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NOTE ON ANTIFERROMAGNETIC GLASSER MODEL

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In recent papers [1,2] a linear chain of classical spins with an interaction defined by the Hamiltonian

$$H = J \sum_{i=1}^{N} \ln(1 - \vec{S}_i \cdot \vec{S}_{i+1}) \tag{1}$$

has been studied. Glasser [1] has considered an open chain of classical one-component (Ising) spins and found that the partition function, as a function of the first spin satisfies, in the thermodynamic limit, certain singular integral equation. He found an exact solution to this equation only for one non-trivial case, $\beta J = 1$, and was lead to the conjecture that the system undergoes an ordering transition when the end spin is held fixed at ∓ 1. Next, Niemeijer and Ruijgrok [2] have considered a closed chain of classical three-component (Heisenberg) spins with periodic boundary conditions and calculate partition function and correlation functions exactly. They have shown that for $J > 0$ a phase transition is found for $\beta J = 1$, where both the specific heat and the magnetic susceptibility diverge. They note, however, that for the antiferromagnetic case, $J < 0$, the free energy is a regular function of the temperature and the specific heat and magnetic susceptibility remain finite. As we shall see, the antiferromagnetic case corresponds to the Hamiltonian

$$H = J \sum_{i=1}^{N} \ln(1 + \vec{S}_i \cdot \vec{S}_{i+1}) \tag{2}$$

with $J > 0$ rather than to the Hamiltonian (1) with $J < 0$.

To see this, consider the following Hamiltonian for N spins with periodic boundary conditions

$$H_\epsilon = J \sum_{i=1}^{N} \ln(1 - \epsilon \vec{S}_i \cdot \vec{S}_{i+1}), \tag{3}$$

where $J > 0$, $\vec{S}_i$ are n-component classical unit vectors and $\epsilon = \mp 1$. We can calculate the partition function along the lines of the well-known continuous analogue of the transfer matrix method. The partition function can be written as

$$Z_N = (4\pi)^{-N} \sum_{\nu=0}^{\infty} c_\nu \lambda_\nu^N , \tag{4}$$

where λ_ν are eigenvalues of the integral equation

$$\int d\vec{S}_2 (1-\epsilon \vec{S}_1 \vec{S}_2)^{-\beta J} \psi_{\nu\mu}(\vec{S}_2) = \lambda_\nu \psi_{\nu\mu}(\vec{S}_1) \quad . \tag{5}$$

and $\mu = 1,2,\ldots,c_\nu$ is the degeneracy of the eigenvalue λ_ν. For $n \geq 3$ the eigenfunctions of (5) are hyperspherical harmonics, while for $n=2$ (plane rotator model) we have

$$\psi_\nu(\theta) = \begin{cases} 1/\sqrt{2\pi} & \nu = 0 \\ e^{\pm i\nu\theta}/\sqrt{\pi} & \nu \neq 0. \end{cases} \tag{6}$$

The eigenvalues of (5) (for $n \geq 2$) are given by

$$\lambda_\nu = \epsilon^\nu \pi^{\frac{n-1}{2}} 2^{n-1-K} \frac{\Gamma(\frac{n-1}{2}-K)\Gamma(\nu+K)}{\Gamma(n+\nu-1-K)\Gamma(K)} \quad \text{for} \quad K = \beta J < \frac{n-1}{2}. \tag{7}$$

For $K < \frac{n-1}{2}$ we have for the free energy per spin in the thermodynamic limit

$$f = kT \ln[\pi^{\frac{3-n}{2}} 2^{K+3-n} \Gamma(n-1-K)/\Gamma(\frac{n-1}{2}-K)]. \tag{8}$$

For $K = \frac{n-1}{2}$ the free energy diverges and for $K > \frac{n-1}{2}$ is complex when defined as the analytic continuation of f for $K > \frac{n-1}{2}$. Note that the free energy does not depend on ϵ. It means that the free energy and the specific heat are the same for ferromagnetic as well as for antiferromagnetic case.

The specific heat per spin is

$$C = kK^2[\zeta(2,\frac{n-1}{2}-K) - \zeta(2,n-1-K)] \quad (K < \frac{n-1}{2}), \tag{9}$$

where $\zeta(n,x)$ is the generalized zeta function. The critical temperature is defined by $K_c = \frac{n-1}{2}$ and when T is lowered towards T_c the specific heat diverges as $C \sim (T-T_c)^{-2}$, implying that its critical exponent $\alpha = 2$.

It is easy to calculate the following expression for the correlation functions

$$\langle \vec{S}_i \cdot \vec{S}_{i+r} \rangle = (\lambda_1/\lambda_0)^r = (\frac{\epsilon K}{n-1-K})^r \quad . \tag{10}$$

One can see that for $\epsilon = +1$ the correlations decrease monotonically with distance as for ferromagnets, while for $\epsilon = -1$ the correlations decay in an oscillatory way with distance,

changing sign at each site, a behaviour characteristic for antiferromagnetic chains. It is useful to define correlation length $\xi(T)$ by

$$1/\xi = -\ln|\lambda_1/\lambda_0| \; . \tag{11}$$

When T is lowered towards T_c, the correlation length diverges as $\xi \sim (T-T_c)^{-1}$, implying that $\nu = 1$. For the critical exponent η one can obtain the non-classical value 1.

The magnetic susceptibility can now be calculated from the fluctuation relation

$$\chi = \frac{\beta}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \langle \vec{S}_i \cdot \vec{S}_j \rangle \quad . \tag{12}$$

In the thermodynamic limit eq. (12) reduces to

$$\chi = \beta \frac{n-1+(\epsilon-1)K}{n-1-(\epsilon+1)K}. \tag{13}$$

From (13) it can be seen that the magnetic susceptibility for the ferromagnetic case, diverges with a critical exponent $\gamma = 1$, which is the classical value, when the temperature approaches the critical temperature from above, whereas for the antiferromagnetic case the susceptibility vanishes when T approaches T_c.

Finally, note that the scaling relation $(2-\eta)\nu = \gamma$ is fulfilled in the present model, while the hyperscaling relation $d\nu = 2-\alpha$ is violated. Note also that the critical exponents do not depend on n. The fact that the system shows a phase transition to the ordered phase as $T \to T_c$ from above is not in contradiction with Van Hove's well-known theorem that states that there can be no phase transition in one-dimensional classical systems with non-singular potentials of finite range, since the potential we are dealing with is obviously singular. For $T < T_c$ the model is not well defined since the free energy becomes complex. This behaviour occurs also in other models, e.g., in the Gaussian model [3].

REFERENCES

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