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RENORMALIZATION-GROUP TREATMENT OF SYSTEMS WITH SUPERCONDUCTING AND OTHER ORDERINGS IN MAGNETIC FIELD

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I. INTRODUCTION

The critical behaviour of systems with two interacting order parameters in the framework of the generalized Ginzburg-Landau models has been intensively studied in the mean field approximation [1] and by the renormalization group (RG) approach [2,3,4,5].

Systems with a fluctuating order parameter coupled to a gauge field, namely, a superconductor in a magnetic field [6] and the transition between nematic and smectic-A liquid crystal mesophases [7] have also been considered. The fluctuations of the gauge field (the vector potential of the magnetic field and the director vector in the smectic-A phase) change the universality at the transition point to: (i) a new Halperin-Lubensky-Ma(HLM)-type second-order phase transition for $n > n_c = 365{,}9$ and (ii) a weakly first-order phase transition for $n < n_c$ [7], where n is the symmetry index of the order parameter.

Recently, the RG approach has been applied by Grewe and Schuh [8] to the problem of coexistence of superconductivity and ferromagnetism in a magnetic field, using the three energy functional proposed by Blount and Varma [9]. In the vicinity of the ferromagnetic to superconducting phase-boundary, the critical fluctuations of the magnetic ordering are absorbed into the fluctuations of the vector potential. Thus one obtains a quantitative modification of the HLM recursion relations. This result is due to the fact that the magnetic field is conjugated to the magnetization. Hence a term linear in the magnetic order parameter appears to be relevant for the result of the RG treatment.

In this paper we apply the RG approach [10] to systems in a magnetic field which contain two interacting order parameters, namely, a superconducting and another (nonmagnetic) one. Systems with such two order parameters are, for instance, a superconductor with a structural distortion associated with doubling of the lattice periodicity [11,12] or a two-band semimetal with both exitonic and superconducting phase transitions [13,14].

II. THE MODEL

We start from a free energy functional of the form:

$$\mathcal{F}[\psi,\phi,\vec{A}] = -\int d\vec{x}\,[a|\psi(\vec{x})|^2 + \gamma|(\nabla - iq_0\vec{A}(\vec{x}))\psi(\vec{x})|^2 + \frac{b_\psi}{2}|\psi(\vec{x})|^4 +$$

$$+\frac{1}{8\pi\mu_0}[\mathrm{rot}\,\vec{A}(\vec{x})]^2 + \frac{r}{2}\phi^2(\vec{x}) + \frac{\kappa}{2}(\nabla\,\phi(\vec{x}))^2 + \frac{b_\phi}{4}\phi^4(\vec{x}) + \frac{c}{2}\phi^2(\vec{x})|\psi(\vec{x})|^2, \tag{1}$$

where $\psi(\vec{x})$ is the superconducting order parameter, $\vec{A}(\vec{x})$ is the vector potential and $\phi(\vec{x})$ is the other (nonmagnetic) order parameter. As usual, $a = a'(T-T_\psi)/T_\psi$, $r = r'(T-T_\phi)/T_\phi$, μ is the magnetic permeability of the system, $q_0 = 2e$ is twice the electron charge, b_ψ, b_ϕ, c, γ and κ are assumed to be analytic functions of the temperature (including the critical points T_ψ and T_ϕ, $T_\psi \simeq T_\phi$). Here $\hbar = c = k_B = 1$. The dimension of space is $d = 4-\epsilon$. The Coulomb gauge $\mathrm{div}\vec{A}(\vec{x}) = 0 = 0$ is assumed. There are several characteristic lengths in model (1), but the quantitative results of the RG treatment do not change if we use a common cut-off for the short-wave-length fluctuations of the fields $\sigma = \psi, \phi, \vec{A}$.

In momentum space we shall work, for convenience, with the dimensionless wave-vectors $\vec{q} = \vec{q}'/q_c$ (q_c is the momentum cut-off). After some simple transformations of the fields, free energy (1) in momentum space representation is

$$\mathcal{F} = \mathcal{F}_0 + \mathcal{F}_{A\psi^2} + \mathcal{F}_{A^2\psi^2} + \mathcal{F}_{\phi^4} + \mathcal{F}_{\phi^2\psi^2} + \mathcal{F}_{\psi^4} \tag{2}$$

where

$$\mathcal{F}_0 = -\sum_{\vec{q}}\{\sum_\alpha (r_\psi+\vec{q}^2)\,\psi^*_\alpha(\vec{q})\,\psi_\alpha(\vec{q}) - \frac{1}{2}\sum_N (r_\phi+\vec{q}^2)\,\phi_N(\vec{q})\,\phi_N(-\vec{q}) -$$

$$-\frac{1}{8\pi\mu_0}\sum_i \vec{q}^2 A_i(\vec{q})\,A_i(-\vec{q}). \tag{3}$$

$$\mathcal{F}_{\psi^4} = -\frac{u_0}{2\Omega}\sum_{\alpha\beta;\vec{q}_1\ldots\vec{q}_3}\psi^*_\alpha(\vec{q}_1)\,\psi^*_\beta(\vec{q}_2)\,\psi_\alpha(\vec{q}_3)\,\psi_\beta(\vec{q}_1+\vec{q}_2-\vec{q}_3), \tag{4}$$

$$\mathcal{F}_{A\psi^2} = -\frac{q_0}{\Omega^{1/2}}\sum_{i\alpha;\vec{q}_1\vec{q}_2}(\vec{q}_1+\vec{q}_2)_i\,A_i(\vec{q}_1-\vec{q}_2)\,\psi^*_\alpha(\vec{q}_1)\,\psi_\alpha(\vec{q}_2), \tag{5}$$

$$\mathcal{F}_{A^2\psi^2} = -\frac{q_0^2}{\Omega}\sum_{i\alpha,\vec{q}_1\ldots\vec{q}_3}A_i(\vec{q}_1)A_i(\vec{q}_2)\psi^*_\alpha(\vec{q}_3)\,\psi_\alpha(\vec{q}_3-\vec{q}_1-\vec{q}_2), \tag{6}$$

$$\mathcal{F}_{\phi^4} = -\frac{v_0}{4\Omega}\sum_{NL;\vec{q}_1\ldots\vec{q}_3}\phi_N(\vec{q}_1)\,\phi_N(\vec{q}_2)\,\phi_L(\vec{q}_3)\,\phi_L(-\vec{q}_1-\vec{q}_2-\vec{q}_3), \tag{7}$$

$$\mathcal{F}_{\phi^2\psi^2} = -\frac{w_0}{2\Omega}\sum_{\alpha N;\vec{q}_1\ldots\vec{q}_3}\phi_N(\vec{q}_1)\phi_N(\vec{q}_2)\,\psi^*_\alpha(\vec{q}_3)\,\psi_\alpha(\vec{q}_3-\vec{q}_1-\vec{q}_2). \tag{8}$$

In (3)-(8) Ω is a dimensionless volume $\Omega = Vq_c^d$ and

$$r_\psi = \frac{a}{\gamma q_c^2}, \quad r_\phi = \frac{r}{\kappa q_c^2},$$

$$u_0 = \frac{b_\psi}{\gamma^2}q_c^{d-4}, \quad v_0 = \frac{b_\phi}{\kappa^2}q_c^{d-4}, \quad w_0 = \frac{c}{\kappa\gamma}q_c^{d-4}. \tag{9}$$

The order parameters $\psi(\vec{x})$ and $\phi(\vec{x})$ are generalized to a ($n/2$)-component complex field and to a m-component real field, respectively. The vector potential $\vec{A}(\vec{x})$ is a d-dimensional vector. The suffixes α, N and i denote the components of the corresponding fields. The Feynman graph rules for model (2) are standard. The free correlation functions

$$G^{(0)}_{\sigma\alpha\alpha'}(\vec{q},\vec{q}') = \langle\sigma^*_\alpha(\vec{q})\,\sigma_{\alpha'}(\vec{q}')\rangle_0, \qquad \sigma \equiv \psi, \phi, \vec{A}, \tag{10}$$

are

$$G^{(0)}_{\sigma\alpha\alpha'}(\vec{q},\vec{q}') = \delta_{\alpha\alpha'}\,\delta(\vec{q}-\vec{q}')\,G^{(0)}_\sigma(\vec{q}) \tag{11}$$

with

$$G^{(0)}_\sigma(\vec{q}) = \frac{1}{r_\sigma+\vec{q}^2}, \qquad \sigma = \psi, \phi, \tag{12}$$

and

$$G_A^{(0)}(\vec{q}) = 4\pi\mu_0/\vec{q}^{\,2}. \tag{13}$$

III. THE RENORMALIZATION-GROUP TRANSFORMATION

Exact recursion relations to order $O(\epsilon)$ for free energy (2) are obtained through a straightforward application of the RG approach [10]. Some complications owing to the inconsistency between the RG procedure and the requirement for a gauge invariance of the model we get over as has been proposed by Halperin et al.[8]. The partial trace of probability distribution $\exp\{\mathcal{F}\}$, taken over the high momentum ($e^{-s}<q<1$) degrees of freedom $\sigma_a(\vec{q})$ is calculated to the first order in the vertex constants for parameters r_ψ and r_ϕ and to the second order for the vertex constants u_0, v_0, w_0 and q_0. The contributions to the q-dependent terms in correlation functions (12)-(13) are also accounted to order $O(\epsilon^2)$. Then we get

$$r'_\psi = e^{s(2-\eta_\psi)}\{r_\psi + \frac{n+2}{2} f(s,r_\psi)\, u_0 + \frac{3}{4\pi} f(s,0)\, q_0^2\mu + \frac{m}{2} f(s,r_\phi)\, w_0\} \tag{14}$$

$$r'_\phi = e^{s(2-\eta_\phi)}\{r_\phi + (m+2) f(s,r_\phi)\, v_0 + \frac{n}{2} f(s,r_\psi)\, w_0\}, \tag{15}$$

$$u'_0 = e^{s(\epsilon-2\eta_\psi)}\{u_0 - \frac{n+8}{2} g(s)\, u_0^2 - \frac{m}{2} g(s)\, w_0^2 - 96\pi^2 q_0^4\mu^4 g(s)\}, \tag{16}$$

$$v'_0 = e^{s(\epsilon-2\eta_\phi)}\{v_0 - (m+8)\, g(s)\, v_0^2 - \frac{n}{4} g(s)\, w_0^2\}, \tag{17}$$

$$w'_0 = e^{s(\epsilon-\eta_\phi-\eta_\psi)}\{w_0 - 2g(s)\, w_0^2 - \frac{n+2}{2} g(s)\, u_0 w_0 - (m+2)\, g(s)\, v_0 w_0\}, \tag{18}$$

$$(\mu')^{-1} = e^{-s\eta_A}\mu^{-1}\{1 + \frac{ns}{12\pi} q_0^2\mu\}, \tag{19}$$

$$q'_0 = e^{\frac{\epsilon-\eta_A}{2}s}\, q_0, \tag{20}$$

$$1 = e^{-s\eta_\psi}\{1 - \frac{3s}{2\pi} q_0^2\mu\}, \tag{21}$$

where

$$g(s) = \frac{s}{8\pi^2}, \quad f(s,r_\sigma) = \frac{1}{8\pi^2}\left(\frac{1-e^{-2s}}{2} - r_\sigma s\right) \tag{22}$$

($0<s<\infty$) and $\eta_\psi, \eta_\phi, \eta_A$ are the anomalous dimensions of the fields. The parameters r_ψ, r_ϕ, u_0, v_0, w_0, and $(q_0^2\mu)$ are assumed of order $O(\epsilon)$. Recursion relations (14)-(21) are direct generalization on those for a superconductor in a magnetic field [8] and those for two coupled fields [2]. Up to order $O(\epsilon)$ we have for the anomalous dimension $\eta_\psi, \eta_\phi = 0$.

The assumption $\eta_A \neq \epsilon$ in eq.(20) would lead to $q_0^* = \infty$ (if $\eta_A < \epsilon$) or to $q_0^* = 0$ (if $\eta_A > \epsilon$). The first case does not give any finite fixed points. In the case $q_0^* = 0$, the free energy differs from the original one (2), because vertices of type $\mathcal{F}_{A\psi^2}$ and $\mathcal{F}_{A^2\psi^2}$ would be absent in the fixed point free energy.

Denoting $t = q_0^2\mu/12\pi\epsilon$, we obtain for η_ψ and η_A (see (19) and (21))

$$\eta_\psi = -18\epsilon t^*, \tag{23}$$

$$\eta_A = n\epsilon t^*. \tag{24}$$

Having the anomalous dimensions η_A, η_ψ and $\eta_\phi = 0$ we can study the recursion relations in a reduced parameter space $\vec{\mu} = (r_\psi, r_\phi, u, v, w, t)$, where

$$u = u_0/8\bar{\epsilon}, \quad v = v_0/4\bar{\epsilon}, \quad w = w_0/8\bar{\epsilon}, \quad (\bar{\epsilon} = 8\pi^2\epsilon). \tag{25}$$

Then we obtain from (14)-(21):

$$r'_\psi = e^{2s}\{1+18\epsilon s t\}\{r_\psi + 32(n+2)\bar{\epsilon} f(s,r_\psi)u + 32m\bar{\epsilon} f(s,r_\phi)w + \\ +18\bar{\epsilon} f(s,0)\}t \,, \tag{26}$$

$$r'_\phi = e^{2s}\{r_\phi + 32(m+2)\bar{\epsilon} f(s,r_\phi)v + 32n\bar{\epsilon} f(s,r_\psi)w\}, \tag{27}$$

$$u' = e^{\epsilon s}\{1 + 36\epsilon s t\}\{u - 4(n+8)\epsilon s u^2 - 4m\epsilon s w^2 - 27\epsilon s t^2\}, \tag{28}$$

$$v' = e^{\epsilon s}\{v - 4(m+8)\epsilon s v^2 - 4n\epsilon s w^2\}, \tag{29}$$

$$w' = e^{\epsilon s}\{1+18\epsilon s t\}\{w - 16\epsilon s w^2 - 4(n+2)\epsilon s u w - 4(m+2)\epsilon s v w\}, \tag{30}$$

$$t' = e^{\epsilon s}\, t\{1 - n s \epsilon t\}. \tag{31}$$

IV. ANALYSIS OF THE FIXED POINTS

From (31) one obtains two types of fixed points $\mu^* = (r_\psi^*,\dots)$ corresponding to $t^* = 0$ and $t^* = 1/n$. In both cases the fixed point values u^* , v^* and w^* are

$$(1+36t^*)u^* = 4(n+8)u^{*2} + 4mw^{*2} + 27t^{*2}, \tag{32}$$

$$v^* = 4(m+8)v^{*2} + 4nw^{*2}, \tag{33}$$

$$(1+18t^*)w^* = 16w^{*2} + 4(n+2)u^*w^* + 4(m+2)v^*w^* \,. \tag{34}$$

If the values w^*, v^* and u^* are known, r_ψ^* and r_ϕ^* are to be

$$r_\psi^* = -2\epsilon[(n+2)u^* + 2mw^* + \frac{9}{4}t^*] \,, \\ r_\phi^* = -2\epsilon[(m+2)v^* + 2nw^* + \frac{9}{4}t^*] \,. \tag{35}$$

The eigenvalues e^{sy_μ} $(\mu = r_\psi,\dots)$ of the linearized transformation matrix for the relevant variables which determine the scaling behaviour of the system are

$$y_t = -\epsilon\,, \quad t^* = \frac{1}{n}\,, \tag{36}$$

$$y_t = \epsilon\,, \quad t^* = 0\,, \tag{37}$$

$$y_{r_\psi, r_\phi} = 2 + \epsilon\{9t^* - 2(n+2)u^* - 2(m+2)v^* \pm \\ \pm\sqrt{[9t^* + (m+2(v^* - 2(n+2)u^*]^2 + 16mnw^{*2}}\} \,. \tag{38}$$

The eigenvalues y_u , y_v and y_w could be found from a third order algebraic equation which is easily obtainable. Using these y_u , y_v and y_w one might obtain only corrections to the various fixed points.

We shall shortly comment the following cases:

A. Case $t^* = 0$

From (23) and (24) we have $\eta_\psi = \eta_A = 0$. This is the above discussed $q_0^* = 0$ case. The fixed points (32)-(35) and the critical exponents (36)-(38) (despite the presence of the parameter in the recursion relations) to the order $O(\epsilon)$ are the same as those described by Kosterlitz et al [2]. According to (37), in a magnetic field the fixed points of a system with two ordering parameters are unstable with respect to perturbations of the magnetic field around the value $\mathcal{H} = 0$.

B. Case $t^* = 1/n$

1. Decoupled behaviour: $w^* = 0$. In this case for $n > n_c = 365{,}9$ one obtains an always unstable Gaussian-HLM fixed point and a Heisenberg-HLM fixed point for $m \neq -8$ and $n > n_c$. The last one is stable if n and m satisfy the condition

$$32 - nm - 2n - 2m + \frac{m+8}{2}[n + 2 + \frac{216}{n} - \frac{n+2}{n}\sqrt{n^2 - 360n - 2160}] + O(\epsilon) < 0. \tag{39}$$

In the particular case when in the original model $u_0 = v_0$, the Caussian-HLM fixed point vanishes, whereas the Heisenberg-HLM fixed point is possible only if

$$108(m+8)^2 - n(n+36)(m+8) + n(n+8) = 0, \qquad n > 365.9. \tag{40}$$

The critical exponents for n and m satisfying (40) are the same for so different fixed points as the Heisenberg and HLM ones.

Here we shall point out the following interesting behaviour of the HLM fixed point when the term $27t^{*2}$ is removed from equation (32) for u^*. The last is possible when the symmetry index n is very large. Then, instead of HLM fixed point, we get a Wilson-like fixed point

$$r_\psi^* = -\frac{n+2}{2(n+8)}(1+\frac{36}{n})\epsilon - \frac{9}{2n}\epsilon, \quad u^* = \frac{1+36/n}{4(n+8)} \tag{41}$$

and, of course, a Gaussian one. The critical exponents corresponding to the fixed point (41) are

$$y_{r_\psi} = 2 + \epsilon[\frac{18}{n} - \frac{n+2}{n+8}(1+\frac{36}{n})],$$

$$y_u = -(1+\frac{36}{n})\epsilon, \tag{42}$$

whereas t^* and y_t are given by (36). The origin of the term $27t^{*2}$ in (32) is due to the presence of the vertex $\mathcal{J}_{A^2\psi^2}$ in (2). For large n its removal breaks down the gauge invariance of model (2).

2. Coupled behaviour: $w^* \neq 0$. The presence of terms with $t^* = 1/n$ in (32)-(34) reflects in the absence of "bicritical" fixed point solution of the type $w^* = u^* = v^*$. The term $4mw^{*2}$ in the equation for u^* (32) modifies the critical value of the symmetry index n from n_c to $n_c' > n_c$ for $m > 0$, and to $n_c' < n_c$ for $m < 0$. When $m = 0$; system (32)-(34) decouples and the solutions for u^*, v^* and w^* can be analytically determined. We have another analytic solution of system (32)-(34) for $n \to \infty$. Then, as seen from (39), the physical system falls into the range of stability of the decoupled fixed points for $m > -2$.

V. SUMMARY AND CONCLUDING REMARKS

We have presented the RG recursion relations for a system containing two order parameters with an interaction of type $\phi^2\psi^2$. Moreover, one of the order parameters (ψ), having a charge q_0, is coupled to a magnetic field. For such a system, exact recursion relations are found, which generalize the recursion relations for two cases: (i) for a system with two coupled order parameters [2] and (ii) for a superconductor in a magnetic field [6].

The critical behaviour of a system with two uncharged order parameters is unstable with respect to perturbations connected with the simultaneous appearance of a charge (in one of the order parameters) and a magnetic field $\mathcal{H} \neq 0$. At $\mathcal{H} \neq 0$, the coupled behaviour $w^* \neq 0$ changes significantly. As a consequence, there is no stable fixed point in the physically interesting region $n < n_c = 365.9$. This is to be interpreted as a signal that: (i) There is no tetracritical behaviour, i.e., no inter-section points of two second order phase boundary lines on the phase diagram of the system exist. Thus, if a mixed "($\psi - \phi$)" phase would occur, it would not be bounded by second order lines only. (ii) Owing to the lack of a bicritical behaviour, the system has no points (on the phase diagram) where a first order transition lines would branch in two second order ones.

These features are consequences of the fact, that the vector potential makes some transition lines of weakly first order.

For systems with two order parameters, an effective extension of the superconducting critical region is possible due to the influence of the other ordering $\phi(x)$ near the point $T_\psi = T_\phi$ (see ref. [15]). Then one might suggest that the range of the weakly first order transition is also extended near this point.

Stable coupled fixed points should be looked for when n and m satisfy the inverse inequality (39). Then one has to find the real roots of an algebraic equation of fourth order in w^* with coefficients being polinoms of n and m.

In the limiting case $n \to \infty$ the effects of the vector potential fluctuations are negligible.

The above-mentioned results obtained for the example of superconductivity are applicable to every system of two ordering parameters one of which is coupled to a gauge field.

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