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SOLITONS AND SPIN COMPLEXES IN THE ANISOTROPIC HEISENBERG CHAIN

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In the paper the results obtained via quasiclassical quantization of the two-parameter soliton solutions of the Landau-Lifshitz equation (LLE) are compared with the exact quantum-mechanical results for spin complexes in the anisotropic Heisenberg chain. For the first time such a comparison has been made in the case of isotropic interaction by Kosevitch et al.[1], who showed that the energy obtained by quasiclassical quantization of the one-soliton solution of LLE is equal to that obtained by Bethe[2], for all number n of spin deviations. Subsequently the same result has been obtained in refs.[3].

It has been conjectured (see ref.[4]) that for all completely integrable systems there is full agreement between the quasiclassical and quantum energies. We shall verify this conjecture in the case of the anisotropic Heisenberg chain (which is known to be completely integrable[5]). Moreover, the presence of the anisotropy will enable us to extend the comparison to other features of the quantum and classical solutions.

The Hamiltonian of a $s=1/2$ ferromagnetic chain with longitudinal exchange anisotropy has the form $\mathcal{H}=-J\sum_\ell[\frac{1}{g}(S_\ell^x S_{\ell+1}^x+S_\ell^y S_{\ell+1}^y)+S_\ell^z S_{\ell+1}^z]$, $J\geq 0$, $g\geq 1$. The n-magnon bound state energy has been found in refs.[6] to be:

$$E_n(k)=J\,\mathrm{th}\sigma\,(\mathrm{ch}\,n\sigma-\cos ka)/\mathrm{sh}\,n\sigma, \tag{1}$$

where $\sigma=\ln(g+\sqrt{g^2-1})$, k is the center-of-mass quasimomentum, and a is the lattice constant. Let us remark two facts following from Eq. (1) (see author's paper in refs.[6]) and having, as we shall see, quasiclassical conterparts:

i) The energy of heavy complexes is nonzero and independent of n and k ($E_n\to J\,\mathrm{th}\sigma$ for $n\gg 1$). This property of the quantum solution is simply related to the existence of an immobile domain wall in the classical model.

ii) For $k=0$ $E_n>E_{n-1}$, while at the zone-boundary ($k=\pi$) $E_n<E_{n-1}$. The intersection of the curves $E_n(k)$ with different n is naturally related to the presence of magnetic soliton without internal pression.

Let us stress that the quantum and classical solutions have properties i), ii) only in the presence of anisotropy.

The quasiclassical consideration of magnetic systems on the basis of LLE [1,7,8]: $\frac{\partial \vec{M}}{\partial t} = \frac{2\mu}{h}[\vec{M} \times \frac{\delta W}{\delta \vec{M}}]$ ($\vec{M}$ is the magnetic momentum density; $W\{\vec{M}\}$ - the energy functional; μ_0, the Bohr magneton) supposes the long-wave approximation. One can convince oneself (this also follows from ref. [7], where the quantum and quasiclassical results are compared for the anisotropic chain in the case of immobile solitons and large n), that the long-wave approximation on our case leads to the condition $\eta = \sqrt{(g-1)/2} \ll 1$. Thereby the energy functional takes the form (compare to ref. [7]):

$$W\{M\} = \frac{1}{2} \int d\xi \, [\alpha (\frac{\partial \vec{M}}{\partial \xi})^2 - \beta M_z^2] \tag{2}$$

where $\alpha = \frac{J a^2}{4\mu_0 M_0}$, $\beta = \frac{J\eta^2}{\mu_0 M_0}$, M_0 is the saturation magnetic density. The constants α and β are chosen such that the frequency of the classical spin wave be equal to the energy of the quantum magnon for $ka \ll 1$. For the explicit form of LLE we refer the reader to ref. [1], where they are written down, together with their one-soliton solutions, in the case of one-ion anisotropy (where W has the same form as (2) but with different α and β). In refs. [1,8] the quasiclassical quantization is also performed, and the following expression for the energy is obtained:

$$E_n(p) = h\omega_0 N_1 \left(\mathrm{th}\frac{n}{n_1} + \frac{2\sin^2\frac{\pi p}{p_0}}{\mathrm{sh}\frac{2n}{n_1}} \right). \tag{3}$$

The parameters ω and V of the soliton solutions of LLE are related to the parameters n and p in eq. (3) through: $h\omega = \frac{\partial E}{\partial n}$, $v = \frac{\partial E}{\partial p}$ (see ref. [1]). In our case $h\omega_0 = 2J\eta^2$, $n_1 = \frac{1}{\eta}$, $\frac{\pi p}{p_0} = ka$. Inserting these values in eq. (3), we obtain $E_n(k) = 2J\eta(\mathrm{ch}\,2n\eta - \cos ka)/\mathrm{sh}\,2n\eta$ which coincides with the asymptotic at $\eta \ll 1$ of the exact quantum result (1) for arbitrary n and k. Thus, at weak anisotropy the quasiclassical energy is exact for all n. For immobile solitons ($k = 0$) and large n the coincidence of the quasiclassical and quantum results had already been shown in ref. [7].

The energy of the domain wall (i.e., of the solution with $v=0$, $\omega=0$) can be written in our case following ref. [1] as $E = J\eta$. Comparing this with the asymptotics of E_n when $n \gg 1$, $\eta \ll 1$ (see i) above), we see that the energy of heavy complexes equals twice the energy of a domain wall. The immobility of the classical wall reflects the fact that in the quantum picture the group velocity of heavy complexes

$v = \frac{\partial E}{\partial p} = 0$. In the model (2) there exist, as first shown in ref. [9], solutions with $\omega=0$, $v \neq 0$ (magnetic solitons without precession). The existence condition for such solutions can be written in our case in terms k and n as (compare ref. [1]): $\mathrm{ch}\, 2n\eta \cos ka = 1$. This condition is identical to the condition $E_{n+1}(k) = E_n(k)$ with $E_n(k)$ taken from (1) and $\eta \ll 1$. It is somewhat strange that this holds not only asymptotically as expected from $\hbar\omega = \frac{\partial E}{\partial n}$ but for all n. Thus the existence of solitons without precession in model (2) is related in the quantum picture to the intersection of the curves $E_n(k)$ with different n. As one can see, the correspondence between the quasiclassical and quantum approaches is more comprehensive in the presence of anisotropy and supports the bound state interpretation of the solitons [1,3,8].

Equality between the quasiclassical and exact quantum energies for all n has been previously obtained within other models [10], in spite of the fact that the quasiclassical approximation is justified a priori only at $n \gg 1$. In our openion, the reason of full agreement for completely integrable systems for small n (including $n=1$) is thus far not completely understood. Neither are reasons of agreement in the range of large quasimomenta k in magnetic systems, where the long-wave approximation is expected to break down [1]. It is beyond doubt that the reason is to be looked for among the features of the quantum system solvable by Bethe's ansatz.

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