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D.I.Uzunov

RENORMALIZATION-GROUP RECURSION RELATIONS FOR THE NONIDEAL BOSE GAS

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1. INTRODUCTION

The critical properties of the superfluid transition in Bose liquids have received a great deal of microscopic theoretical investigations starting, for instance, from the famous papers of A.Z.Patashinsky and V.L.Pokrovsky [1], and A.A.Migdal [2]. The most useful model for explicit calculations and for the obtaining nontrivial qualitative results near the λ-point is the nonideal Bose gas(NBG). In recent years,considerable efforts have been made to investigate the critical properties of the NBG by means of the Wilson renormalization group (WRG) in its field version to first order in the ϵ-expansion as well as by the (1/n)-expansion [3-15]. The WRG method [16-20] is a special powerful tool for quantitative investigations in, and only in, the scaling region near a critical point. As a field-theoretic method it enables us to derive from the field equations [1,2,21] the selected solutions, namely, scaling type solutions.

In this paper we present the recursion relations for the NBG to second order in $\epsilon = d_c - d$, where d is the dimensionality of the system, and d_c is the corresponding borderline dimension ($d_c = 4$ for $T_c \neq 0$ and $d_c = 2$ in limit $T_c \to 0$). The general recursion relations are studied in two cases:

(i) The finite-temperature critical behaviour ($T_c \neq 0$), or the classical case. In this case the recursion relations coincide with those obtained previously [22] for the Ginzburg-Landau classical Hamiltonian. For $T_c \neq 0$, $d_c = 4$ and the well-known universality appears: there is no any influence of the quantum correlations on the scaling behaviour. These features, discussed previously in refs. [1,2,23], and by the WRG to order ϵ^1 in refs. [3,5,6,24-26,28], are a consequence of the well-known fact (mentioned firstly by L.D.Landau, see refs. [1,2]) that the nonzero Bose-Matsubara frequencies are not relevant for the description of finite-temperature critical behaviours. But the Matsubara frequency enables us [26] to obtain the dynamic critical exponent z (see refs. [7-9], where the (1/n)-expansion has been applied, and refs. [27,28], where the case of superconductors has been considered to second order in ϵ). The dynamic critical exponent for the NBG has been calculated for $T_c \neq 0$ to second order in ϵ in ref. [7].

(ii) In the quantum limit $T_c \to 0$ the striking and most interesting (in my opinion) situation is recovered /29/. In this limit the perturbation series for the NBG (but not for other quantum mechanical models /26–30/) are drastically simplified. In result they can be exactly summed /29/. Then the critical behaviour appears to be the Gaussian one. Note that the λ-line $T_c(P)$ in He_4 has no point $T_c(P_0) = 0$. The results in limit $T_c \to 0$ are, before all, of a methodical interest. They cannot be referred to the λ-points in He_4.

In some previous investigations there are errors in the calculations /3,11–14/. These errors lead to uncorrect predictions (only) for the quantum case.

II. THE NONIDEAL BOSE GAS IN ITS FUNCTIONAL FORMULATION

With the same success one can start with the operator formalism /21/ but I think the functional formulation (see refs. /31,32/)is more appropriate for a combination with the WRG. Using the results in refs. /31,32/, the grand canonical partition function $Z = \mathrm{Tr}\{e^S\}$ of the NBG is expressed by the action $S = S_0 + S_I$, where

$$S_0 = \sum_{\alpha,q} \{ i\omega_\ell - a(\underset{\sim}{k}) \} \phi^*_\alpha(q)\phi_\alpha(q), \tag{1}$$

and

$$S_I = -\frac{v}{2\beta V} \sum_{q_1, q_2, q_3; \alpha\beta} \phi^*_\alpha(q_1)\phi^*_\beta(q_2)\phi_\alpha(q_3)\phi_\beta(q_1+q_2-q_3) \tag{2}$$

are the free and the interaction parts of the action S, respectively. In (1)–(2) the (Matsubara) frequency-momentum representation is used, $q = (\underset{\sim}{k}, \omega_\ell)$, of the system in volume $V = L^d$ with periodic boundary conditions ($\underset{\sim}{k} = \{k_i\}$, $k_i = 2\pi\ell_i/L$, $\ell_i = 0 \pm 1, \ldots$). In (1)–(2), $\omega = 2\pi\ell T$ (with $\ell = 0 \pm 1, \ldots$, $\beta = 1/T$, $k_B = h = 1$) is the Matsubara frequency, v is the scattering length, $a(\underset{\sim}{k}) = \frac{k^2}{2m} + r$, $|\underset{\sim}{k}| \equiv k$, is the mass of the bosons, $r = -\mu$, μ is the chemical potential assumed to be $\mu \sim (T_c - T)$ near the transition point T_c. We shall make difference between T and T_c only in r. The field vector $\underset{\sim}{\phi}(q) = \{\phi_\alpha(q), \quad \alpha = 1, \ldots, \frac{n}{2}\}$ is a Fourier transform of

$$\underset{\sim}{\psi}(\underset{\sim}{x}, \tau) = (\beta V)^{-1/2} \sum_q e^{i\underset{\sim}{k}\cdot\underset{\sim}{x} - i\omega_\ell\tau}\, \underset{\sim}{\phi}(q), \tag{3}$$

namely, the complex Bose (commuting) function depending on th space vector $\underset{\sim}{x}$ and the imaginary time τ, $0 < \tau < \beta$. The tra symbol in Z denotes:

$$\mathrm{Tr} = \prod_{\alpha=1}^{n/2} \prod_{k<\Lambda} \prod_{\omega_\ell} \int D\phi^*_\alpha(q) D\phi_\alpha(q). \tag{4}$$

The bare Green function $G_0(q) = -\langle\phi_\alpha(q)\phi^*_\alpha(q)\rangle$ is equal to

$$G_0^{-1}(q) = i\omega_\ell - a(\underset{\sim}{k}). \tag{5}$$

The Feynman diagrams for the model (1)–(2) are standard /21,31/. Note, the summation in frequencies ω_ℓ of an internal line formed by the legs of one and the same action is performed (see refs. /21,31/) by the rule

$$\sum_{\omega_\ell} e^{i\omega_\ell\tau} G_0(q), \quad \tau = +0. \tag{6}$$

III. RECURSION RELATIONS TO ORDER ϵ^2

The exact recursion relations to order ϵ^1 are presented in ref. /29/. Using the diagrams in figs. 1 and 2 we obtain the recursion relations to first order in ϵ:

$$\omega'_\ell = e^{s(2-\eta)}\omega_\ell, \tag{7}$$

$$m' = e^{s\eta} m, \tag{8}$$

$$r' = e^{s(2-\eta)}\{r + \frac{n+2}{2} I_1(r)(vT)\}, \tag{9}$$

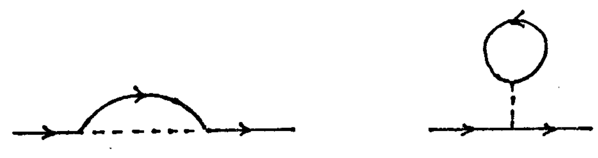

Fig.1

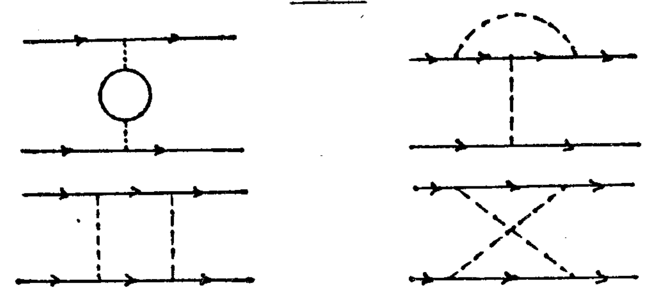

Fig.2

$$(vT)' = e^{s(4-d-2\eta)}\{(vT) - \frac{1}{2}[(n+6)I_2(r) + 2\tilde{I}_2(r)](vT)^2\}, \tag{10}$$

with the integrals

$$I_1(r) = \sum_{\substack{\omega_\ell \\ \tau=+0}} \int' d\underline{k}\, e^{i\omega_\ell \cdot \tau} G_0(q) = \frac{1}{T}\int' d\underline{k}\, n(\underline{k}), \tag{11}$$

$$I_2(r) = \sum_{\omega_\ell} \int' d\underline{k}\, G_0^2(q) = \frac{1}{(2T)^2}\int' d\underline{k}\, \frac{1}{\mathrm{sh}^2[\frac{a(\underline{k})}{2T}]}, \tag{12}$$

$$\tilde{I}_2(r) = \sum_{\omega_\ell} \int' d\underline{k}\, G_0(q) G_0(-q) = \frac{1}{2T}\int' d\underline{k}\, \frac{\coth[\frac{a(\underline{k})}{2T}]}{a(\underline{k})},$$

$$\int' d\underline{k} = K_{d-2}\int_{e^{-s}\Lambda}^{\Lambda} k^{d-1}dk \int_0^{\pi} \sin^{d-2}\theta\, d\theta \int_0^{2\pi} d\phi; \quad K_d = \frac{2^{1-d}\pi^{-d/2}}{\Gamma(\frac{d}{2})}. \tag{13}$$

In (11)-(13), $n^{-1}(k) = e^{\beta a(k)} - 1$, $\int' d\underline{k} = \int_{e^{-s}\Lambda}^{\Lambda} d\underline{k}$, where e^s with $0 < s < \infty$ is the rescaling factor, η is the anomalous dimension of the field $\phi(q)$. I do not make rescaling in the frequency ω_ℓ here (see ref. [26]), realizing in this way an extremely close application of the original WRG. With or without rescaling in ω_ℓ one obtains the same results. As I have pointed out in ref. [29], the summation (6) leads to a difference between the integrand $n(k)$ in I_1 and the corresponding integrand in refs. [11,12], but coincides with the result in refs. [3,6]. On the other hand, the integrand in $\tilde{I}_2$ differs from the corresponding function in ref. [3], but coincides with the result in refs. [11,12]. These differences disappear in classical limit $T_c \neq 0$, whereas in the quantum limit the final results are significantly different. The true results for the NBG are presented here.

Now I shall briefly sketch out the derivation of the recursion relations to order ϵ^2. The diagrams contributing in the renormalized Green function $G^{-1}(q') = e^{-s(2-\eta)}\{i\omega_\ell - a(ke^{-s})\}$ are presented in figure 3. From figs. (3a)-(3d) one finds

$$-(\frac{vT}{2})^2 (n+2)^2 \int' d\underline{k}_1 d\underline{k}_2 \sum_{\omega_{\ell_1}\omega_{\ell_2}} e^{i\omega_{\ell_1}\tau} G_0(q_1) G_0^2(q_2)|_{\tau=+0}\, e^{s(2-\eta)} H_0' =$$

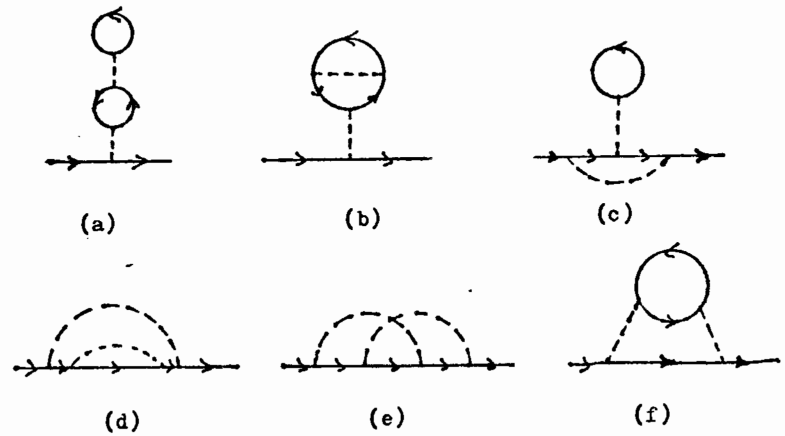

Fig.3

$$= e^{s(2-\eta)} (\frac{n+2}{2})^2 (v_0 T)^2 I_1(r) I_2(r); \quad H_0' = \sum_{\alpha,q}\{i\omega_\ell - \frac{(ke^{-s})^2}{2m} - r\}|\phi_\alpha'(q)|^2$$

$$\phi_\alpha(\omega_\ell, \underline{k}e^{-s}) = \exp[s(1-\eta/2)]\,\phi_\alpha'(\omega_\ell, \underline{k}). \tag{14}$$

For the q-dependent contributions in $G(q)$ figs. (3e) and (3f) yield

$$-\frac{1}{2}(vT)^2 (n+2) \sum_{\omega_{\ell_1}\omega_{\ell_2}} \int' d\underline{k}_1 d\underline{k}_2 G_0(q_1) G_0(q_2) G_0(q_1 + q - q_2) e^{s(2-\eta)} H_0' =$$

$$= e^{s(2-\eta)}(\frac{n+2}{2})(vT)^2 K(r,q) H_0', \tag{15}$$

where

$$K(r,q') = \int' d\underline{k}_1 d\underline{k}_2 S_K(q_1, q_2; q'; r); \quad q' = (\underline{k}e^{-s}, \omega_\ell) \tag{16}$$

is given by the sum

$$S_K = -\sum_{\omega_{\ell_1}\omega_{\ell_2}} G_0(q_1) G_0(q_2) G_0(q_1 - q_2 + q') \tag{17}$$

which, after the summation, is

$$S_K = (\frac{1}{2T})^2 \frac{1}{(i\omega_\ell - a_k - a_1 + a_3^-)}[\coth(\frac{a_1}{2T}) + \coth(\frac{a_1 - a_3^-}{2T})] \times \tag{18}$$

$$\times\ [\coth(\frac{a_k}{2T})-\coth(\frac{a_3^-}{2T})] \qquad (18)$$

with

$$a_j = \frac{1}{2m}\underset{\sim}{k}_j^2 + r \equiv \gamma \underset{\sim}{k}_j^2 + r, \quad j=1,2; \qquad (19)$$

$$a_3^{\pm} = \gamma(k_1 \pm k_2)^2 + r \qquad (20)$$

and

$$a_k = \gamma(\underset{\sim}{k}' - \underset{\sim}{k}_2)^2 + r \quad , \quad \underset{\sim}{k}' = \underset{\sim}{k}e^{-s}. \qquad (20')$$

Representing $K(r,q)$ as

$$K(r,\underset{\sim}{k},\omega_\ell) = \Delta K_k(r) + \Delta K_\omega(r) + K(r,0,0) \qquad (21)$$

with

$$\Delta K_\omega(r) = K(r,\underset{\sim}{k},\omega_\ell) - K(r,\underset{\sim}{k},0) \qquad (22)$$

and

$$\Delta K_k(r) = K(r,\underset{\sim}{k},0) - K(r,0,0), \qquad (23)$$

the recursion relations for the parameters of the bare Green function to order ϵ^2 are

$$\omega_\ell' = e^{s(2-\eta)}\omega_\ell\{1 + \frac{n+2}{2}(vT)^2[\frac{\Delta K_\omega(r)}{i\omega_\ell}]\}, \qquad (24)$$

$$\gamma' = e^{-s\eta}\gamma\{1 - \frac{n+2}{2}(vT)^2[\frac{\Delta K_k(r)}{(\underset{\sim}{k}e^{-s})^2}]\}, \qquad (25)$$

and

$$r' = e^{s(2-\eta)}\{r + (\frac{n+2}{2})(vT)I_1(r) - (\frac{n+2}{2})^2(vT)^2 I_1(r)I_2(r) - \qquad (26)$$

$$-(\frac{n+2}{2})(vT)^2 K(r,0,0)\}.$$

What follows is to find out the second order contributions to the recursion relation for the interaction constant v. They are presented diagrammatically in figs.4 and 5. The diagrams in fig.4 yield:

$$-(\frac{vT}{2})^3[(n^2+6n+16)I_2^2(r) + (12n+32)J_1(r) + \qquad (27)$$

$$+ 4(n+2)J_2(r) + 24J_3(r) + 4(n+2)J_4(r) + 4\tilde{I}_2^2(r)]$$

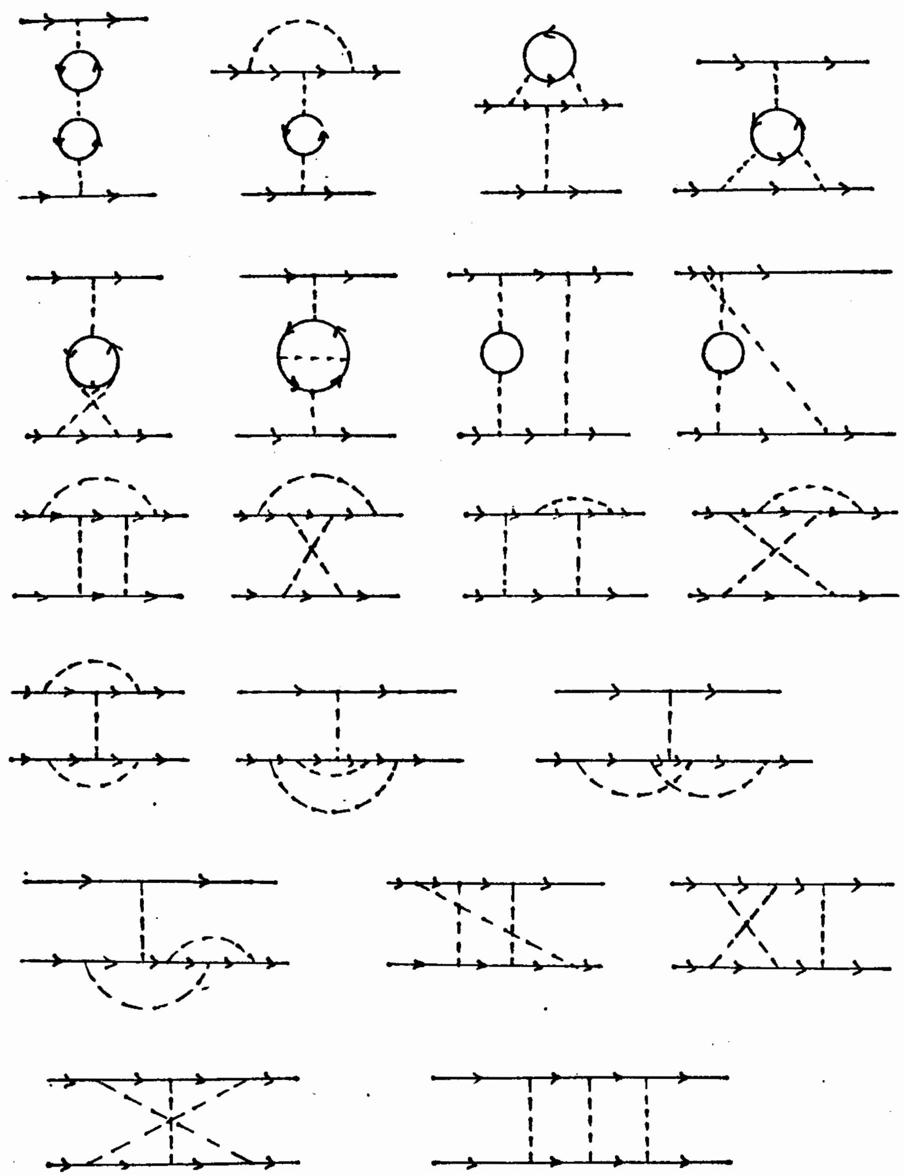

Fig.4

with

$$J_j(r) = \int' d\underset{\sim}{k}_1 d\underset{\sim}{k}_2 S_j^J(\underset{\sim}{k}_1,\underset{\sim}{k}_2,r), \quad j=1,\ldots,5, \qquad (28)$$

where

$$S_1^J = \sum_{\omega_{\ell_1}\omega_{\ell_2}} G_0^2(q_1)G_0(q_2)G_0(q_2-q_1), \quad (29)$$

$$S_2^J = \sum_{\omega_{\ell_1}\omega_{\ell_2}} G_0^2(q_1)G_0(q_2)G_0(q_1+q_2), \quad (30)$$

$$S_3^J = \sum_{\omega_{\ell_1}\omega_{\ell_2}} G_0^2(q_1)G_0(q_2)G_0(q_1-q_2), \quad (31)$$

$$S_4^J = \sum_{\omega_{\ell_1}\omega_{\ell_2}} G_0(q_1)G_0(-q_1)G_0(q_2)G_0(q_2-q_1), \quad (32)$$

$$S_5^J = \sum_{\omega_{\ell_1}\omega_{\ell_2}} G_0(q_1)G_0(-q_1)G_0(q_2)G_0(q_1+q_2). \quad (33)$$

After the summations in Eqs. (29)-(33) one easily obtains

$$S_1^J = \frac{1}{(2T)^2}\left[\coth\left(\frac{a_2}{2T}\right) - \coth\left(\frac{a_3^-}{2T}\right)\right]\times$$
$$\times\left\{B_1\left[\coth\left(\frac{a_1}{2T}\right) - \coth\left(\frac{a_2-a_3^-}{2T}\right)\right] + \frac{A_1 - Ba_1}{2T\,\mathrm{sh}^2\left(\frac{a_1}{2T}\right)}\right\} \quad (34)$$

with

$$A_1 = \frac{2a_1 - a_2 + a_3^-}{(a_2-a_3^-)^2 + a_1^2 - 2a_1(a_2-a_3^-)} \quad (35)$$

and

$$B_1 = \frac{1}{(a_2-a_3^-)^2 + a_1^2 - 2a_1(a_2-a_3^-)}, \quad (36)$$

$$S_2^J = \frac{1}{(2T)^2}\left[\coth\left(\frac{a_2}{2T}\right) - \coth\left(\frac{a_3^+}{2T}\right)\right]\times$$
$$\times\left\{B_2\left[\coth\left(\frac{a_1}{2T}\right) + \coth\left(\frac{a_2-a_3^+}{2T}\right)\right] + \frac{A_2 - B_2a_1}{2T\,\mathrm{sh}^2\left(\frac{a_1}{2T}\right)}\right\} \quad (37)$$

with

$$A_2 = \frac{a_2 + 2a_1 - a_3^+}{2a_1(a_3^+ - a_2) - a_1^2 - (a_2 - a_3^+)^2} \quad (38)$$

and

$$B_2 = \frac{1}{2a_1(a_3^+ - a_2) - a_1^2 - (a_2 - a_3^+)^2}, \quad (39)$$

$$S_3^J = \frac{1}{(2T)^2}\left[\coth\left(\frac{a_2}{2T}\right) + \coth\left(\frac{a_3^-}{2T}\right)\right]\times$$
$$\times\left\{B_3\left[\coth\left(\frac{a_1}{2T}\right) - \coth\left(\frac{a_2+a_3^-}{2T}\right)\right] + \frac{A_3 - B_3a_1}{2T\,\mathrm{sh}^2\left(\frac{a_1}{2T}\right)}\right\}, \quad (40)$$

$$A_3 = (2a_1 - a_2 - a_3^-)B_3 \quad (41)$$

with

$$B_3 = \frac{1}{2a_1(a_2+a_3^-) - (a_2+a_3^-)^2 - a_1^2}, \quad (42)$$

and

$$S_4^J = \frac{1}{(2T)^3}\,\frac{\left[\coth\left(\frac{a_2}{2T}\right) - \coth\left(\frac{a_3^-}{2T}\right)\right]}{\left[a_1^2 - (a_2-a_3^-)^2\right]}\left\{\left(\frac{a_2-a_3^-}{a_1}\right)\coth\left(\frac{a_1}{2T}\right) - \coth\left(\frac{a_2-a_3^-}{2T}\right)\right\}, \quad (43)$$

$$S_5^J(a_1,a_2,a_3^+) = S_4^J(a_1,a_2,a_3^+). \quad (44)$$

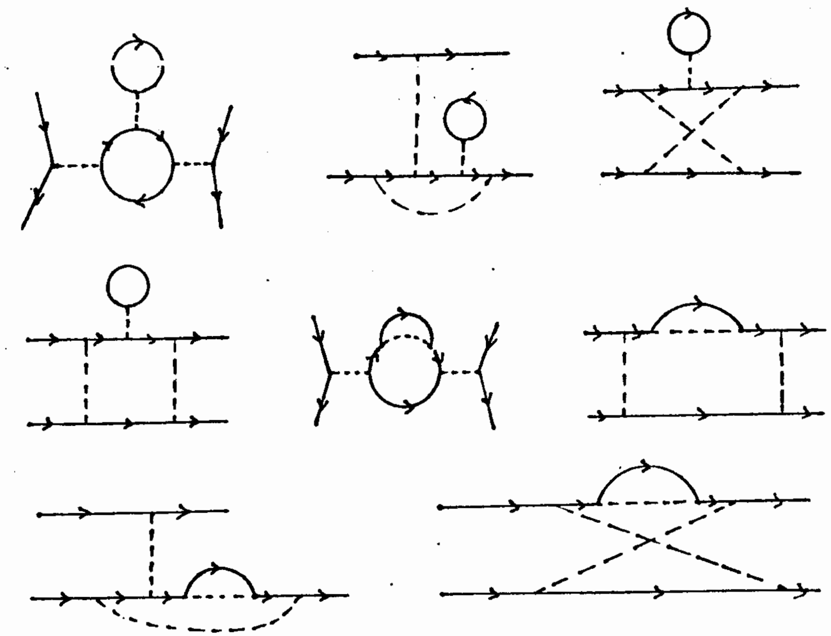

Fig5

From the diagrams in fig.5 one finds

$$-2\left(\frac{vT}{2}\right)^3[(n+2)(n+6)I_1(r)I_3(r)+2(n+2)I_1(r)\tilde{I}_3(r)] \tag{45}$$

with

$$I_3(r) = \sum_{\omega\ell} \int' d\underline{k}\, G_0^3(q) = \frac{1}{2}\frac{d}{dr}[I_2(r)], \tag{46}$$

and

$$\tilde{I}_3(r) = \sum_{\omega\ell} \int' d\underline{k}\, G_0^2(q)G_0(-q) = \frac{1}{2}\frac{d}{dr}[\tilde{I}_2(r)]. \tag{47}$$

Finally for the renormalized interaction constant one obtains:

$$\begin{aligned}(vT)' = e^{s(4-d-2\eta)}\{&(vT) - \frac{1}{2}(vT)^2[(n+6)I_2(r) + 2\tilde{I}_2(r)] + \\ &+\frac{(vT)^3}{4}[(n^2+6n+16)I_2^2(r)+4\tilde{I}_2^2(r)+(12n+32)J_1(r)+ \\ &+4(n+2)J_2(r)+24J_3(r)+4(n+2)J_4(r)+16J_5(r)+ \\ &+2(n+2)(n+6)I_1(r)I_3(r)+4(n+2)I_1(r)\tilde{I}_3(r)]\}.\end{aligned} \tag{48}$$

The recursion relations (24)-(26) and (48) for the parameters of the Hamiltonian (1)-(2) are very complicated for to be investigated in a common way. But fortunately, one has to account for explicitly that he is really interested in two cases: $T_c \neq 0$ and $T_c = 0$.

IV. DISCUSSION

Using the results in the previous section one can easily obtain the results in ref./22/ and the universality (see also refs./16-20/). Here, despite of its irrelevance, the frequency ω_ℓ gives us the possibility of obtaining dynamic critical exponent to order ϵ^2. This result is obtained previously through a direct calculation in ref./7/. The full information for the static critical behaviour is obtained in ref./22/.

The quantum-mechanical limit $T_c \to 0$ is obtained by the approximation $\coth x \sim 1$. Then one finds that the perturbation contributions in γ, r and ω_ℓ vanish and as a consequence: scaling laws of Gaussian type. But the corresponding fixed point is not a true Gaussian fixed point. It has nonzero value /29/

$$v^* = \frac{2\pi}{m}\epsilon\{1+\epsilon[\ln(\frac{\Lambda}{2\sqrt{\pi}}) + \frac{C_E}{2}]\} \tag{49}$$

(C_E is the Euler constant). The Gaussian type fixed point that is stable for $d<2$ is considered to infinite order in $\epsilon = 2-d$ in ref./29/.

Here I wish to note that the described situation in limit $T_c \to 0$ can be obtained not only with the WRG but also by the standard perturbation techniques. The limit $T_c \to 0$ is visualized in fig.6.

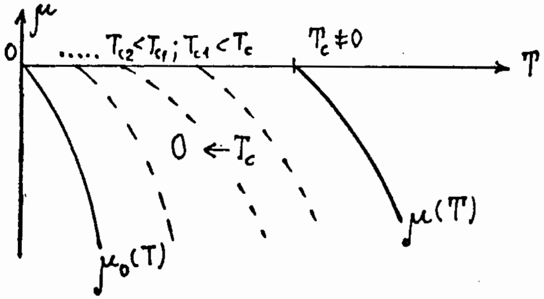

Fig.6

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Узунов Д.И. Ренормгрупповые рекуррентные соотношения для неидеального бозе-газа E17-82-21

Получены ренормгрупповые рекуррентные соотношения во втором порядке по ϵ ($\epsilon = d_c - d$, d - размерность пространства, d_c - критическая размерность) для неидеального бозе-газа. Наличие в модели мацубаровских частот дает возможность интерпретировать температуру T как существенный параметр теории. Рассматриваются два случая: 1/ $T_c \neq 0$ (T_c - температура перехода), в котором имеет место универсальность. 2/ $T_c \to 0$, в котором возможно точное суммирование ряда теории возмущений, что приводит к модифицированному, благодаря наличию взаимодействия, гауссовскому критическому поведению.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Uzunov D.I. Renormalization-Group Recursion Relations for the Nonideal Bose Gas E17-82-21

The renormalization-group recursion relations to second order in $\epsilon = d_c - d$ (where d is the dimension of space, and d_c is the borderline dimension) for the nonideal Bose gas in its functional formulation are presented. The presence of the Matsubara frequency allows us to understand the temperature T as an additional (to $(T-T_c)$, where T_c is the transition temperature) relevant parameter in the model. The two possible cases are considered: the nonzero ($T_c \neq 0$) and the zero-temperature ($T_c = 0$) critical behaviours. The last case where $d_c = 2 + 0$ ($\epsilon = 2 - d$), has two interesting properties: (i) in limit $T_c \to 0$ (as we understand the case $T_c = 0$) the infinite perturbation series for the nonideal Bose gas can be exactly summed owing to great simplifications and, (ii) the result from this infinite summation is a Gaussian critical behaviour with corrections to the scaling laws.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1982