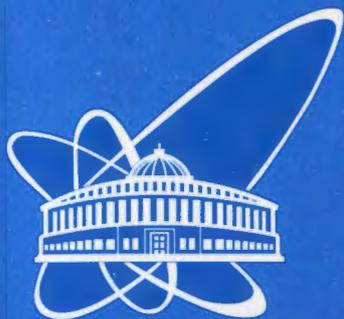


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NEW FLUCTUATION-DRIVEN PHASE TRANSITIONS
AND CRITICAL PHENOMENA IN UNCONVENTIONAL
SUPERCONDUCTORS

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1. INTRODUCTION

In this work an entirely new critical behavior in unconventional ferromagnetic superconductors and superfluids is established and described. This phenomenon corresponds to an isotropic ferromagnetic order in real systems but does not belong to any known universality class [1] and, hence, it might be of considerable experimental and theoretical interest. Due to crystal and magnetic anisotropy a new type of fluctuation-driven first order phase transitions occur, as shown in the present paper. These novel fluctuation effects can be observed near finite and zero temperature («quantum») phase transitions [1, 2] in a wide class of ferromagnetic systems with unconventional (spin-triplet) superconductivity or superfluidity.

The present investigation has been performed on the concrete example of intermetallic compounds UGe_2 and URhGe , where the remarkable phenomenon of coexistence of itinerant ferromagnetism and unconventional spin-triplet superconductivity [3] has been observed [4]. For example, in UGe_2 , the coexistence phase occurs [4] at temperatures $0 \leq T < 1$ K and pressures $1 < P < P_0 \sim 1.7$ GPa. A fragment of (P, T) phase diagrams of itinerant ferromagnetic compounds [4] is sketched in Fig. 1, where the lines $T_F(P)$ and $T_c(P)$ of the paramagnetic

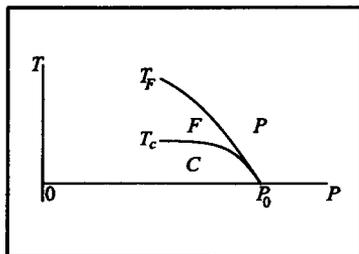
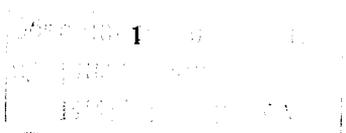


Fig. 1. (P, T) diagram with a zero-temperature multicritical point $(P_0, 0)$. Para- (P), ferromagnetic (F), and coexistence (C) phases, separated by the lines $T_f(P)$ and $T_c(P)$ of P-F and F-C phase transitions, respectively

(P)-to-ferromagnetic (F) and ferromagnetic-to-coexistence phase (C) transitions are very close to each other and intersect at very low temperature or terminate at the absolute zero $(P_0, 0)$. At low temperature, where the phase transition lines are close enough to each other, the interaction between the real magnetization vector $\mathbf{M}(\mathbf{r}) = \{M_j(\mathbf{r}); j = 1, \dots, m\}$ and the complex order parameter vector of the



spin-triplet Cooper pairing [3], $\psi(\mathbf{r}) = \{\psi_\alpha(\mathbf{r}) = (\psi'_\alpha + i\psi''_\alpha); \alpha = 1, \dots, n/2\}$ ($n = 6$) cannot be neglected [1] and, as shown here, this interaction produces new fluctuation phenomena.

Both thermal fluctuations at finite temperatures ($T > 0$) and quantum fluctuations (correlations) near the P -driven quantum phase transition at $T = 0$ should be considered but at the first stage the quantum effects [2] can be neglected as irrelevant to finite temperature phase transitions ($T_F \sim T_c > 0$). The present treatment of a recently derived free energy functional [5] by the standard Wilson-Fisher renormalization group (RG) [1] shows that unconventional ferromagnetic superconductors with an isotropic magnetic order ($m = 3$) exhibit a quite particular multicritical behavior for any $T > 0$, whereas the magnetic anisotropy ($m = 1, 2$) generates fluctuation-driven first order transitions [1]. Thus the phase transition properties of spin-triplet ferromagnetic superconductors are completely different from those predicted by mean-field theories [5, 6]. The results can be used in the interpretation of experimental data for phase transitions in itinerant ferromagnetic compounds [7].

The study presents for the first time an example of complex quantum criticality characterized by a double-rate quantum critical dynamics. In the quantum limit ($T \rightarrow 0$) the fields M and ψ have different dynamical exponents, z_M and z_ψ , and this leads to two different upper critical dimensions: $d_U^M = 6 - z_M$ and $d_U^\psi = 6 - z_\psi$. The complete consideration of the quantum fluctuations of both fields M and ψ requires a new RG approach in which one should either consider the difference ($z_M - z_\psi$) as an auxiliary small parameter or create a completely new theoretical paradigm of description. The considered problem is quite general and presents a challenge to the theory of quantum phase transitions [2]. The results can be applied to any natural system within the same class of symmetry although this paper is based on the example of itinerant ferromagnetic compounds.

2. RENORMALIZATION-GROUP INVESTIGATION

The relevant part of the fluctuation Hamiltonian of unconventional ferromagnetic superconductors [5, 6] can be written in the form

$$\mathcal{H} = \sum_{\mathbf{k}} \left[(r + k^2) |\psi(\mathbf{k})|^2 + \frac{1}{2} (t + k^2) |M(\mathbf{k})|^2 \right] + \frac{ig}{\sqrt{V}} \sum_{\mathbf{k}_1, \mathbf{k}_2} M(\mathbf{k}_1) \cdot [\psi(\mathbf{k}_2) \times \psi^*(\mathbf{k}_1 + \mathbf{k}_2)], \quad (1)$$

where $V \sim L^d$ is the volume of the d -dimensional system, the length unit is chosen so that the wave vector \mathbf{k} is confined below unity ($0 \leq k = |\mathbf{k}| \leq 1$);

$g \geq 0$ is a coupling constant, describing the effect of the scalar product M and the vector product ($\psi \times \psi^*$) for symmetry indices $m = (n/2) = 3$, and the parameters $t \sim (T - T_f)$ and $r \sim (T - T_s)$ are expressed by the critical temperatures of the generic ($g \equiv 0$) ferromagnetic and superconducting transitions. As mean-field studies indicate [5, 6], $T_s(P)$ is much lower than $T_c(T)$ and $T_F(P) \neq T_f(P)$.

The fourth order terms ($M^4, |\psi|^4, M^2|\psi|^2$) in the total free energy (*effective* Hamiltonian) [5, 6] have not been included in Eq. (1) as they are irrelevant to the present investigation. The simple dimensional analysis shows that the g -term in Eq. (1) corresponds to a scaling factor $b^{3-d/2}$ and, hence, becomes relevant below the upper borderline dimension $d_U = 6$, while fourth order terms are scaled by a factor b^{4-d} as in the usual ϕ^4 -theory and are relevant below $d < 4$ ($b > 1$ is a scaling number) [1]. Therefore we should perform the RG investigation in spatial dimensions $d = 6 - \epsilon$, where the g -term in Eq. (1) describes the only relevant fluctuation interaction. Moreover, the total fluctuation Hamiltonian [5, 6] contains off-diagonal terms of the form $k_i k_j \psi_\alpha \psi_\beta^*$; $i \neq j$ and/or $\alpha \neq \beta$. Using a convenient loop expansion these terms can be completely integrated out from the partition function to show that they modify the parameters (r, t, g) of the theory but they do not affect the structure of the model (1). So, such terms change auxiliary quantities, for example, the coordinates of the RG fixed points (FPs) but they do not affect the main RG results for the stability of the FPs and the values of the critical exponents. Here we ignore these off-diagonal terms.

One may consider several cases: (i) uniaxial magnetic symmetry, $M = (0, 0, M_3)$, (ii) tetragonal crystal symmetry when $\psi = (\psi_1, \psi_2, 0)$, (iii) XY magnetic order ($M_1, M_2, 0$), and (iv) the general case of cubic crystal symmetry and isotropic magnetic order ($m = 3$) when all components of the three-dimensional vectors M and ψ may have nonzero equilibrium and fluctuation components. The latter case is of major interest to real systems where fluctuations of all components of the fields are possible despite the presence of spatial crystal and magnetic anisotropy that nullifies some of the equilibrium field components. In one-loop approximation, the RG analysis reveals different pictures for anisotropic (i)–(iii) and isotropic (iv) systems. As usual, a Gaussian («trivial») FP ($g^* = 0$) exists for all $d > 0$ and, as usual [1] this FP is stable for $d > 6$ where the fluctuations are irrelevant. In the reminder of this paper the attention will be focussed on spatial dimensions $d < 6$, where the critical behavior is usually governed by nontrivial FPs ($g^* \neq 0$). In the cases (i)–(iii) only negative («unphysical» [9]) FP values of g^2 have been obtained for $d < 6$. For example, in the case (i) the RG relation for g takes the form

$$g' = b^{3-d/2-\eta} g (1 + g^2 K_d \ln b), \quad (2)$$

where g' is the renormalized value of g , $\eta = (K_{d-1}/8)g^2$ is the anomalous dimension (Fisher's exponent) [1] of the field M_3 ; $K_d = 2^{1-d} \pi^{-d/2} / \Gamma(d/2)$.

Using Eq. (2) one obtains the FP coordinate $(g^2)^* = -96\pi^3\epsilon$. For $d < 6$ this FP is unphysical and does not describe any critical behavior. For $d > 6$ the same FP is physical but unstable towards the parameter g as one can see from the positive value $y_g = -11\epsilon/2 > 0$ of the respective stability exponent y_g defined by $\delta g' = b^{y_g}\delta g$. Therefore, a change of the order of the phase transition from the second order in mean-field («fluctuation free») approximation to a fluctuation-driven first order transition when the fluctuation g -interaction is taken into account takes place. This conclusion is supported by general concepts of RG theory [1] and by the particular property of these systems to exhibit first order phase transitions [6] in mean-field approximation for broad variations of T and P .

In the case (iv) of isotropic systems the RG equation for g is degenerate and the ϵ -expansion breaks down. A similar situation is known from the theory of disordered systems [9] but here the physical mechanism and details of description are different. Namely for this degeneration one should consider the RG equations up to the two-loop order. The derivation of the two-loop terms in the RG equations is quite nontrivial because of the special symmetry properties of the interaction g -term in Eq. (1). For example, some diagrams with opposite arrows of internal lines, as the couple shown in Fig. (2), have opposite signs and compensate each other. The terms bringing contributions to the g -vertex are shown diagrammatically in Fig. 3. The RG analysis is carried out by a completely new $\epsilon^{1/4}$ -expansion for the FP values and $\epsilon^{1/2}$ -expansion for the critical exponents; again $\epsilon = (6-d)$. The RG equations are quite lengthy and here only the equation for g is discussed. It has the form

$$g' = b^{(\epsilon-2\eta_\psi-\eta_M)/2}g [1 + Ag^2 + 3(2B + C)g^4], \quad (3)$$

where

$$A = \frac{K_d}{2} [2\ln b + \epsilon(\ln b)^2 + (1 - b^2)(2r + t)], \quad (4)$$

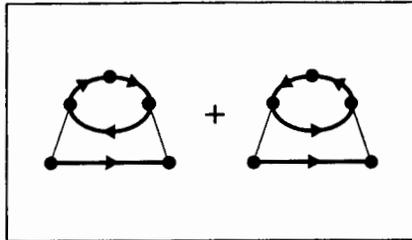


Fig. 2. A sum of g^5 -diagrams equal to zero. The thick and thin lines correspond to correlation functions $\langle |\psi_\alpha|^2 \rangle$ and $\langle |M_j|^2 \rangle$, respectively; vertices (\bullet) represent g -term in Eq. (1)

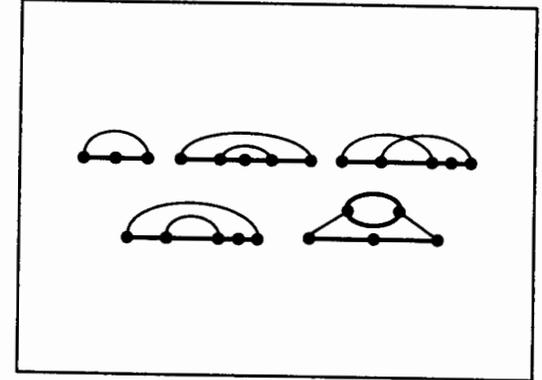


Fig. 3. Diagrams for g' of third and fifth order in g . The arrows of the thick lines have been omitted

$$B = \frac{K_{d-1}K_d}{192} [9(b^2 - 1) - 11\ln b - 6(\ln b)^2], \quad (5)$$

$$C = \frac{3K_{d-1}K_d}{64} [\ln b + 2(\ln b)^2], \quad (6)$$

η_M and η_ψ are the anomalous dimensions of the fields M and ψ , respectively. The one-loop approximation gives correct results to order $\epsilon^{1/2}$ and the two-loop approximation brings such results up to order ϵ . In Eq. (4), r and t are small expansion quantities with equal FP values $t^* = r^* = K_d g^2$. Using the condition for invariance of the two k^2 -terms in Eq. (1) one obtains $\eta_M = \eta_\psi \equiv \eta$, where

$$\eta = \frac{K_{d-1}}{8} g^2 \left(1 - \frac{13}{96} K_{d-1} g^2 \right). \quad (7)$$

Equation (3) yields a new FP

$$g^* = 8(3\pi^3)^{1/2} (2\epsilon/13)^{1/4}, \quad (8)$$

which corresponds to the critical exponent $\eta = 2(2\epsilon/13)^{1/2} - 2\epsilon/3$ (for $d = 3$, $\eta \approx -0.64$).

The eigenvalue problem for the RG stability matrix $\hat{\mathcal{M}} = [(\partial\mu_i/\partial\mu_j); (\mu_1, \mu_2, \mu_3) = (r, t, g)]$ can be solved by the expansion of the matrix elements up to order $\epsilon^{3/2}$. When the eigenvalues $\lambda_j = A_j(b)b^{y_j}$ of $\hat{\mathcal{M}}$ are calculated dangerous large terms of type b^2 and $b^2(\ln b)$, ($b \gg 1$) [8] in the off-diagonal elements of the matrix $\hat{\mathcal{M}}$ ensure the compensation of redundant large terms of the same type in the diagonal elements \mathcal{M}_{ii} . This compensation is crucial for the validity of scaling for this type of critical behavior. Such a problem does not appear in standard cases of RG analysis [1, 8]. As in the usual ϕ^4 -theory [8]

the amplitudes A_j depend on the scaling factor b : $A_1 = A_2 = 1 + (27/13)b^2\epsilon$, $A_3 = 1 - (81/52)\epsilon(\ln b)^2$. The critical exponents $y_t = y_1$, $y_r = y_2$ and $y_g = y_3$ are b -invariant:

$$y_r = 2 + 10\sqrt{\frac{2\epsilon}{13}} + \frac{197}{39}\epsilon, \quad (9)$$

$y_t = y_r - 18(2\epsilon/13)^{1/2}$, and $y_g = -\epsilon > 0$ for $d < 6$. The correlation length critical exponents $\nu_\psi = 1/y_r$ and $\nu_M = 1/y_t$ corresponding to the fields ψ and M are

$$\nu_\psi = \frac{1}{2} - \frac{5}{2}\sqrt{\frac{2\epsilon}{13}} + \frac{103}{156}\epsilon, \quad (10)$$

$$\nu_M = \frac{1}{2} + 2\sqrt{\frac{2\epsilon}{13}} - \frac{5\epsilon}{156}. \quad (11)$$

These exponents describe a quite particular multicritical behavior which differs from the numerous examples known so far. For $d = 3$, $\nu_\psi = 0.78$ which is somewhat above the usual value $\nu \sim 0.6 \div 0.7$ near a standard phase transition of the second order [1], but $\nu_M = 1.76$ at the same dimension ($d = 3$) is unusually large. The fact that Fisher's exponent [1] η is negative for $d = 3$ does not create troubles because such cases are known in complex systems, for example, in conventional superconductors [10]. Perhaps, a direct extrapolation of the results from the present ϵ -series is not completely reliable because of the fact that the series has been derived under the assumptions of $\epsilon \ll 1$ and under the conditions $\epsilon^{1/2}b < 1$, $\epsilon^{1/2}(\ln b) \ll 1$ provided $b > 1$. These conditions are stronger than those in the usual ϕ^4 -theory [1, 8]. Using the known relation [1] $\gamma = (2 - \eta)\nu$, the susceptibility exponents for $d = 3$ take the values $\gamma_\psi = 2.06$ and $\gamma_M = 4.65$. These values exceed even those corresponding to the Hartree approximation [1] ($\gamma = 2\nu = 2$ for $d = 3$) and can be easily distinguished in experiments. Note, that here we follow the interpretation of the asymptotic ϵ -series in the way given by Lawrie et al. [9]. This point of view is quite comprehensive, in particular, for an avoiding artificial conclusions from the RG analysis of complex systems with competing effects, such as the systems described by Eq. (1).

Notes about the quantum effects on the phase transitions. The critical behavior discussed so far may occur in a close vicinity of finite temperature multicritical points ($T_c = T_f > 0$) in systems possessing the symmetry of the model (1). In certain systems, as shown in Fig. 1, this multicritical points may occur at $T = 0$. In the quantum limit ($T \rightarrow 0$), or, more generally, in the low-temperature limit [$T \ll \mu$; $\mu \equiv (t, r)$; $k_B = 1$] the thermal wavelengths of the fields M and ψ exceed the inter-particle interaction radius and the quantum correlations fluctuations become essential for the critical behavior [2, 11]. The quantum effects can be considered by RG analysis of a comprehensively generalized version of the model (1), namely, the action \mathcal{S} of the referent quantum system. The generalized

action is constructed with the help of the substitution $(-\mathcal{H}/T) \rightarrow S[M(q), \psi(q)]$. Now the description is given in terms of the (Bose) quantum fields $M(q)$ and $\psi(q)$ which depend on the $(d+1)$ -dimensional vector $q = (\omega_l, \mathbf{k})$; $\omega_l = 2\pi lT$ is the Matsubara frequency ($\hbar = 1$; $l = 0, \pm 1, \dots$). The \mathbf{k} -sums in Eq. (1) should be substituted by respective q -sums and the inverse bare correlation functions $(r+k^2)$ and $(t+k^2)$ in Eq. (1) contain additional ω_l -dependent terms, for example, [2, 11]

$$\langle |\psi_\alpha(q)|^2 \rangle^{-1} = |\omega_l| + k^2 + r. \quad (12)$$

The bare correlation function $\langle |M_j(q)|^2 \rangle$ contains a term of type $|\omega_l|/k^\theta$, where $\theta = 1$ and $\theta = 2$ for clean and dirty itinerant ferromagnets, respectively [11]. The quantum dynamics of the field ψ is described by the bare value $z = 2$ of the dynamical critical exponent $z = z_\psi$, whereas the quantum dynamics of the magnetization corresponds to $z_M = 3$ (for $\theta = 1$), or, to $z_M = 4$ (for $\theta = 2$). This means that the classical-to-quantum dimensional crossover at $T \rightarrow 0$ is given by $d \rightarrow (d+2)$ and, hence, the system exhibits a simple mean-field behavior for $d \geq 4$. Just below the upper quantum critical dimension $d_U^{(0)} = 4$ the relevant quantum effects at $T = 0$ are represented by the field ψ , whereas the quantum (ω_l -) fluctuations of the magnetization are relevant for $d < 3$ (clean systems), or, for even for $d < 2$ (dirty limit) [11]. This picture is confirmed by the analysis of singularities of the relevant perturbation integrals. Therefore the quantum fluctuations of the field ψ have a dominating role below spatial dimensions $d < 4$, and for dimensions $3 < d < 4$ (clean systems), or, for $2 < d < 4$ in case of dirty limit, they are the only quantum fluctuations in these systems.

Taking into account the quantum fluctuations of the field ψ and completely neglecting the ω_l -dependence of the magnetization M , $\epsilon_0 = (4-d)$ analysis of the generalized action \mathcal{S} has been performed within the one-loop approximation (order ϵ_0^1). In the classical limit ($r/T \ll 1$), one rederives the results already reported above together with an essentially new result, namely, the value of the dynamical exponent $z_\psi = 2 - (2\epsilon/13)^{1/2}$ which describes the quantum dynamics of the field ψ . In the quantum limit ($r/T \gg 1$, $T \rightarrow 0$), the static phase transition properties are affected by the quantum fluctuations, in particular, in isotropic systems ($n/2 = m = 3$). For this case, the one-loop RG equations corresponding to $T = 0$ are not degenerate and give definite results. The RG equation for g ,

$$g' = b^{\epsilon_0/2} g \left(1 + \frac{g^2}{24\pi^3} \ln b \right), \quad (13)$$

yields two FPs: (a) a Gaussian FP ($g^* = 0$), which is unstable for $d < 4$, and (b) a FP $(g^2)^* = -12\pi^3\epsilon_0$ which is unphysical [$(g^2)^* < 0$] for $d < 4$ and unstable for $d \geq 4$. Thus the new stable critical behavior corresponding to $T > 0$ and $d < 6$ disappears in the quantum limit $T \rightarrow 0$. At the absolute zero and any dimension $d > 0$ the P -driven phase transition (Fig. 1) is of the first order.

This can be explained as a mere result of the limit $T \rightarrow 0$. The only role of the quantum effects is the creation of the new unphysical FP (b). In fact, the referent classical system described by \mathcal{H} from Eq. (1) also loses its stable FP (8) in the zero-temperature (*classical*) limit $T \rightarrow 0$ but does not generate any new FP because of the lack of g^3 -term in the equation for g' ; see Eq. (13). At $T = 0$ the classical system has a purely mean-field behavior [2] which is characterized by a Gaussian FP ($g^* = 0$) and is unstable towards T perturbations for $0 < d < 6$. This is a usual classical zero temperature behavior where the quantum correlations are ignored. For the standard ϕ^4 -theory this picture holds for $d < 4$. One may suppose that the quantum fluctuations of the field ψ are not enough to ensure a stable quantum multicritical behavior at $T_c = T_F = 0$ and that the lack of such a behavior in result of neglecting the quantum fluctuations of M . One may try to take into account these quantum fluctuations by the special approaches from the theory of disordered systems, where additional expansion parameters are used to ensure the marginality of the fluctuating modes at the same borderline dimension d_U (see, e.g., Ref. [6]). It may be conjectured that the techniques known from the theory of disordered systems with extended impurities cannot be straightforwardly applied to the present problem and, perhaps, a completely new supposition should be introduced.

3. FINAL REMARKS

The present results may be of use in interpretations of recent experiments [7] in UGe_2 , where the magnetic order is uniaxial (Ising symmetry) and the experimental data, in accord with the present consideration, indicate that the C-P phase transition is of the first order. Systems with isotropic magnetic order are needed for an experimental test of the new multicritical behavior. Besides, the present investigation exhibits several new essential problems which are a challenge to the theory of quantum phase transitions.

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REFERENCES

1. *Uzunov D. I.* Theory of Critical Phenomena. Singapore: World Scientific, 1993.
2. *Shopova D. V., Uzunov D. I.* // Phys. Rep. C. 2003. V. 379. P. 1.
3. *Sigrist M., Ueda K.* // Rev. Mod. Phys. 1991. V. 63. P. 239.

4. For UGe_2 , see: *Saxena S. S. et al.* // Nature. 2000. V. 406. P. 587;
Huxley A. et al. // Phys. Rev. B. 2001. V. 63. P. 144519;
For URhGe, see: *Aoki D. et al.* // Nature. 2001. V. 413. P. 613.
5. *Machida K., Ohmi T.* // Phys. Rev. Lett. 2001. V. 86. P. 850;
Walker M. B., Samokhin K. V. // Phys. Rev. Lett. 2002. V. 88. P. 207001.
6. *Shopova D. V., Uzunov D. I.* // Phys. Lett. A. 2003. V. 313. P. 139;
Shopova D. V., Uzunov D. I. // Phys. Rev. B. 2005. V. 72. P. 024531;
Shopova D. V., Uzunov D. I. // Bulg. J. Phys. 2005. V. 32. P. 81;
Shopova D. V., Uzunov D. I. Progress in Ferromagnetism Research / Ed. by V. N. Murray. N. Y.: Nova Science Publishers, 2006; Chapter 10.
7. *Pfleiderer C., Huxley A. D.* // Phys. Rev. Lett. 2002. V. 89. P. 147005;
Harada A. et al. // J. Phys. Soc. Jap. 2005. V. 74. P. 2675.
8. *Bruce A. D., Droz M., Aharony A.* // J. Phys. C: Solid State Physics. 1974. V. 7. P. 3673.
9. *Lubensky T. C.* // Phys. Rev. B. 1975. V. 11. P. 3573;
Lawrie I. D., Millev Y. T., Uzunov D. I. // J. Phys. A: Math. Gen. 1987. V. 20. P. 1599;
(Err.) *ibid*, 1987. V. 20. P. 6159.
10. *Halperin B. I., Lubensky T. C., Ma S. K.* // Phys. Rev. Lett. 1974. V. 32. P. 292.
11. *Hertz J. A.* // Phys. Rev. B. 1976. V. 14. P. 1165.

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