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P-POLARIZED NONLINEAR SURFACE AND GUIDED WAVES IN LAYERED STRUCTURES

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1. INTRODUCTION

In the last years considerable interest has been roused for the theoretical and experimental study of surface polaritons [1-6]. The linear surface polaritons (LSP) are polaritons which propagate along the interface between two adjacent media, i.e., solutions of Maxwell's equations for which the amplitude of the electric and magnetic fields tends to zero in an exponential manner as one moves away from the interface into either medium.

As is well known the LSP are the admixture of the optical phonons and photons, and like the bulk polaritons show strong dispersion in the long-wavelength limit [7]. The LSP associated with optical phonons have been observed experimentally in layered structures by various methods, including surface reflection Raman scattering method [8-10] and attenuated total reflection method [6].

In recent years another class of normal modes of thin crystals, which are usually called linear guided wave polaritons (LGWP), has also been intensively investigated [11-14]. The observation of LGWP by Raman scattering experiments has been first reported in ref. [15]. The LGWP modes propagate with a real wave vector k parallel to the surface, and the normal component $k_\perp$ of the wave vector is imaginary outside the slab and real inside. Thus, the electromagnetic fields decay exponentially outside the slab but have the character of standing waves within the film.

Recently, much attention has been given to the theoretical investigation of nonlinear surface waves. In two seminal notes Tomlinson [16] and Agranovich et al. [17] have obtained an exact solution of Maxwell's equations which describe the propagation of s and p-polarized nonlinear surface waves, respectively, in the case when one of the two dielectric media in contact across a planar interface is optically unaxial and characterized by the diagonal dielectric tensor:

$$\epsilon_{11}(\omega, |\vec{E}|^2) = \epsilon_{22}(\omega, |\vec{E}|^2) = \epsilon_\perp(\omega) + a(\omega)(|E_1|^2 + |E_2|^2)\,, \qquad (1)$$

$$\epsilon_{33}(\omega, |\vec{E}|^2) = \epsilon_\parallel(\omega).$$

Maradudin[18] has studied in detail the dispersion relation of these s-polarized nonlinear surface polaritons (NSP) and has found that in the case $\alpha(\omega) < 0$ the electric field in the nonlinear medium is singular. Later on Lomtev[19] has generalized the results of Agranovich et al.[17] to the case of two optically unaxial nonlinear media characterized by dielectric tensors of the form (1).

It turned out also that the Maxwell equations, which describe the nonlinear surface waves in the system with the same geometry as in[17] but with a stronger nonlinearity of the form $\epsilon_{11} = \epsilon_{22} = \epsilon_\perp + \alpha(|E_1|^2 + |E_2|^2) + \beta(|E_1|^4 + |E_2|^4)$, are also exactly solvable[20]. In ref.[21] we discussed shortly the propagation characteristics of p-polarized NSP in thin dielectric films of thickness d characterized by the diagonal dielectric tensor (1).

It is the purpose of this paper to study in detail the propagation characteristics of p-polarized NSP and of p-polarized nonlinear guided wave polaritons (NGWP) in the following three-layer structures: i) dielectric medium (vacuum, for example) - optically linear medium (of thickness d) - optically uniaxial nonlinear crystal and ii) dielectric medium (vacuum, for example) - optically uniaxial nonlinear crystal (of thickness d) - optically linear medium. In this work we shall find the exact solutions of Maxwell's equations that describe the nonlinear surface waves in these layered systems.

The paper is organized as follows: In the next section we obtain the electromagnetic NSP and NGWP modes in the three-layer structure i) and calculate the power carried in the nonlinear surface waves. In Sect.3 we study the propagation characteristics of NSP and NGWP in the layered system ii) and we calculate the power carried by the NSP modes. Finally, in Sect.4 the results are briefly discussed.

2. NSP AND NGWP IN THE THREE-LAYER STRUCTURE: DIELECTRIC MEDIUM - DIELECTRIC FILM-OPTICALLY UNIAXIAL NONLINEAR CRYSTAL

We start by obtaining the electromagnetic modes of the three-layer structure consisting of a dielectric medium with isotropic dielectric constant $\epsilon_1(\omega)$ (vacuum, for example) in the region I $(-\infty < z \le 0)$, a dielectric film with isotropic, frequency - dependent dielectric constant $\epsilon_2(\omega)$ in the region II $(0 \le z \le d)$ and an anisotropic substrate (optically uniaxial nonlinear crystal) described by the diagonal dielectric tensor (1), in the region III $(z \ge d)$. We restrict ourselves to p-polarized NSP which are characterized by $E_2 = 0$, $H_1 = H_3 = 0$, i.e., the waves propagate in the x direction, with electric vector in the xz plane (TM waves).

a) The Case of NSP

The cartezian components of $\vec{E}$ and $\vec{H}$ are:

$$E_{1,3} = \mathcal{E}_{1,3}(z)\exp(-i\omega t + ikx), \qquad H_2 = \mathcal{H}_2(z)\exp(-i\omega t + ikx), \tag{2}$$

Maxwell's equations are

$$\frac{d\mathcal{E}_1}{dz} - ik\mathcal{E}_3 = i\frac{\omega}{c}\mathcal{H}_2, \qquad \frac{d\mathcal{H}_2}{dz} = i\frac{\omega}{c}D_1, \qquad k\mathcal{H}_2 = -\frac{\omega}{c}D_3. \tag{3}$$

From (3) it follows that

$$\frac{d^2\mathcal{E}_1^{I}}{dz^2} - k_1^2\mathcal{E}_1^{I} = 0, \quad k_1^2 = k^2 - \frac{\omega^2}{c^2}\epsilon_1,$$
$$\frac{d^2\mathcal{E}_1^{II}}{dz^2} - k_2^2\mathcal{E}_1^{II} = 0, \quad k_2^2 = k^2 - \frac{\omega^2}{c^2}\epsilon_2, \tag{4}$$
$$\frac{d^2\mathcal{E}_1^{III}}{dz^2} - \frac{k_3^2}{\epsilon_\parallel}[\epsilon_\perp + \alpha(\mathcal{E}_1^{III})^2]\mathcal{E}_1^{III} = 0, \quad k_3^2 = k^2 - \frac{\omega^2}{c^2}\epsilon_\parallel .$$

We look for solutions localized near the surfaces of the thin film with fields that fall to zero as $|z| \to +\infty$. The solutions of eqs. (4) in the case of $\alpha < 0$, $k_1^2 > 0$, $k_2^2 > 0$, $k_3^2 > 0$ are given by

$$\mathcal{E}_1^{I}(z) = Ae^{k_1 z}, \qquad -\infty < z \le 0,$$
$$\mathcal{E}_1^{II}(z) = B_1 e^{k_2 z} + B_2 e^{-k_2 z}, \qquad 0 \le z \le d, \tag{5}$$
$$\mathcal{E}_1^{III}(z) = \left(\frac{2\epsilon_\perp}{|\alpha|}\right)^{1/2}\{\cosh[k_3(\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2}(z - z_0)]\}^{-1}, \qquad z \ge d,$$

where A, B_1, B_2 and z_0 are to be determined from the boundary conditions imposed upon the system. Thus, the interface between the dielectric medium II and the nonlinear uniaxial crystal III can support an optical surface wave that propagates along the interface with a constant shape and amplitude.

For TM waves the boundary conditions are reduced to four equations, i.e., $\mathcal{E}_1$ and D_3 are continuous across the interfaces $z=0$ and $z=d$. Then, we have the following set of equations for the coefficients A, B_1, B_2 and the point z_0, where $\mathcal{E}_1^{III}(z)$ has a maximum $\mathcal{E}^{III}_{1,max} = (\frac{2\epsilon_\perp}{|\alpha|})^{1/2}$:

$$A = B_1 + B_2 , \quad \frac{\epsilon_1}{k_1} A = \frac{\epsilon_2}{k_2}(B_1 - B_2) ,$$
$$B_1 e^{k_1 d} + B_2 e^{-k_2 d} = u , \quad \frac{\epsilon_2}{k_2}(B_1 e^{k_2 d} - B_2 e^{-k_2 d}) = uv\frac{\epsilon_\parallel}{k_3} , \tag{6}$$

where

$$u = (\frac{2\epsilon_\perp}{|\alpha|})^{1/2} \{ \cosh[k_3(\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2} (z_0 - d)]\}^{-1} ,$$
$$v = (\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2} \tanh[k_3 (\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2} (z_0 - d)] . \tag{7}$$

The system (6) has a nontrivial solution if and only if

$$(\frac{\epsilon_1}{k_1} + \frac{\epsilon_2}{k_2})(\frac{\epsilon_2}{k_2} - \frac{\epsilon_\parallel}{k_3} v) + e^{-2k_2 d}(\frac{\epsilon_1}{k_1} - \frac{\epsilon_2}{k_2})(\frac{\epsilon_2}{k_2} + \frac{\epsilon_3}{k_3} v) = 0. \tag{8}$$

We remark that if $\epsilon_2 < 0$ and $\alpha \to 0$, then $z_0 \to -\infty$ and $v \to -(\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2}$. Then (8) is reduced to the dispersion relation for LSP obtained by Mills and Maradudin /7/:

$$(\frac{\epsilon_1}{k_1} + \frac{\epsilon_2}{k_2})[\frac{\epsilon_2}{k_2} + \frac{(\epsilon_\parallel \epsilon_\perp)^{1/2}}{k_3}] + e^{-2k_2 d}(\frac{\epsilon_1}{k_1} - \frac{\epsilon_2}{k_2})[\frac{\epsilon_2}{k_2} - \frac{(\epsilon_\parallel \epsilon_\perp)^{1/2}}{k_3}] = 0. \tag{9}$$

For $d \to \infty$ eq. (9) gives two independent equations $\frac{\epsilon_1}{k_1} + \frac{\epsilon_2}{k_2} = 0$ and $\frac{\epsilon_2}{k_2} + \frac{(\epsilon_\parallel \epsilon_\perp)^{1/2}}{k_3} = 0$, from which we determine the two surface modes $\omega_1 = \omega_1(k)$ and $\omega_2 = \omega_2(k)$. It has been shown in ref. /7/ that for finite d there are also only two surface modes which are related to the interfaces I-II and II-III, respectively. If we take $\mathcal{E}_1^I(0) = \mathcal{E}_0$ as an independent amplitude, then:

$$A = \mathcal{E}_0 , \quad B_1 = \frac{1}{2}\mathcal{E}_0 \frac{k_2}{\epsilon_2}(\frac{\epsilon_1}{k_1} + \frac{\epsilon_2}{k_2}), \quad B_2 = -\frac{1}{2}\mathcal{E}_0 \frac{k_2}{\epsilon_2}(\frac{\epsilon_1}{k_1} - \frac{\epsilon_2}{k_2}). \tag{10}$$

From (6) and (7) we have:

$$2\frac{\epsilon_2}{k_2}(\frac{2\epsilon_\perp}{|\alpha|})^{1/2} \{\cosh[k_3(\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2}(z_0 - d)]\}^{-1} = \tag{11}$$
$$= \mathcal{E}_0 [e^{k_2 d}(\frac{\epsilon_1}{k_1} + \frac{\epsilon_2}{k_2}) - e^{-k_2 d}(\frac{\epsilon_1}{k_1} - \frac{\epsilon_2}{k_2})].$$

The coupled equations (8) and (11) give the point z_0 and the dispersion relation of the two surface modes $\omega_1 = \omega_1(k, \mathcal{E}_0)$ and $\omega_2 = \omega_2(k, \mathcal{E}_0)$ which are related to the interfaces I-II and II-III, respectively. When $\epsilon_1 = \epsilon_2$ and $d \to 0$, we have from (8) and (11) the following equations obtained by Agranovich et al. /17/:

$$\frac{\epsilon_2}{k_2} = \frac{\epsilon_\parallel}{k_3}(\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2} \tanh[k_3 (\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2} z_0] ,$$
$$\mathcal{E}_0 = (\frac{2\epsilon_\perp}{|\alpha|})^{1/2} \{\cosh[k_3 (\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2} z_0]\}^{-1} . \tag{12}$$

It is also of interest to calculate the time averaged power carried in the nonlinear surface wave in the x direction per unit distance in the y direction:

$$P = -\frac{c}{4\pi}\int_{-\infty}^{\infty} E_3 H_2^* dz . \tag{13}$$

We find that $P = P_1 + P_2 + P_3$, where

$$P_1 = \frac{1}{8\pi}\mathcal{E}_0^2 k k_1 k_3^{-4} \omega \epsilon_\parallel ,$$

$$P_2 = \frac{1}{8\pi} k k_2 k_3^{-4} \omega \epsilon_\parallel [B_1^2(e^{2k_2 d} - 1) - 4B_1 B_2 k_2 d + B_2^2(1 - e^{-2k_2 d})], \tag{14}$$

$$P_3 = \frac{1}{6\pi} k \omega k_3^{-3} \epsilon_\parallel^{1/2} \epsilon_\perp^{3/2} (|\alpha|)^{-1} \{1 + \tanh^3[k_3(\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2}(z_0 - d)]\} .$$

Here P_1, P_2 and P_3 represent the powers carried in the media I, II and III, respectively.

In the case $\alpha(\omega) > 0$ the solution of Maxwell's equations in the region III is given by

$$\mathcal{E}_1^{III}(z) = (\frac{2\epsilon_\perp}{\alpha})^{1/2} \{\sinh[k_3(\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2} (z - z_0)]\}^{-1} . \tag{15}$$

The boundary conditions that $\mathcal{E}_1$ and D_3 must be continuous at $z=0$ and $z=d$ give four relations:

$$A = B_1 + B_2 , \quad \frac{\epsilon_1}{k_1} A = \frac{\epsilon_2}{k_2}(B_1 - B_2),$$

$$e^{k_2 d} B_1 + e^{-k_2 d} B_2 = u', \quad \frac{\epsilon_2}{k_2}(e^{k_2 d} B_1 - e^{-k_2 d} B_2) = u'v' \frac{\epsilon_\parallel}{k_3}, \tag{16}$$

where

$$u' = \left(\frac{2\epsilon_\perp}{\alpha}\right)^{1/2} \{\sinh[k_3 (\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2} (z_0 - d)]\}^{-1},$$

$$v' = -\left(\frac{\epsilon_\perp}{\epsilon_\parallel}\right)^{1/2} \{\tanh[k_3 (\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2} (z_0 - d)]\}^{-1}. \tag{17}$$

The system (16) has a nontrivial solution if and only if

$$\left(\frac{\epsilon_1}{k_1} + \frac{\epsilon_2}{k_2}\right)\left(\frac{\epsilon_2}{k_2} - \frac{\epsilon_\parallel}{k_3} v'\right) + e^{-2k_2 d}\left(\frac{\epsilon_1}{k_1} - \frac{\epsilon_2}{k_2}\right)\left(\frac{\epsilon_2}{k_2} + \frac{\epsilon_\parallel}{k_3} v'\right) = 0 . \tag{17'}$$

If $\epsilon_2 < 0$ and $\alpha \to 0$ then $z_0 \to +\infty$ and $v' \to -(\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2}$. Then (17) is reduced to the dispersion relation for LSP /7/.

In the case $\alpha(\omega) > 0$, we see that the electric field in the nonlinear crystal has a singularity at $z = z_0$. As has been pointed out by Maradudin /18/, the fact that the electric field is signular is as artifact of our use of a real, local dielectric tensor.

The singularity would be removed if we had a nonlinear crystal with intrinsic damping.

b) The Case of NGWP

The geometry of the system is the same as in the case a). We consider only the modes which have exponentially decreasing fields outside the slab and oscillatory solutions inside the film. If k_2 is imaginary the solutions inside the slab are periodic. Thus we take $k_2 = ik_\perp$ and have $k^2 + k_\perp^2 = \frac{\omega^2}{c^2}\epsilon_2(\omega)$, $k_1^2 = k^2 - \frac{\omega^2}{c^2}\epsilon_1(\omega)$, where k_1 and $k_\perp$ must be real. If region I is the vacuum $\epsilon_1(\omega) = 1$ and if region II is a crystal with $\epsilon_2(\omega) = \epsilon_\infty(\omega^2 - \omega_\ell^2)/(\omega^2 - \omega_t^2)$, NGWP exist only in the regions, where $ck/\sqrt{\epsilon_2(\omega)} < \omega < \omega_t$ and $\omega_\ell < \omega < ck$. Thus, we see that the frequencies of NSP and NGWP fall into nonoverlapping regions of the (ω, k)-plane.

The solutions of Maxwell's equation that describe the p-polarized NGWP in the three-layer system are:

$$\mathcal{E}_1^{I}(z) = \mathcal{E}_0 e^{k_1 z}, \quad -\infty < z \le 0,$$

$$\mathcal{E}_1^{II}(z) = B_1 \sin(k_\perp z) + B_2 \cos(k_\perp z), \quad 0 \le z \le d, \tag{18}$$

$$\mathcal{E}_1^{III}(z) = \left(\frac{2\epsilon_\perp}{|\alpha|}\right)^{1/2} \{\cosh[k_3 (\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2} (z - z_0)]\}^{-1}, \quad z \ge d,$$

where B_1, B_2 and z_0 are to be determined from the boundary conditions.

The requirement that $\mathcal{E}_1$ and D_3 be continuous at $z = 0$ and $z = d$ yields the equations

$$B_1 \sin(k_\perp d) + B_2 \cos(k_\perp d) = \left(\frac{2\epsilon_\perp}{|\alpha|}\right)^{1/2} \{\cosh[k_3 (\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2} (d - z_0)]\}^{-1}, \tag{19}$$

$$\frac{\epsilon_2}{k_\perp} \frac{[B_1 - B_2 \tan(k_\perp d)]}{[B_1 \tan(k_\perp d) + B_2]} = \frac{(\epsilon_\parallel \epsilon_\perp)^{1/2}}{k_3} \tanh[k_3 (\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2} (d - z_0)], \tag{20}$$

where

$$B_1 = -\frac{\epsilon_1}{\epsilon_2} \frac{k_\perp}{k_1} \mathcal{E}_0, \quad B_2 = \mathcal{E}_0 .$$

Eqs. (19) and (20) determine the point z_0, where the electric field in the nonlinear medium has a maximum and the dispersion of NGWP, $\omega_m = \omega_m(k, \mathcal{E}_0)$, labelled by mode number m.

If $\alpha \to 0$ then $z_0 \to -\infty$ and eq. (20) becomes

$$\frac{\epsilon_2}{k_\perp} \frac{[B_1 - B_2 \tan(k_\perp d)]}{[B_1 \tan(k_\perp d) + B_2]} = \frac{(\epsilon_\parallel \epsilon_\perp)^{1/2}}{k_3} . \tag{21}$$

Eq. (21) describes the dispersion of p-polarized LGWP /11-14/.

If $\epsilon_\parallel = \epsilon_\perp = \epsilon_3$ (an isotropic substrate) eq. (21) can be transformed into the form /14/:

$$\left(-k_\perp + \frac{k_1 k_3 \epsilon_2^2}{\epsilon_1 \epsilon_3}\right) \tan(k_\perp d) = -\epsilon_2 \left(\frac{k_1}{\epsilon_1} + \frac{k_3}{\epsilon_3}\right) k_\perp . \tag{22}$$

A graphic study of eq. (22) shows that the allowed values of $k_\perp$ are given by $k_\perp d = (m + \delta)\pi/2$, where m is odd integer and δ is a small positive number less than unity. Thus the propagation modes described by (22) are LGWP whose amplitude along the z-direction normal to the surface is oscillatory inside the film. The electromagnetic fields associated with the LGWP have standing wave character in the z-direction, but they propagate parallel to the surface and transport energy along the film as they proceed. Thus eqs. (19) and (20) describe the corresponding p-polarized NGWP modes in the three layer structure i).

The guided wave polaritons in the far infrared do not seem to be very familiar waves, but the waveguide modes in the visible have been studied in detail in the last years [22,23]. It was found that a thin layer of dielectric film which has a refractive index larger than that of the surroundings is a perfect optical waveguide. The waveguid modes (zigzag waves) are waves bounded in a film with refractive index n_2 (n_3 is the refractive index of the air, $n_3 < n_1 < n_2$). Thus eq.(22) becomes [22]:

$$\tan(k_\perp d) = k_\perp \frac{[(n_2/n_1)^2 k_1 + (n_2/n_3)^2 k_3]}{[k_\perp^2 - (n_2^2/n_1 n_3)^2 k_1 k_3]} . \qquad (23)$$

Different waveguide modes are indexed by the mode number m ($m = 0, 1, 2, \ldots$) and the eigenvalues of the waveguide modes range from $\frac{\omega}{c} n_1$ to $\frac{\omega}{c} n_2$ ($\frac{\omega}{c} n_1 < k < \frac{\omega}{c} n_2$).

3. NSL AND NGWP IN THE THREE-LAYER STRUCTURE: DIELECTRIC MEDIUM-OPTICALLY UNIAXIAL NONLINEAR FILM-LINEAR ISOTROPIC CRYSTAL

a) The Case of NSP

The geometry of our three-layer system is the following. Region I ($-\infty < z \leq 0$) is a dielectric medium with isotropic dielectric constant $\epsilon_1(\omega)$ (vacuum, for example), region II ($0 \leq z \leq d$) is a thin dielectric film (uniaxial crystal) described by the diagonal dielectric tensor (1) and region III ($z \geq d$) is an isotropic substrate with dielectric constant
From (3) it follows that:

$$\frac{d^2 \mathcal{E}_1^{I}}{dz^2} - k_1^2 \mathcal{E}_1^{I} = 0, \quad k_1^2 = k^2 - \frac{\omega^2}{c^2}\epsilon_1 , \qquad (24)$$

$$\frac{d^2 \mathcal{E}_1^{II}}{dz^2} - \frac{k_2^2}{\epsilon_\parallel}[\epsilon_\perp + \alpha(\mathcal{E}_1^{II})^2]\mathcal{E}_1^{II} = 0, \quad k_2^2 = k^2 - \frac{\omega^2}{c^2}\epsilon_\parallel , \qquad (25)$$

$$\frac{d^2 \mathcal{E}_1^{III}}{dz^2} - k_3^2 \mathcal{E}_1^{III} = 0, \quad k_3^2 = k^2 - \frac{\omega^2}{c^2}\epsilon_3 . \qquad (26)$$

We seek solutions of Maxwell's equations which exponentially decrease outside the slab. Eqs. (24) and (26) can be easily integrated to obtain $\mathcal{E}_1^{I}(z) = \mathcal{E}_0 e^{k_1 z}$, $z < 0$ and $\mathcal{E}_1^{III}(z) = A e^{-k_3 z}$, $z > d$.

In the following we put $\mathcal{E}_1^{II}(z) = y(z)$. Then we integrate (25) to obtain:

$$y'^2 - \frac{k_2^2}{\epsilon_\parallel}(\epsilon_\perp y^2 + \frac{\alpha}{2} y^4) = c_0 . \qquad (27)$$

The constant c_0 is obtained from the continuity conditions of $\mathcal{E}_1(z)$ and $D_3(z)$ at $z = 0$:

$$c_0 = \mathcal{E}_0^2 [(\frac{\epsilon_1}{\epsilon_\parallel} \frac{k_2^2}{k_1^2})^2 - b\mathcal{E}_0^2 - a] , \qquad (28)$$

where $b = \frac{\alpha}{2}\frac{k_2^2}{\epsilon_\parallel}$ and $a = \frac{\epsilon_\perp}{\epsilon_\parallel} k_2^2$.

We remark that the NSP exist only in the regions of the (ω, k)-plane, where $k_1^2 > 0$, $k_2^2 > 0$ and $k_3^2 > 0$.

On further integrating (27), we have:

$$\int (c_0 + ay^2 + by^4)^{-1/2} dy = z - z_0 . \qquad (29)$$

There are now four cases to consider:

a_1) The case $a > 0$, $b = -|b| < 0$, $c_0 > 0$ ($\alpha < 0$).
Let $y = 1/v$, then eq.(28) can be reduced to the form

$$\int^{u} (c_0 v^4 + av^2 - |b|)^{-1/2} dv = z - z_0 . \qquad (30)$$

The solution of (30) can be expressed in terms of Jacobi elliptic functions [24]. Next we have:

$$\int^{u} (c_0 v^4 + av^2 - |b|)^{-1/2} dv = (a^2 + 4|b| c_0)^{-1/4} \, nc^{-1}\{[\frac{2|b|}{a + (a^2 + 4|b| c_0)^{1/2}}]^{-1/2} u/m\},$$

$$\text{where } m = \frac{[a + (a^2 + 4|b| c_0)^{1/2}]}{2(a^2 + 4|b| c_0)^{1/2}} . \qquad (31)$$

Here

$$u = \int_0^{\phi} (1 - m \sin^2\theta)^{-1/2} d\theta, \quad cn(u/m) = \cos\phi,$$

$$sn(u/m) = \sin\phi, \quad dn(u/m) = (1 - m\sin^2\phi)^{1/2} ,$$

$$nc(u/m) = (cn\, u)^{-1} , \quad nd(u/m) = (dn\, u)^{-1}$$

are the Jacobi elliptic functions [24].

Finally the solution of (25) is given by

$$\mathcal{E}_1^{II}(z) = \delta\, \mathrm{cn}[\gamma(z - z_0)/m],$$
$$\delta = \left[\frac{a + (a^2 + 4|b|c_0)^{1/2}}{2|b|}\right]^{1/2}, \quad \gamma = (a^2 + 4|b|c_0)^{1/4}. \tag{32}$$

The function $\mathrm{cn}(u/m)$ has the period $4K(m)$, where $K(m) = \int_0^{\pi/2} (1 - m\sin^2\theta)^{-1/2}\, d\theta$ is the complete elliptic integral of the first kind. From (32) we see that $[\mathcal{E}_1^{II}(z)]^2$ has the maxima at the points $z_n = z_0 + n\ell_1$, $n = 0,1,2,\ldots$, where the distance between two maxima is $\ell_1 = 2K(m)/\gamma$ and $\mathcal{E}^{II}_{1,max} = \delta$.

Thus, the electric field in the nonlinear slab is an oscillatory function of the variable z. The finite size of the nonlinear uniaxial crystal gives rise to standing rather than to travelling waves in the z direction.

We remark that if $c_0 = 0$, then $m = 1$, $\mathrm{cn}(u/1) = (\cosh u)^{-1}$. In this particular case we have a nonperiodic solution of the form:

$$\mathcal{E}_1^{II}(z) = \left(\frac{a}{|b|}\right)^{1/2} \{\cosh[a(z - z_0)]\}^{-1}. \tag{32a}$$

From the boundary conditions at $z = 0$ and $z = d$, we get the following equations for z_0 and the dispersion of p-polarized NSP modes $\omega_m = \omega_m(k, \mathcal{E}_0)$, labelled by a positive integer m:

$$\mathcal{E}_0 = \delta\, \mathrm{cn}(\gamma z_0), \quad A = \delta e^{k_3 d}\, \mathrm{cn}[\gamma(d - z_0)],$$
$$\frac{\epsilon_1}{\epsilon_\parallel}\frac{k_2^2}{k_1} = \gamma\, \mathrm{sn}(\gamma z_0)\, \mathrm{dn}(\gamma z_0)\, \mathrm{nc}(\gamma z_0), \tag{33}$$
$$\frac{\epsilon_3}{\epsilon_\parallel}\frac{k_2^2}{k_3} = \gamma\, \mathrm{sn}[\gamma(d - z_0)]\, \mathrm{dn}[\gamma(d - z_0)]\, \mathrm{nc}[\gamma(d - z_0)].$$

The time averaged power carried in the nonlinear surface wave is $P = P_1 + P_2 + P_3$, where

$$P_1 = \frac{1}{8\pi}\mathcal{E}_0^2\, k k_1 k_2^{-4} \omega\epsilon_\parallel, \quad P_2 = \frac{1}{4\pi}\omega\epsilon_\parallel k k_2^{-4} \delta^2 \gamma(q + r + s),$$
$$P_3 = \frac{1}{4\pi} A^2 k k_3 k_2^{-4} \omega\epsilon_\parallel (1 - e^{-2k_3 d}), \tag{34}$$
$$q = \frac{(1-m)}{3m}\gamma d, \quad r = \frac{(2m-1)}{3m}\{E[\gamma(d - z_0)/m] + E(\gamma z_0/m)\},$$
$$s = -\frac{1}{3}\{\mathrm{sn}[\gamma(d - z_0)]\,\mathrm{cn}[\gamma(d - z_0)]\,\mathrm{dn}[\gamma(d - z_0)] + \mathrm{sn}(\gamma z_0)\mathrm{cn}(\gamma z_0)\,\mathrm{dn}(\gamma z_0)\}$$

and $E(u/m) = \int_0^u (1 - t^2)^{-1/2}(1 - mt^2)^{1/2} dt$ is the elliptic integral of the second kind

Here P_1, P_2, P_3 are the powers in the media I, II and III, respectively.

In obtaining this expression for the average power we used the following integrals:

$$\int_0^u \mathrm{sn}^2(t/m)\, dt = \frac{1}{m}[u - E(u/m)],$$

$$\int_0^u \mathrm{sn}^4(t/m)\, dt = \frac{2(m+1)}{3m^2}[u - E(u/m)] + \frac{1}{3m}\mathrm{sn}\,u\,\mathrm{cn}\,u\,\mathrm{dn}\,u - \frac{1}{3m}u.$$

a2) The case $a > 0$, $b = -|b| < 0$, $c_0 < 0$ $(\alpha < 0)$.

Taking into account

$$\int_0^u (-|c_0|v^4 + av^2 - |b|)^{-1/2} dv = \left[\frac{2}{a + (a^2 - 4|b||c_0|)^{1/2}}\right]^{1/2} \mathrm{nd}^{-1} \times$$
$$\times \left\{\left[\frac{a + (a^2 - 4|b||c_0|)^{1/2}}{2|b|}\right]^{1/2} u/p\right\}, \tag{35}$$

where $p = \dfrac{2(a^2 - 4|b||c_0|)^{1/2}}{[a + (a^2 - 4|b||c_0|)^{1/2}]}$ we finally have

$$\mathcal{E}_1^{II}(z) = \rho\, \mathrm{dn}[\beta(z - z_0)/p], \tag{36}$$

where

$$\rho = \left[\frac{a + (a^2 - 4|b||c_0|)^{1/2}}{2|b|}\right]^{1/2}, \quad \beta = \left[\frac{a + (a^2 - 4|b||c_0|)^{1/2}}{2}\right]^{1/2}.$$

As is well known[24] the Jacobi elliptic function $\mathrm{dn}(u/p)$ has the period $2K(p)$.

Thus, $[\mathcal{E}_1^{II}(z)]^2$ is periodic in the variable z and has maxima at the points $z_n = z_0 + n\ell_2$, $n = 0,1,2,\ldots$, where $\ell_2 = 2K(p)/\beta$ is the distance between two consecutive maxima and $\mathcal{E}^{II}_{1,max} = \rho$.

The boundary conditions reduce to four equations, i.e., $\mathcal{E}_1$ and D_3 are continuous across the interfaces $z = 0$ and $z = d$:

$$\mathcal{E}_0 = \rho\, \mathrm{dn}(\beta z_0), \quad A = \delta e^{k_3 d}\, \mathrm{dn}[\beta(d - z_0)],$$
$$\frac{\epsilon_1}{\epsilon_\parallel}\frac{k_2^2}{k_1} = p\beta\, \mathrm{sn}(\beta z_0)\, \mathrm{cn}(\beta z_0)\mathrm{nd}(\beta z_0), \tag{37}$$
$$\frac{\epsilon_3}{\epsilon_\parallel}\frac{k_2^2}{k_3} = p\beta\, \mathrm{sn}[\beta(d - z_0)]\, \mathrm{cn}[\beta(d - z_0)]\, \mathrm{nd}[\beta(d - z_0)].$$

From (37) we obtain z_0 and the dispersion relation of the surface modes $\omega_m = \omega_m(k, \mathcal{E}_0)$. If $c_0 = 1$ then $p=1$ and $dn(u/1) = (\cosh u)^{-1}$. Thus, we get the nonperiodic solution (32a).

a3) The case $a>0$, $b>0$, $c_0>0$ $(\alpha>0)$.

The solution of eq. (25) is:

$$\mathcal{E}_1^{II}(z) = \delta' cs[\gamma'(z - z_0)/m']$$

$$cs(u/m') = cn(u/m')/sn(u/m'),$$

where

$$m' = \frac{2(a^2 - 4bc_0)^{1/2}}{[a+(a^2-4bc_0)^{1/2}]}, \quad \delta' = \left[\frac{2c_0}{a-(a^2-4bc_0)^{1/2}}\right]^{1/2}, \qquad (38)$$

$$\gamma' = \left[\frac{a + (a^2 - 4bc_0)^{1/2}}{2}\right]^{1/2}.$$

In this case we see that the electric field in the nonlinear film has a singularity at $z=z_0$ which would be removed if, e.g., damping were introduced into the diagonal dielectric tensor.

The equations for z_0 and for the normal modes $\omega_m=\omega_m(k, \mathcal{E}_0)$ are:

$$\mathcal{E}_0 = \delta' cs(\gamma' z_0), \quad A = \delta' e^{k_3 d} cs[\gamma'(d - z_0)],$$

$$\frac{\epsilon_1}{\epsilon_\parallel}\frac{k_2^2}{k_1} = \gamma' ns(\gamma' z_0) ds(\gamma' z_0) sc(\gamma' z_0), \qquad (39)$$

$$\frac{\epsilon_3}{\epsilon_\parallel}\frac{k_2^2}{k_3} = \gamma' ns[\gamma'(d - z_0)] ds[\gamma'(d - z_0)] sc[\gamma'(d - z_0)].$$

a4) The case $a>0$, $b>0$, $c_0=-|c_0|<0$ $(\alpha>0)$.

In this case we get:

$$\mathcal{E}_1^{II}(z) = \rho' ds[\beta'(z - z_0)/p'] \qquad (40)$$

$$ds(u/p') = dn(u/p')/sn(u/p'),$$

where

$$p' = \frac{[a + (a^2 + 4b|c_0|)^{1/2}]}{2(a^2 + 4b|c_0|)^{1/2}}, \quad \rho' = \frac{(a^2 + 4b|c_0|)^{1/4}}{b^{1/2}},$$

$$\beta' = (a^2 + 4b|c_0|)^{1/2}.$$

The point z_0, where the electric field has a singularity, and the branches $\omega_m = \omega_m(k, \mathcal{E}_0)$ of the dispersion relation are determined by:

$$\mathcal{E}_0 = \rho' ds(\beta' z_0), \quad A = \rho' e^{k_3 d} ds[\beta'(d - z_0)],$$

$$\frac{\epsilon_1}{\epsilon_\parallel}\frac{k_2^2}{k_1} = \beta' cs(\beta' z_0) ns(\beta' z_0) sd(\beta' z_0), \qquad (41)$$

$$\frac{\epsilon_3}{\epsilon_\parallel}\frac{k_2^2}{k_3} = \beta' cs[\beta'(d-z_0)] ns[\beta'(d-z_0)] sd[\beta'(d-z_0)].$$

b) The Case of NGWP

The NGWP modes propagate with a real wave vector k parallel to the surface and k_2 is imaginary, $k_2=ik_\perp$, where $k_\perp$ is the component of the wave vector perpendicular to the surface. Therefore $a = -\frac{\epsilon_\perp}{\epsilon_\parallel}k_\perp < 0$, and in the following there are two cases to consider.

b_1) The case $a=-|a|<0$, $b=-|b|<0$, $c_0>0$.

The solution of eq. (25) is

$$\mathcal{E}_1^{II}(z) = \tilde{\delta}\, cn[\tilde{\gamma}(z - z_0)/\tilde{m}],$$

where

$$\tilde{m} = \frac{[(|a|^2 + 4|b|c_0)^{1/2} - |a|]}{2(|a|^2 + 4|b|c_0)^{1/2}}, \qquad (42)$$

$$\tilde{\delta} = \left[\frac{(|a|^2 + 4|b|c_0)^{1/2} - |a|}{2|b|}\right]^{1/2}, \quad \tilde{\gamma} = (|a|^2 + 4|b|c_0)^{1/4}.$$

The equations for z_0 and the dispersion of NGWP modes $\omega_m = \omega_m(k, \mathcal{E}_0)$ labeled by a positive integer m are:

$$\mathcal{E}_0 = \tilde{\delta}\, cn(\tilde{\gamma} z_0), \quad A = \tilde{\delta}\, e^{k_3 d}\, cn[\tilde{\gamma}(d - z_0)],$$

$$-\frac{\epsilon_1}{\epsilon_\parallel}\frac{k_\perp^2}{k_1} = \tilde{\gamma}\, sn(\tilde{\gamma} z_0) dn(\tilde{\gamma} z_0) nc(\tilde{\gamma} z_0), \qquad (43)$$

$$-\frac{\epsilon_3}{\epsilon_\parallel}\frac{k_\perp^2}{k_3} = \tilde{\gamma}\, sn[\tilde{\gamma}(d - z_0)] dn[\tilde{\gamma}(d - z_0)] nc[\tilde{\gamma}(d - z_0)].$$

If $a=0$, then $b=0$ and $\tilde{m}=0$, $\tilde{\delta}=(\frac{c_0}{|a|})^{1/2}$, $\tilde{\gamma}=|a|$. Because $\mathrm{cn}(u/0)=\cos u$, the solution (42) becomes:

$$\mathcal{E}_1^{II}(z)=\tilde{B}_1\sin[(\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2}k_\perp z]+\tilde{B}_2\cos[(\frac{\epsilon_\perp}{\epsilon_\parallel})^{1/2}k_\perp z]\,. \qquad (44)$$

Thus, eq. (44) described the LGWP, i.e., the guided wave polaritons in linear theory. Eq. (42) describes the nonlinear analogue of what are usually called guided wave polaritons /11-15/.

The power carried by the NGWP modes is $P=P_1+P_2+P_3$,

where

$$P_1=\frac{1}{8\pi}\mathcal{E}_0^2 kk_1k_\perp^{-4}\omega\epsilon_\parallel\,,\quad P_2=\frac{1}{4\pi}\omega\epsilon_\parallel kk_\perp^{-4}\tilde{\delta}^2\tilde{\gamma}(\tilde{q}+\tilde{r}+\tilde{s})\,,$$

$$P_3=\frac{1}{4\pi}A^2kk_3k_\perp^{-4}\omega\epsilon_\parallel(1-e^{-2k_3d})\,,$$

are the powers in the media I,II and III, respectively. Here

$$\tilde{q}=\frac{(1-\tilde{m})}{3\tilde{m}}\tilde{\gamma}d,\quad \tilde{r}=\frac{(2\tilde{m}-1)}{3\tilde{m}}\{E[\tilde{\gamma}(d-z_0)/\tilde{m}]+E(\tilde{\gamma}z_0/\tilde{m})\}\,,$$

$$\tilde{s}=-\frac{1}{3}\{\mathrm{sn}[\tilde{\gamma}(d-z_0)]\,\mathrm{cn}[\tilde{\gamma}(d-z_0)]\,\mathrm{dn}[\tilde{\gamma}(d-z_0)]\;+$$

$$+\;\mathrm{sn}(\tilde{\gamma}z_0)\,\mathrm{cn}(\tilde{\gamma}z_0)\,\mathrm{dn}(\tilde{\gamma}z_0)\}\,.$$

b_2) The case $a=-|a|<0$, $b>0$, $c_0=-|c_0|<0$.

We find that:

$$\mathcal{E}_1^{II}(z)=\tilde{\rho}'\,\mathrm{ds}[\tilde{\beta}'(z-z_0)/\tilde{p}'],$$

where

$$\tilde{p}'=\frac{[(|a|^2+4b|c_0|)^{1/2}-|a|]}{2(|a|^2+4b|c_0|)^{1/2}},$$

$$\tilde{\rho}'=\frac{(|a|^2+4b|c_0|)^{1/4}}{b^{1/2}},\quad \tilde{\beta}'=(|a|^2+4b|c_0|)^{1/2}\,. \qquad (45)$$

The boundary conditions give the following equations for z_0 and the normal modes $\omega_m=\omega_m(k,\mathcal{E}_0)$

$$\mathcal{E}_0=\tilde{\rho}'\,\mathrm{ds}(\tilde{\beta}'z_0),\quad \Lambda=\tilde{\rho}'e^{k_3d}\;\mathrm{ds}[\tilde{\beta}'(d-z_0)]\;-$$

$$-\frac{\epsilon_1}{\epsilon_\parallel}\frac{k_\perp^2}{k_1}=\tilde{\beta}'\,\mathrm{cs}(\tilde{\beta}'z_0)\,\mathrm{ns}(\tilde{\beta}'z_0)\,\mathrm{sd}(\tilde{\beta}'z_0)\;- \qquad (46)$$

$$-\frac{\epsilon_3}{\epsilon_\parallel}\frac{k_\perp^2}{k_3}=\tilde{\beta}'\,\mathrm{cs}[\tilde{\beta}'(d-z_0)]\,\mathrm{ns}[\tilde{\beta}'(d-z_0)]\,\mathrm{sd}[\tilde{\beta}'(d-z_0)]\,.$$

4. CONCLUSIONS

We conclude with a few comments about the results we have obtained in this paper. In Sect.2 we have studied the propagation characteristics of the nonlinear analogue of what are usually called surface polaritons /7-10/ and guided wave polaritons /11-15/ in linear theory in the case of an isotropic linear slab placed on an anisotropic nonlinear substrate.

In Sect.3 we have written down the exact oscillatory solutions of Maxwell's equations that describe the propagation of NSP and of NGWP in a nonlinear film placed on a linear substrate. These solutions are expressed in terms of Jacobi elliptic functions and are periodic in the space variable z normal to the surfaces.

The surprising result we have obtained is the oscillatory behaviour of the solutions within the slab even in the case of NSP. This is closely related to the fact that the finite size of the nonlinear crystal gives rise to standing waves in the z direction perpendicular to the surfaces. The physical distinction between NSP and NGWP oscillatory solutions is that the frequencies of propagations of NSP and NGWP fall into non-overlapping domains of the (ω,k)-space.

In some regions of the (ω,k)-plane we have obtained singular electric fields in the nonlinear media. Such a behaviour of the electromagnetic fields is an artifact of the use of a real dielectric tensor. The fact that the nonlinear surface modes $\omega_m=\omega_m(k,\mathcal{E}_0)$ are functions of the amplitude $\mathcal{E}_0$ of the electric field is a common characteristics of nonlinear phenomena.

The case of s-polarized NSP and NGWP in the same three-layer structures will be analysed in a future work.

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p-поляризованные нелинейные поверхностные и связанные волны в слоистых структурах

Найдены точные решения уравнений Максвелла, которые отвечают p-поляризованным нелинейным поверхностным поляритонам и p-поляризованным нелинейным связанным волнам поляритонов в слоистых структурах для двух случаев: 1/ пленка поверхностно-активного вещества, помещенная на подложку, диэлектрические свойства которой описываются тензором $\epsilon_{11} = \epsilon_{22} = \epsilon_{\perp} + \alpha(|E_1|^2 + |E_2|^2)$, $\epsilon_{33} = \epsilon_{\parallel}$; 2/ пленка, диэлектрические свойства которой описываются тензором такого же вида /оптически одноосный нелинейный кристалл/, помещенная на оптически однородную подложку с диэлектрической проницаемостью ϵ_3. Получены также аналитические формулы для потока энергии переносимого поверхностными волнами.

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p-Polarized Nonlinear Surface and Guided Waves in Layered Structures

We found an exact solution of Maxwell's equations, which describes the propagation of p-polarized nonlinear surface polaritons and of p-polarized nonlinear guided wave polaritons in two cases: i) in a film of a surface active material placed on a substrate described by a diagonal dielectric tensor whose elements depend on the amplitude of the electric field according to $\epsilon_{11} = \epsilon_{22} = \epsilon_{\perp} + \alpha(|E_1|^2 + |E_2|^2)$, $\epsilon_{33} = \epsilon_{\parallel}$, and ii) in a film, described by the same dielectric tensor (optically uniaxial nonlinear crystal), placed on a substrate with dielectric constant ϵ_3 (optically linear medium). The power carried in the surface waves has also been exactly calculated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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