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STRONGLY NONLINEAR SURFACE POLARITONS

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1. An exact solution of the Maxwell equations corresponding to the surface nonlinear H-wave, propagating along the x axis has been found in [1] under the assumption that one of the contiguous dielectric media is optically unaxial and is specified by the diagonal dielectric tensor

$$\epsilon_{11}(\omega) = \epsilon_{22}(\omega) = \epsilon_{\perp}(\omega) + \alpha\,(|E_1|^2 + |E_2|^2), \quad \epsilon_{33}(\omega) = \epsilon_{\parallel}(\omega)\,.$$

It was also mentioned that it is expedient to study a stronger nonlinearity of $\epsilon_{ij}(\omega, |E|^2)$ to elucidate a qualitative and quantitative behaviour of nonlinear surface polaritons (NSP). In this paper we find an exact solution of the Maxwell equation corresponding to the surface nonlinear H-wave for a more general dependence of $\epsilon_{11}(\omega, |E|^2)$ on the amplitude E_1 and discuss the modifications of the properties of NSP.

As in ref. [1] we consider medium I, corresponding to $z < 0$, to be isotropic and linear with the dielectric constant $\epsilon_1(\omega)$ and optically uniaxial medium II to fill the halfspace $z > 0$ and

$$\epsilon_{11}(\omega, |E|^2) = \epsilon_{22}(\omega, |E|^2) = \epsilon_{\perp}(\omega) + \alpha\,(|E_1|^2 + |E_2|^2) + \beta(|E_1|^4 + |E_2|^4),$$
$$\epsilon_{33}(\omega) = \epsilon_{\parallel}(\omega).$$

The dependence of the electric and magnetic fields on the coordinates and time is:

$$\begin{aligned} E_1(x,z,t) &= \mathcal{E}_1(z)\, e^{-i\omega t + ikx}, \\ E_3(x,z,t) &= \mathcal{E}_3(z)\, e^{-i\omega t + ikx}, \\ H_2(x,z,t) &= \mathcal{H}_2(z)\, e^{-i\omega t + ikx}. \end{aligned} \tag{1}$$

The field amplitudes in (1) satisfy the Maxwell equations:

$$\begin{aligned} \frac{d\mathcal{E}_1}{dz} - ik\mathcal{E}_3 &= i\frac{\omega}{c}\mathcal{H}_2\,, \\ \frac{d\mathcal{H}_2}{dz} &= i\frac{\omega}{c}\mathcal{D}_1\,, \\ k\mathcal{H}_2 &= -\frac{\omega}{c}\mathcal{D}_3\,, \end{aligned} \tag{2}$$

$$\mathfrak{D}^{I}_{1} = \epsilon_1 \mathcal{E}^{I}_{1}, \quad \mathfrak{D}^{I}_{3} = \epsilon_1 \mathcal{E}^{I}_{3},$$

$$\mathfrak{D}^{II}_{1} = \epsilon_{11} \mathcal{E}^{II}_{1}, \quad \mathfrak{D}^{II}_{3} = \epsilon_{\parallel} \mathcal{E}^{II}_{3}.$$

Excluding $\mathcal{H}_2(z)$ and $\mathcal{E}_3(z)$, from the system of equations (2) we get the following equations for $\mathcal{E}^{I}_{1}(z)$ and $\mathcal{E}^{II}_{1}(z)$

$$\frac{d^2 \mathcal{E}^{I}_{1}}{dz^2} - k_1^2 \mathcal{E}^{I}_{1} = 0,$$
$$\frac{d^2 \mathcal{E}^{II}_{1}}{dz^2} - \frac{k_2^2}{\epsilon_\parallel}[\epsilon_\perp + \alpha(\mathcal{E}^{II}_{1})^2 + \beta(\mathcal{E}^{II}_{1})^4]\mathcal{E}^{II}_{1} = 0, \tag{3}$$

where $k_1^2 = k^2 - \frac{\omega^2}{c^2}\epsilon_1$, $k_2^2 = k^2 - \frac{\omega^2}{c^2}\epsilon_\parallel$.

Note that for the case when $\epsilon_{11}(\omega, \mathcal{E}_1)$ depends on $|E_1|^{2n}$, $n > 2$ the procedure is the same, i.e., we arrive at equations of the form (3) with the corresponding function $\epsilon_{11}(\omega, \mathcal{E}_1)$.

If for $z < 0$, $\mathcal{E}^{I}_{1}(z)$, as in ref. /1/, is given by the formula $\mathcal{E}^{I}_{1}(z) = \mathcal{E}^{I}_{1}(0)e^{k_1 z}$, then for $z > 0$ the exact solution vanishing at $z \to +\infty$ has the form:

$$\mathcal{E}^{II}_{1}(z) = \left(\frac{4\epsilon_\perp}{|\alpha|}\right)^{1/2}\{\nu^{1/2}\cosh[2k_2\left(\frac{\epsilon_\perp}{\epsilon_\parallel}\right)^{1/2}(z - z_0)] + 1\}^{-1/2}, \tag{4}$$

$$\nu = 1 - \frac{16\beta\epsilon_\perp}{3\alpha^2}.$$

For $\beta = 0$ ($\nu = 1$) we have the solution obtained in ref. /1/. Consider the second equation of (3) for the most general form of the dependence of ϵ_{11} on $\mathcal{E}^2_1$, $\epsilon_{11} = \sum_{k=0}^{N} \alpha_k \mathcal{E}^{2k}_{1}$. For the solution $\mathcal{E}(z) \to 0$ as $z \to +\infty$ we get

$$z - z_0 = \int \left(q \sum_{k=0}^{N} \frac{\alpha_k}{k+1} \mathcal{E}_1^{2(k+1)}\right)^{-1/2} d\mathcal{E}_1,$$

where q is a constant, that determines $\mathcal{E}^{II}_{1} = \mathcal{E}^{II}_{1}(z)$.

For $N = 1$ we get the result of ref. /1/ whereas for $N = 2$ our result is (4) (see, for example, ref. /2/). Since $\beta = \beta(\omega)$ and depends on medium, both cases $\beta > 0$ and $\beta < 0$ are possible, the latter requires $\nu > 0$. In the case when this condition is violated other nonlinear contributions to $\epsilon_{ij}(\omega, |E|^2)$ should be taken into account.

2. Now we discuss the characteristics of the NSP(4) due to the contribution of the term $\beta|E_1|^4$. The continuity condition at $z = 0$ for the electric field results in the following

equation for z_0

$$\mathcal{E}(0) = \mathcal{E}^{I}_{1}(0) = \mathcal{E}^{II}_{1}(0) = \left(\frac{4\epsilon_\perp}{|\alpha|}\right)^{1/2}\{1 + \nu^{1/2}\cosh[2k_2\left(\frac{\epsilon_\perp}{\epsilon_\parallel}\right)^{1/2} z_0]\}^{-1/2}. \tag{5}$$

One can easily show that if $\beta > 0$ ($\nu < 1$), then $|z_0(\beta,\alpha)| > |z_0(0,\alpha)|$ and if $\beta < 0$ ($\nu > 1$), then $|z_0(\beta,\alpha)| < |z_0(0,\alpha)|$; "drawing in" of NSP is possible in comparison with ref. /1/, the maximum value of the NSP amplitude $\mathcal{E}^{II}_{1,max}(\beta,\alpha) = \left(\frac{2\epsilon_\perp}{|\alpha|}\right)^{1/2}\left(\frac{2}{1+\nu^{1/2}}\right)^{1/2}$ being at $z = z_0$.

For $\beta > 0$ ($\nu < 1$), $\mathcal{E}^{II}_{1,max}(\beta,\alpha) > \mathcal{E}^{II}_{1,max}(0,\alpha)$ and conversely for $\beta < 0$ ($\nu > 1$), $\mathcal{E}^{II}_{1,max}(\beta,\alpha) < \mathcal{E}^{II}_{1,max}(0,\alpha)$.

From the continuity condition of the magnetic field at $z = 0$ and using eqs. (2) and (4), we derive the following dispersion equation $\omega = \omega(k, \mathcal{E}(0))$:

$$\frac{\epsilon_1}{k_1} = \frac{(\epsilon_\perp \epsilon_\parallel)^{1/2}}{k_2} \frac{\sinh[2k_2\left(\frac{\epsilon_\perp}{\epsilon_\parallel}\right)^{1/2} z_0]}{\cosh[2k_2\left(\frac{\epsilon_\perp}{\epsilon_\parallel}\right)^{1/2} z_0] + \nu^{-1/2}}. \tag{6}$$

It follows from (6) that for $\epsilon_1 > 0$ (particularly when nonlinear medium is in contact with vacuum), we have $z_0 > 0$; and for $\epsilon_1 < 0$, $z_0 < 0$.

Therefore for $\beta < 0$, $\mathcal{E}^{II}_{1}(z)$ has the maximum value at the point z_0 which is closer to the surface than in the case of ref. /1/. However $\mathcal{E}^{II}_{1,max}(\beta,\alpha) < \mathcal{E}^{II}_{1,max}(0,\alpha)$, i.e., we have a certain "drawing in" of the NSP to the surface. For $\beta > 0$ and $\epsilon_1 > 0$ the NSP are "drawn in" inside medium II and having achieved the maximum value at $z = z_0$ vanish monotonically.

As in ref. /1/ using eqs. (5) and (6), we can construct $n^2(\omega) = \frac{k^2 c^2}{\omega^2}$. In our case this quantity is given by

$$n^2(\omega) = \frac{\epsilon_1[(\epsilon_\parallel p)^{1/2} - \epsilon_\parallel]}{(\epsilon_\parallel p)^{1/2} - \epsilon_1}, \quad p = \epsilon_\perp - \frac{|\alpha|\mathcal{E}^2(0)}{2} + \frac{\beta\mathcal{E}^4(0)}{3}. \tag{7}$$

The existence of NSP for ϵ_1, $\epsilon_\parallel$, $\epsilon_\perp > 0$ (linear surface polaritons do not exist in this case) requires the fulfillment of the following inequalities:

a) For $\epsilon_1 > \epsilon_\parallel$, $p > \frac{\epsilon_1^2}{\epsilon_\parallel}$; for $\beta > 0$ the condition for $\mathcal{E}(0)$ (at fixed ϵ_1, $\epsilon_\parallel$, $\epsilon_\perp$) is less critical than in ref. /1/ whereas for $\beta < 0$ it is more critical.

b) For $\epsilon_1 < \epsilon_\parallel$, $p < \frac{\epsilon_1^2}{\epsilon_\parallel}$, the situation is opposite.

The concrete formulae (3)-(7) and the characteristics of NSP coincide with the results of ref. /1/ for $\beta = 0$.

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