^{-\mathbf{r}}}{1 - e^{-\beta}}.$$

Taking into account (8), (9) and (10) we obtain

$$\mathbf{G}(r-\sigma) = \mathbf{3}\delta(r-\sigma) - \langle \dot{\mathbf{x}}(r) \dot{\mathbf{x}}(\sigma) \rangle_{\mathbf{S}} , \qquad (11)$$

where

where  

$$\langle \mathbf{A}[\mathbf{x}] \rangle_{\mathbf{S}} = \frac{\int \mathbf{D} \mathbf{x} e^{-\mathbf{S}[\mathbf{x}]} \mathbf{A}[\mathbf{x}]}{\int \mathbf{D} \mathbf{x} e^{-\mathbf{S}[\mathbf{x}]} \mathbf{A}[\mathbf{x}]}, \qquad (12)$$

$$\mathbf{S}[\mathbf{x}] = \frac{1}{2} \int_{0}^{\beta} d\mathbf{r} \mathbf{x}^{2}(\mathbf{r}) - \mathbf{V}[\mathbf{x}]$$
and  

$$Z_{\mathbf{u}} = Z_{\mathbf{u}} = e^{-\beta \mathbf{F}},$$

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where F is the polaron free energy given by  $^{/19/}$ 

$$\mathbf{F} = -\frac{1}{\beta} \ln \int \mathbf{D} \mathbf{x} e^{-\mathbf{g}[\mathbf{x}]} . \tag{13}$$

Formulae (11)-(13) are exact expressions and can be used to calculate  $G(r-\sigma)$  in different approximations. They represent a considerable simplification of the problem, since the phonon variables have been exactly excluded and the system is now described by the electron-electron interaction  $V[\vec{x}]$ . The generalization of (11)-(13) to include the interaction with several phonon modes is trivial.

## 4. APPROXIMATE EXPRESSION FOR THE CONDUCTIVITY

The path integral in (11) can be calculated only approximately. To do this in the framework of the variational principle for the free energy, we calculate (11) using the trial action  $^{10/10}$ 

$$\mathbf{S}_{0}[\mathbf{x}] = \frac{1}{2} \int_{0}^{\beta} dr \, \mathbf{x}^{2}(r) + \int_{-\infty}^{+\infty} d\mathbf{w} \, \mathbf{A}(\mathbf{w}) \int_{0}^{\beta} dr \, \int_{0}^{r} d\sigma \, \mathbf{e}^{-\mathbf{w}(r-\sigma)} | \mathbf{x}(r) - \mathbf{x}(\sigma) |^{2} ,$$

where  $A(w) = e^{\beta w} A(-w) > 0$ . To calculate  $\langle \vec{x}(r) | \vec{x}(\sigma) \rangle_{S_p}$  we must take the second variational. derivative of the functional:

$$\mathbf{J}[\vec{\mathbf{F}}] = \langle \mathbf{e} \stackrel{\beta}{\underset{o}{\overset{\circ}{\mathbf{f}}}} \vec{\mathbf{f}} \cdot \vec{\mathbf{F}}(r) \cdot \vec{\mathbf{x}}(r)}{\underset{s_{0}}{\overset{\beta}{\mathbf{f}}}} = e \stackrel{\beta}{\underset{o}{\overset{\beta}{\mathbf{f}}}} \frac{\beta}{dr} \stackrel{\beta}{\underset{o}{\overset{\beta}{\mathbf{f}}}} d\sigma \mathbf{H}(r-\sigma) \cdot \vec{\mathbf{F}}(r) \cdot \vec{\mathbf{F}}(\sigma)}, \quad (14)$$

where

$$H(r - \sigma) = \frac{1}{\beta} \sum_{n=1}^{\infty} \frac{\omega_n^2 \cos \omega_n (r - \sigma)}{Z_0(i\omega_n)} ,$$
  
$$Z_0(i\omega_n) = (i\omega_n)^2 \left\{ 1 + 4 \int_{-\infty}^{\infty} \frac{dw}{w} \frac{A(w)}{w^2 + \omega_n^2} \right\} .$$

The resulting expression for  $G(\omega_n)$  is:

$$\vec{G}(\omega_{n}) = 3 \left[ 1 + \frac{\omega_{n}^{2}}{Z_{0}(i\omega_{n})} \right].$$
(15)

To determine  $Z_0(i\omega_n)$  we use the variational principle for the polaron free energy /19/

$$F \leq F_0 + \frac{1}{\beta} < S - S_0 >_{S_0},$$

where  $F_0$  is the free energy related to the action  $S_0[\vec{x}]$ . The condition of minimum of the r.h.s. of this inequality leads to the following equation for  $Z_0(i\omega_n)^{10/2}$ 

$$Z_{0}(i\omega_{n}) = -\omega_{n}^{2} - \frac{2a}{3\sqrt{\pi}} \int_{0}^{\beta/2} dr C(r) (1 - \cos\omega_{n}r) \Phi^{-3/2}(r), \qquad (16)$$

where

$$\Phi(r) = \frac{4}{\beta} \sum_{n=1}^{\infty} \frac{\cos \omega_n r - 1}{Z_0(i\omega_n)} .$$

It is very difficult to solve this equation to obtain the best  $Z_0(i\omega_n)$ . The FHIP results are obtained if we solve (16) by successive approximations starting with the expression of  $\Phi(r)$ corresponding to the Feynman one-oscillator model, for which /19/

$$A_{0}(w') = \frac{w^{3}(\nu^{2}-1)}{4} \frac{1}{1-e^{-\beta w'}} [\delta(w'-w) - \delta(w'+w)],$$
  

$$\Phi_{0}(r) = \frac{1}{\nu^{2}} f(r); f(r) = r(1-\frac{r}{\beta}) + \frac{\nu^{2}-1}{w\nu} \frac{1-e^{-w\nu r} - e^{-w\nu(\beta-r)} + e^{-\beta w\nu}}{1-e^{-\beta w\nu}}.$$

Here  $w = w(a,\beta)$  and  $\nu = \nu(a,\beta)$  are determined from the condition of minimum for the polaron free energy  $^{/19/}$ . The resulting expression for  $Z_0(i\omega_p)$  is:

$$Z_{0}(i\omega_{n}) = -\omega_{n}^{2} - \frac{2a\nu^{3}}{3\sqrt{\pi}} \int_{0}^{\beta/2} dr C(r) (1 - \cos\omega_{n}r) f^{-3/2}(r).$$
(17)

The analytic prolongation of  $\tilde{g}_{R}(i\omega_{p})$  to the upper half plane is obtained <sup>/10/</sup>changing  $Z_{0}(i\omega_{n})$  in formula (15) by:

$$Z_{0}(\omega) = \omega^{2} - \frac{2a\nu^{3}}{3\sqrt{\pi}} \int_{0}^{\infty} dt (1 - e^{i\omega t}) \operatorname{Im} \{C(-it)f^{-3/2}(-it)\}.$$
(18)

This is the FHIP expression for the polaron impedance. The Green function  $\tilde{g}_{p}(\omega)$  is given by:

$$\vec{g}_{R}(\omega) = 3 \left[ 1 - \frac{\omega^2}{Z_0(\omega)} \right]$$

and the conductivity obtained by putting this expression into

$$\sigma(\omega) = \frac{ie^2 \omega}{Z_0(\omega)} .$$
(19)

The treatment here is much simpler than in  $^{\prime 1\prime}$  and the expressions (18) and (19) are obtained "automatically" without any complementary physical considerations. Of course the expression for the drift mobility which follows from (19) is the same as in  $^{\prime 1\prime}$  and in the low temperature limit differs from the correct result by the factor  $3/2\beta$ .

To obtain (19) we have made two approximations. The first while changing S by  $S_0$  to obtain (15) and (16) from (11). The second while solving (16) in first order to obtain (17). Since the last approximation is exact to order a it cannot be the source of the above-mentioned discrepancy which is already present in lowest order. We can then conclude that in order to overcome this difficulty the important thing is not to improve the Feynman one-oscillator model by using a more accurate quadratic functional or even solving (16) exactly. On the other hand, one could try to improve (15) by considering other terms in an expansion of  $\langle \dot{\mathbf{x}}(r) \, \dot{\mathbf{x}}(\sigma) \rangle_{S}$  in powers of S-S<sub>0</sub>

$$\langle \vec{x}(r) \vec{x}(\sigma) \rangle_{s} = \langle \vec{x}(r) \vec{x}(\sigma) \rangle_{s} + \sum_{n=1}^{\infty} \frac{\langle \vec{x}(r) \vec{x}(\sigma); (s-s_{0})^{n} \rangle_{s}}{n! \langle e^{s_{0}-s} \rangle_{s_{0}}}$$
 (20)

By using (14) and (16) it is possible to show that the first correction (corresponding to n = 1) is equal to zero. Then (15) is exact to first order in powers of  $S-S_0$ . Since  $(S-S_0)S_0$  is proportional to *a* (for small *a*) one could conclude that (15) is also exact in lowest order. However, this point requires a more detailed analysis because for a <<1 and  $\beta >> 1$ ,  $(S-S_0)S_0$  is proportional to  $a\beta$  and the expansion (20) could not be valid at low temperatures. Then other approximation schemes should be developed on the basis of the exact expression (11).

### 5. POLARON EFFECTIVE MASS AND LIFETIME

### AT FINITE TEMPERATURES

Returning to the FHIP expression for the impedance (18) and deforming the contour of integration over t to Ret =  $0.0 < \text{Imt} \le \beta/2$  and  $0 \le \text{Ret} < \infty$ , Imt =  $\beta/2^{1/7}$  we obtain the following expres-

sions for the real and imaginary parts of the polaron impedance at finite temperatures:

$$\operatorname{ReZ}_{0}(\omega) = \omega^{2} - \frac{2a\nu^{3}}{3\sqrt{\pi}} \left\{ \int_{0}^{\beta/2} dt [1 - \cosh \omega t] C(t) t^{-3/2}(t) - \frac{\sinh \frac{\beta \omega}{2}}{\sinh \frac{\beta}{2}} \int_{0}^{\infty} dt g^{-3/2}(t) \cos t \sin \omega t \right\}.$$

$$\operatorname{Im Z}_{0}(\omega) = \frac{2a\nu^{3}}{3\sqrt{\pi}} - \frac{\sinh \beta \omega/2}{\sinh \beta/2} \int_{0}^{\infty} dt g^{-3/2}(t) \cos t \cos \omega t,$$

$$g(t) = \frac{t^{2}}{\beta} + \frac{\beta}{4} + \frac{\nu^{2} - 1}{w\nu} - \frac{\cosh \frac{\beta w\nu}{2} - \cos w\nu t}{\sinh \frac{\beta w\nu}{2}}.$$

$$(21)$$

For a quasiparticle with effective mass M and lifetime  $\tau$  the impedance is

$$Z_{0}(\omega) = M\omega^{2} + i\frac{M\omega}{r} . \qquad (22)$$

Then at low temperatures e  $\beta/2$  >>1, when r >>1, and low frequencies  $\omega <<1$  the polaron behaves as a quasiparticle with

$$M = 1 + \frac{a\nu^{3}}{3\sqrt{\pi}} \int_{0}^{\beta/2} dt \frac{t^{2} e^{-t}}{\left[t(1-\frac{t}{\beta}) + \frac{\nu^{2}-1}{w\nu}(1-e^{-w\nu t})\right]^{3/2}},$$
  
$$\frac{M}{r} = \frac{2a\beta^{3/2} \nu^{3}}{3\sqrt{\pi}\sinh\beta/2} \frac{K_{1}\left[\frac{\beta}{2}\sqrt{1+\frac{4(\nu^{2}-1)}{\beta w\nu}}\right]^{3/2}}{\sqrt{1+\frac{4(\nu^{2}-1)}{\beta w\nu}}},$$

where  $K_1(z)$  is the cylindric function of imaginary argument. This expression for the polaron effective mass at low temperatures is exactly the same as obtained in the previous paper 20/by a quite different method. From the general results for the impedance in a magnetic field of reference  $^{10/}$  it can be shown that M is also the cyclotron mass at low temperatures and weak magnetic fields. In the weak coupling limit M coincides with the mass parameter M<sub>11</sub> of reference  $^{9/}$  if the last is taken at low temperature, low frequency and weak magnetic field.

At higher temperatures the lifetime rapidly decreases and the quasiparticle picture is destroyed. Then the parameter Mobtained from (21) and (22) does not coincide with the cyclotron mass (which is always greater than unity) and cannot be interpreted as the polaron effective mass. The generalization of the present approach to include the action of a static magnetic field and the detailed analysis of the temperature and magnetic field dependence of the polaron cyclotron mass will be given in a next paper.

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Федянин В.К., Горшков С.Н., Родригес К. Е17-82-435 Метод континуального интегрирования в задаче об электропроводности полярона

Предлагается новый подход к вычислению электропроводности полярона методом континуального интегрирования. С помощью формулы Кубо электропроводность полярона выражается через двухвременную коммутаторную функцию Грина, которая может быть получена путем аналитического продолжения температурной функции Грина типа ток-ток. Последняя выражается точно через интеграл по электронным траекториям. Вычисление этого интеграла в рамках вариационного принципа для свободной энергии полярона приводит в нулевом порядке к результатам Фейнмана, Хеллварса, Иддингса и Плацмана. Получены выражения для эффективной массы и времени жизни полярона при низких температурах; проведено сравнение с результатами других работ. Обсуждается источник появления неправильной низкотемпературной зависимости в выражении для дрейфовой подвижности.

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Fedyanin V.K., Gorshkov S.N., Rodriguez C. E17-82-435 Path Integral Approach to Polaron Conductivity

A new approach to calculate the polaron conductivity using path integral methods is proposed. By using Kubo's formula the conductivity is expressed in terms of a double time retarded commutator Green function which can be obtained by analytical prolongation of the current-current Matsubara Green function. The last is exactly expressed by a simple path integral over electron trajectories. Calculation of this integral in the framework of the variational principle for the polaron free energy leads in a very simple way to the FHIP results. The expressions for the low temperature polaron effective mass and lifetime in this approximation are given and compared with previous calculations. The source of the discrepancy in the low temperature expression for the drift mobility is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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