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STABILITY OF INFRARED FIXED POINT IN THEORY OF DEVELOPED TURBULENCE WITH WEAK ANISOTROPY

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1 Introduction

Traditional approach to the description of fully developed turbulence is based on the stochastic Navier-Stokes equation [1]. The complexity of this equation leads to great difficulties which defend to solve it even in the simplest case when one assumes the isotropy of the system under consideration. On the other hand, the isotropic turbulence is almost delusion and if exists is still rather rare. Therefore, if one wants to model more or less realistic developed turbulence then is pushed to consider anisotropically forced turbulence rather than isotropic one. This, of course, rapidly increases complexity of the corresponding differential equation which has to involve itself a part responsible for description of the anisotropy. The exact solution of the stochastic Navier-Stokes equation does not exist and one is forced to find out some convenient methods to touch the problem at least step by step.

One suitable and also powerful tool in the theory of developed turbulence is so-called renormalization group (RG) method[1]. During last two decades the RG technique was widely used in this field of science and gives answers on some principal questions (e.g. the fundamental description of the infra-red (IR) scale invariance) and is also useful for the calculations of many universal parameters (e.g. critical dimensions of the fields and their gradiens etc.). Detailed survey of this questions one can find in Refs. [4, 5] and Refs. therein.

In early papers the RG approach has been applied only to the isotropic models of developed turbulence. However, the method can be (with some modifications) also used in the theory of anisotropic developed turbulence. Crucial question immediately arises here, namely, whether the principal properties of isotropic case and anisotropic one are the same at least on qualitative level. If they are then it is possible to consider the isotropic case as first step in the investigation of the real systems. On this way of transition from the isotropic developed turbulence into anisotropic one we have to learn whether the scaling regime does remain stable under this transition. That means, whether the stable fixed points of the RG equations remain stable under influence of anisotropy.

During last decade a few papers have appeared where the above question has been considered. In some cases it have been found out that stability really takes place (see,e.g.[6, 7]). On the other hand, the existence of the systems without such stability was proved too. As was shown in Ref. [8] in the anisotropic magnetohydrodynamic developed turbulence stable regime generally does not exist. In the papers [7, 9] were investigated d-dimensional models with $d > 2$ for two cases, namely weak anisotropy [7] and strong one [9] and has been shown that the stability of the isotropic fixed point loses for dimensions $d < d_c = 2.68$. It has been also shown that stability of the fixed points even for dimension $d = 3$ take place only for sufficiently weak anisotropy. The only problem in these investigations is that it is impossible to use them in the case $d = 2$ because new ultra-violet (UV) divergences appear in the Green functions

[1]Here we consider quantum-field renormalization group approach [2] rather than Wilson renormalization group technique [3]

when one considers $d = 2$ and they were not taken into account in the papers [7, 9].

In the paper [10] has been given correct treatment of the two-dimensional isotropic turbulence. The correctness in the renormalization procedure has been reached by introduction into the model a new local term (with new coupling constant) which allows to remove additional UV-divergences. From this point of view, earlier obtained results for anisotropical developed turbulence presented in [11] and based on the paper [12] (the results of last paper are in conflict with [10]) can not be consider as right because they are inconsistent with the basic requirement of the UV-renormalization, namely the requirement of the localness of the counterterms [13, 14].

The authors of the recent paper [15] have used the double-expansion procedure introduced in [10] (this procedure is an combination of the well-known Wilson's dimensional regularization procedure and analytical one) in combination with minimal substraction (MS) scheme [?] for investigation of developed turbulence with weak anisotropy for $d = 2$. The main result of the paper was the conclusion that the two-dimensional fixed point is not stable under weak anisotropy. It means that 2d turbulence is very sensitive to the anisotropy and none stable scaling regimes exist in this case. In the case $d = 3$ for both isotropic turbulence and anisotropic one, as it was mentioned above, the existence of the stable fixed point, which governs the Kolmogorov asymptotic regime, has been established by means of the RG approach using analytical regularization procedure [6, 7, 9]. One can also make analytical continuation from $d = 2$ to the three-dimensional turbulence (in the same sense as in the theory of critical phenomena) and verify whether the stability of the fixed point (or, equivalently, stability of the Kolmogorov scaling regime) is restored. From the analysis made in Ref. [15] follows that it is impossible to restore the stable regime by transition from dimension $d = 2$ to $d = 3$. We suppose that main reason for above described discrepancy is related to the straightforward application of the standard MS scheme. In the standard MS scheme one works only with divergent part of the Green functions and its tensor part is neglected. In the case of isotropic models the stability of the fixed points is independent of dimension d of the space. However, in anisotropic models the stability of fixed points depends on the dimension d, and the tensor structure becomes to be important.

In present paper we suggest to apply modified MS scheme, where modification is based on the fact that the tensor structure is left d-dependent. We affirm that after such modification d-dependence is correctly taken into account and can be used in investigation whether it is possible to restore the stability of the anisotropic developed turbulence for some dimension d_c when going from two-dimensional system to three-dimensional one. Thus, after renormalization which one makes for the value $d = 2$ we left the d-dependence in the tensor parts of the counterterms. It allows one to look beyond dimension $d = 2$ and investigates behaviour of the system under continual transition from $d = 2$ to $d = 3$. One can hope that after such procedure the restoration of the stable regime at some critical dimension d_c will appear. In present paper we show that restoration really takes place. The value for the critical dimension is $d_c = 2.44$ (in this case $\epsilon = 2$, details see below in the text). Below

critical dimension the stable regime of the fixed point of the isotropic developed turbulence is lost by influence of weak anisotropy.

The paper is organized as follows. In Section 2 we give the quantum field functional formulation of the problem of the fully developed turbulence with weak anisotropy. The RG analysis is given in Section 3 when we discuss the stability of the obtained fixed point under weak anisotropy. In Section 4 we discuss our results. Appendix I contains expressions for divergent parts of the important graphs. At the end, Appendix II contains analytical expressions for fixed point and equation which describes its stability in the limit of the weak anisotropy.

2 Description of Model. UV-divergencies

In this section we give description of the model. As was already discussed in previous section we work with fully developed turbulence and assume weak anisotropy of the system. It means that parameters which describe deviations from the fully isotropic case are sufficiently small and allow one to forget about corrections of higher degrees (than linear) which are made by them.

In the statistical theory of anisotropically developed turbulence the turbulent flow can be described by a random velocity field $\vec{v}(\vec{x},t)$ and its evolution is given by the randomly forced Navier-Stokes equation

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} - \nu_0 \Delta \vec{v} - \vec{f}^A = \vec{f}, \qquad (1)$$

where we assume incompressibility of the fluid, which is mathematically given by the well-known conditions $\vec{\nabla}\cdot\vec{v} = 0$ and $\vec{\nabla}\cdot\vec{f} = 0$. In eq.(1) the parameter ν_0 is the kinematic viscosity (hereafter all parameters with subscript 0 denote bare parameters of unrenormalized theory, see below)), the term $\vec{f}^A$ is related to anisotropy and will be specified later. The large-scale random force per mass unit $\vec{f}$ is assumed to have Gaussian statistics defined by the averages

$$\langle f_i \rangle = 0, \ \langle f_i(\vec{x}_1,t) f_j(\vec{x}_2,t)\rangle = D_{ij}(\vec{x}_1 - \vec{x}_2, t_1 - t_2). \qquad (2)$$

The two point correlation matrix

$$D_{ij}(\vec{x},t) = \delta(t) \int \frac{d^d\vec{k}}{(2\pi)^d} \tilde{D}_{ij}(\vec{k}) \exp(i\vec{k}\cdot\vec{x}) \qquad (3)$$

is convenient to parametrized in the following way [6, 8]

$$\tilde{D}_{ij}(\vec{k}) = g_0 \nu_0^3 k^{4-d-2\epsilon} [(1 + \alpha_{10}\xi_k^2) P_{ij}(\vec{k}) + \alpha_{20} R_{ij}(\vec{k})], \qquad (4)$$

where a vector $\vec{k}$ is the wave vector, d is dimension of the space (in our case: $2 \leq d$), $\epsilon \geq 0$ is dimensionless parameter of the model. If dimension of the system is taken

$d > 2$ then the physical value of this parameter is $\epsilon = 2$ (so-called energy pumping regime). The situation is more complicated when $d = 2$. In this case the new integrals of motion arise, namely the enstrophy, and all its powers (for details see Ref. [16]) which leads to ambiguity in determination of the inertial range and this freedom is in RG method given by the value of the parameter ϵ. The value $\epsilon = 3$ corresponds to so-called enstrophy pumping regime. This problem of uncertainty is not possible to solve in the framework of the RG technique. On the other hand, the value of ϵ is not important for stability of the fixed point when $d = 2$. Thus, it is not important from our point of view what is the value of ϵ in the case $d = 2$. Its value $\epsilon = 0$ corresponds to a logarithmic perturbation theory for calculation of Green functions, when g_0, which plays the role of bare coupling constant of the model, becomes to be dimensionless. The problem of the continuation from $\epsilon = 0$ to the physical values has been discussed in [17]. $(d \times d)$-matrices P_{ij} and R_{ij} are the transverse projection operators and in the wave-number space are defined by the relations

$$P_{ij}(\vec{k}) = \delta_{ij} - \frac{k_i k_j}{k^2}, \quad R_{ij}(\vec{k}) = \left(n_i - \xi_k \frac{k_i}{k}\right)\left(n_j - \xi_k \frac{k_j}{k}\right), \tag{5}$$

where ξ_k is given by the equation $\xi_k = \vec{k} \cdot \vec{n}/k$. In eq.(5) the unit vector $\vec{n}$ specifies the direction of the anisotropy axis. The tensor $\tilde{D}_{ij}$ given by eq.(4) is the most general form with respect to the condition of incompressibility of the system under consideration and contains two dimensionless free parameters α_{10} and α_{20}. From the positiveness of the correlator tensor D_{ij} one immediately gets restrictions on the above parameters, namely $\alpha_{10} \geq -1$ and $\alpha_{20} \geq 0$. In what follows we assume that these parameters are small enough and generate only small deviations from isotropy case.

Using well-known the Martin–Siggia-Rose formalism of the stochastic quantization [18, 19, 20, ?] one can transform the stochastic problem (1) with the correlator (3) into the quantum field model of the fields $\vec{v}$ and $\vec{v}'$. Here $\vec{v}'$ is an independent of the $\vec{v}$ auxiliary incompressible field which we have to introduce when transform the stochastic problem into the functional form.

The action of the fields $\vec{v}$ and $\vec{v}'$ is given in the form

$$\begin{aligned} S &= \frac{1}{2}\int d^d\vec{x}_1 dt_1 d^d\vec{x}_2 dt_2 \left[v'_i(\vec{x}_1,t_1) D_{ij}(\vec{x}_1 - \vec{x}_2, t_1 - t_2) v'_j(\vec{x}_2, t_2)\right] \\ &+ \int d^d\vec{x} dt \left\{\vec{v}'(\vec{x},t)\left[-\partial_t \vec{v} - (\vec{v}\cdot\vec{\nabla})\vec{v} + \nu_0 \vec{\nabla}^2 \vec{v} + \vec{f}^A\right](\vec{x},t)\right\}. \end{aligned} \tag{6}$$

The functional formulation gives the possibility to use the quantum field theory methods including the RG technique to solve the problem. By means of the RG approach it is possible to extract large-scale asymptotic behaviour of the correlation functions after an appropriate renormalization procedure which is needed to remove UV-divergences.

Now we can return back to give explicit form of the anisotropic dissipative term $\vec{f}^A$. When $d > 2$ the UV-divergences are only present in the one-particle-irreducible Green function $< \vec{v}'\vec{v} >$. To remove them one needs to introduce into the action in addition to the counterterm $\vec{v}'\vec{\nabla}^2\vec{v}$ (the only counterterm needed in isotropic model) the following ones $\vec{v}'(\vec{n}\cdot\nabla)^2\vec{v}$, $(\vec{n}\cdot\vec{v}')\vec{\nabla}^2(\vec{n}\cdot\vec{v})$ and $(\vec{n}\cdot\vec{v}')(\vec{n}\cdot\vec{\nabla})^2(\vec{n}\cdot\vec{v})$. These additional terms are needed to remove divergences related to anisotropic structures. In this case ($d > 2$) one can use the above action (6) with (4) to solve the anisotropic turbulent problem. Therefore, in order to arrive to the multiplicative renormalizable model we have to take the term $\vec{f}^A$ in the form

$$\vec{f}^A = \nu_0\left[\chi_{10}(\vec{n}\cdot\vec{\nabla})^2\vec{v} + \chi_{20}\vec{n}\vec{\nabla}^2(\vec{n}\cdot\vec{v}) + \chi_{30}\vec{n}(\vec{n}\cdot\vec{\nabla})^2(\vec{n}\cdot\vec{v})\right]. \tag{7}$$

Bare parameters χ_{10}, χ_{20} and χ_{30} characterize the weight of the individual structures in the (7).

The more complicated situation arises in the specific dimensional case $d = 2$. In the case $d = 2$ one can find out that new divergences arise. They are related to the 1-irreducible Green function $< \vec{v}'\vec{v}' >$ which is finite when $d > 2$. Here one comes to a problem how to remove these divergences because the term in our action, which contains structure of this type is nonlocal, namely $\vec{v}'k^{4-d-2\epsilon}\vec{v}'$. The only correct way how to solve the above problem is to introduce into the action new local term of the form $\vec{v}'\vec{\nabla}^2\vec{v}'$ (isotropic case) [10]. In the anisotropic case we have to introduce additional counterterms $\vec{v}'(\vec{n}\cdot\nabla)^2\vec{v}'$, $(\vec{n}\cdot\vec{v}')\vec{\nabla}^2(\vec{n}\cdot\vec{v}')$ and $(\vec{n}\cdot\vec{v}')(\vec{n}\cdot\vec{\nabla})^2(\vec{n}\cdot\vec{v}')$. In the paper [10, 12] a double-expansion method with a simultaneous deviation $2\delta = d - 2$ from the spatial dimension $d = 2$ and also a deviation ϵ from the k^2 form of the forcing pair correlation function proportional to $k^{2-2\delta-2\epsilon}$ was proposed. We shall follow the formulation founded on two-expansion parameters in present paper.

In this case kernel (4) corresponding to the correlation matrix $D_{ij}(\vec{x}_1 - \vec{x}_2, t_2 - t_1)$ in action (6) is replaced by the expression

$$\begin{aligned} \tilde{D}_{ij}(\vec{k}) &= g_{10}\nu_0^3 k^{2-2\delta-2\epsilon}[(1+\alpha_{10}\xi_k^2)P_{ij}(\vec{k}) + \alpha_{20}R_{ij}(\vec{k})] \\ &+ g_{20}\nu_0^3 k^2[(1+\alpha_{30}\xi_k^2)P_{ij}(\vec{k}) + (\alpha_{40}+\alpha_{50}\xi_k^2)R_{ij}(\vec{k})], . \end{aligned} \tag{8}$$

Here P_{ij}, R_{ij} are given by relations (5), g_{20}, α_{30}, α_{40} and α_{50} are new parameters of the model. One can see that in such formulation the counterterm $\vec{v}'\vec{\nabla}^2\vec{v}'$ and all anisotropic terms can be taken into account by renormalization of the coupling constant g_{20}, and the parameters α_{30}, α_{40} and α_{50}.

The action (6) with kernel $\tilde{D}_{ij}(\vec{k})$ (8) is given in the form convenient for the realization of the quantum field perturbation analysis with standard Feynman diagram technique. From the quadratic part of the action one obtains the matrix of bare propagators (in the wave-number - frequency representation):

$$=< v_i v_j >_0 \equiv \Delta_{ij}^{vv}(\vec{k},\omega_k),$$

$$=< v_i v'_j >_0 \equiv \Delta_{ij}^{vv'}(\vec{k},\omega_k),$$

$$=< v'_i v'_j >_0 \equiv \Delta_{ij}^{v'v'}(\vec{k},\omega_k) = 0,$$

where

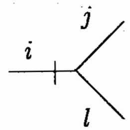
$$\Delta_{ij}^{vv}(\vec{k},\omega_k) = -\frac{K_3}{K_1K_2}P_{ij} + \frac{1}{K_1(K_2+\tilde{K}(1-\xi_k^2))}\left[\frac{\tilde{K}K_3}{K_2}+\frac{\tilde{K}(K_3+K_4(1-\xi_k^2))}{(K_1+\tilde{K}(1-\xi_k^2))}-K_4\right]R_{ij}$$

$$\Delta_{ij}^{vv'}(\vec{k},\omega_k) = \frac{1}{K_2}P_{ij} - \frac{\tilde{K}}{K_2(K_2+\tilde{K}(1-\xi_k^2))}R_{ij}\,, \tag{9}$$

with

$$\begin{aligned} K_1 &= i\omega_k + \nu_0 k^2 + \nu_0\chi_{10}(\vec{n}\cdot\vec{k})^2\,, \\ K_2 &= -i\omega_k + \nu_0 k^2 + \nu_0\chi_{10}(\vec{n}\cdot\vec{k})^2\,, \\ K_3 &= -g_{10}\nu_0^3 k^{2-2\delta-2\epsilon}(1+\alpha_{10}\xi_k^2) - g_{20}\nu_0^3k^2(1+\alpha_{30}\xi_k^2)\,, \\ K_4 &= -g_{10}\nu_0^3k^{2-2\delta-2\epsilon}\alpha_{20} - g_{20}\nu_0^3k^2(\alpha_{40}+\alpha_{50}\xi_k^2)\,, \\ \tilde{K} &= \nu_0\chi_{20}k^2 + \nu_0\chi_{30}(\vec{n}\cdot\vec{k})^2\,. \end{aligned} \tag{10}$$

The propagators are written in the form suitable also for strong anisotropy when parameters α_{i0} are not small. In case of weak anisotropy one can make the expansion and work only with linear terms with respect to all parameters which characterize anisotropy. The interaction vertex in our model is given by the expression



$$\equiv V_{ijl} = i(k_j\delta_{il} + k_l\delta_{ij})$$

Here wave vector $\vec{k}$ corresponds to the field $\vec{v}$'. Now one can use the above introduced Feynman rules for computation all needed graphs.

3 RG-analysis and Stability of the Fixed Point

Using the standard analysis of the quantum field theory (see e.g. [4, 5, 13, 14]) one can find out that the UV divergences of one-particle-irreducible Green functions $< vv' >_{IR}$ and $< v'v' >_{IR}$ are quadratic in the wave vector (possible UV-divergences proportional to the frequency are absent because of Galilean invariance). The last one takes place only in the case when dimension of the space is two. All needed terms for removing of the divergences are included in the action (6) with (7) and kernel (8). This leads to the fact that our model is multiplicatively renormalizable. Thus, one can immediately write down the renormalized action (wave-number - frequency

representation with $\vec{\nabla} \to i\vec{k}, \partial_t \to -i\omega_k$ and all needed integrations and summations are assumed):

$$\begin{aligned} S^R(v,v') = {}& \frac{1}{2}v'_i\Big[g_1\nu^3\mu^{2\epsilon}k^{2-2\delta-2\epsilon}\left(\left(1+\alpha_1\xi_k^2\right)P_{ij}+\alpha_2R_{ij}\right) \\ &+ g_2\nu^3\mu^{-2\delta}k^2\left(\left(Z_5+Z_6\alpha_3\xi_k^2\right)P_{ij}+\left(Z_7\alpha_4+Z_8\alpha_5\xi_k^2\right)R_{ij}\right)\Big]v'_j \\ &+ v'_i\Big[(i\omega_k - Z_1\nu k^2)P_{ij} - \nu k^2\left(Z_2\chi_1\xi_k^2P_{ij}+\left(Z_3\chi_2+Z_4\chi_3\xi_k^2\right)R_{ij}\right)\Big]v_j \\ &+ \frac{1}{2}v'_iv_jv_lV_{ijl}\,, \end{aligned} \tag{11}$$

where μ is a scale setting parameter with the same canonical dimension as the wave number. Quantities g_i, χ_i, α_3, α_4, α_5 and ν are the renormalized counterparts of bare ones and Z_i are renormalization constants, which are expressed via UV divergent parts of the functions $< vv' >_{IR}$ and $< v'v' >_{IR}$. Their general form in one loop approximation is

$$Z_i = 1 - F_i\,\mathrm{Poles}_i^{\delta,\epsilon}\,. \tag{12}$$

In standard MS scheme the amplitudes F_i are only some functions of g_i, χ_i, α_3, α_4, α_5 and are independent of d, ϵ. The pure divergent parts of Feynman diagrams are expressed by terms $\mathrm{Poles}_i^{\delta,\epsilon}$ containing only divergent parts which are given by linear combinations of the poles $\frac{1}{\epsilon}$, $\frac{1}{\delta}$ and $\frac{1}{2\epsilon+\delta}$ (for $\delta \to 0, \epsilon \to 0$). The amplitudes $F_i = F_i^1F_i^2$ are an product of two multipliers F_i^1, F_i^2. One of them, say, F_i^1 is an multiplier originates from divergent part of Feynman diagrams, and the second one F_i^2 is connected only with tensor nature of the diagrams. We explain that on following simple example. Consider the integral

$$I(\mathbf{k},\mathbf{n}) \equiv n_in_jk_lk_m\int d^d\mathbf{q}\frac{1}{q^{2+4\delta}}\Big(\frac{q_iq_jq_lq_m}{q^4} - \frac{\delta_{ij}q_lq_m+\delta_{il}q_jq_m+\delta_{jl}q_iq_m}{q^2}\Big)$$

(summations over repeated indices are implied).

We have

$$I(\mathbf{k},\mathbf{n}) \equiv n_in_jk_lk_mS_{ijlm}\int dq\frac{1}{q^{1+2\delta}}\,,$$

where

$$S_{ijlm} = \frac{S_d}{d(d+2)}(\delta_{ij}\delta_{lm}+\delta_{il}\delta_{jm}+\delta_{im}\delta_{jl} - (d+2)(\delta_{ij}\delta_{lm}+\delta_{il}\delta_{jm}+\delta_{im}\delta_{jl}))\,,$$

and $S_d = 2\pi^{d/2}/\Gamma(d/2)$ is the surface of unit d-dimensional sphere. The purely UV divergent part manifests itself as pole in $2\delta = d-2$, therefore, we find

$$\text{UV div. part of } I = \frac{1}{2\delta}(F_1^2k^2 + F_2^2\mathbf{nk}),$$

where $F_1^2 = F_2^2/2 = (1-d)S_d/d(d+2)$.

In standard MS scheme one puts $d=2$ in F_1^2, F_2^2, therefore d-dependence of these multipliers is ignored. For the theories with vector fields, and, consequently, with tensor diagrams, where the value of fixed points and/or their stability depend on the dimension d, the procedure, which eliminates the dependence of multipliers of type F_1^2, F_2^2 on d, is not quite correct because one is not able to control the stability of fixed point when drives to the $d=3$. In the analysis of Feynman diagrams we propose slightly to modify the MS scheme in such a way that we keep the d-dependence in F in (12). The following calculations of RG functions ($\beta-$ functions and anomalous dimensions) allow to arrive to the results, which are in the qualitative agreement with the results obtained recently in the framework of simple analytical regularization scheme [], i.e. we obtain fixed point which is not stable for $d=2$, but, whose stability is restored for an critical dimension $2<d_c<3$.

The transition from the action (6) into the renormalized one (11) is given by the introduction of the following renormalization constants Z:

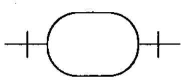
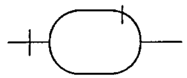
$$\nu_0 = \nu Z_\nu\,, \quad g_{10} = g_1\mu^{2\epsilon}Z_{g_1}\,, \quad g_{20} = g_2\mu^{-2\delta}Z_{g_2}\,, \quad \chi_{i0} = \chi_i Z_{\chi_i}\,, \quad \alpha_{(i+2)0} = \alpha_{i+2}Z_{\alpha_{i+2}}\,, \tag{13}$$

where $i=1,2,3$. By comparison of the corresponding terms in the action (11) with definitions of the renormalization constants Z for the charges (13) one can immediately write relations between them. Namely, we have

$$\begin{aligned} Z_\nu &= Z_1\,, \\ Z_{g_1} &= Z_1^{-3}\,, \\ Z_{g_2} &= Z_5 Z_1^{-3}\,, \\ Z_{\chi_i} &= Z_{1+i}Z_1^{-1}\,, \\ Z_{\alpha_{i+2}} &= Z_{i+5}Z_5^{-1}\,, \end{aligned} \tag{14}$$

where again $i=1,2,3$.

In one-loop approximation divergent one-irreducible Green function $< v'v >_{IR}$ and $< v'v' >_{IR}$ are represented by the Faynman graphs:

$< v'v' >_{IR}=$  , $< v'v >_{IR}=$  (15)

The divergent parts of these diagrams $\Gamma^{v'v'}$, $\Gamma^{v'v}$ have the structure:

$$\begin{aligned} \Gamma^{v'v'} &= \frac{1}{2}\nu^3 A \\ &\times \left[\frac{g_1^2}{4\epsilon+2\delta}\left(a_1\delta_{ij}k^2 + a_2\delta_{ij}(\vec{n}\cdot\vec{k})^2 + a_3 n_i n_j k^2 + a_4 n_i n_j (\vec{n}\cdot\vec{k})^2\right)\right. \\ &+ \frac{g_1 g_2}{2\epsilon}\left(b_1\delta_{ij}k^2 + b_2\delta_{ij}(\vec{n}\cdot\vec{k})^2 + b_3 n_i n_j k^2 + b_4 n_i n_j (\vec{n}\cdot\vec{k})^2\right) \\ &+ \left.\frac{g_2^2}{-2\delta}\left(c_1\delta_{ij}k^2 + c_2\delta_{ij}(\vec{n}\cdot\vec{k})^2 + c_3 n_i n_j k^2 + c_4 n_i n_j (\vec{n}\cdot\vec{k})^2\right)\right], \end{aligned}$$

$$\begin{aligned} \Gamma^{v'v} &= -\nu A \\ &\times \left[\frac{g_1}{2\epsilon}\left(d_1\delta_{ij}k^2 + d_2\delta_{ij}(\vec{n}\cdot\vec{k})^2 + d_3 n_i n_j k^2 + d_4 n_i n_j (\vec{n}\cdot\vec{k})^2\right)\right. \\ &+ \left.\frac{g_2}{-2\delta}\left(e_1\delta_{ij}k^2 + e_2\delta_{ij}(\vec{n}\cdot\vec{k})^2 + e_3 n_i n_j k^2 + e_4 n_i n_j (\vec{n}\cdot\vec{k})^2\right)\right], \end{aligned} \tag{16}$$

where parameter A and functions a_i, b_i, c_i, d_i and e_i are given in the Appendix I ($i=1...4$). The counterterms are built up from these divergent parts which lead to the following equations for renormalization constants

$$\begin{aligned} Z_1 &= 1 - A\left(\frac{g_1}{2\epsilon}d_1 + \frac{g_2}{-2\delta}e_1\right), \\ Z_{1+i} &= 1 - \frac{A}{\chi_i}\left(\frac{g_1}{2\epsilon}d_{1+i} + \frac{g_2}{-2\delta}e_{1+i}\right), \\ Z_5 &= 1 + \frac{A}{2}\left(\frac{g_1^2}{g_2}\frac{a_1}{4\epsilon+2\delta} + \frac{g_1}{2\epsilon}b_1 + \frac{g_2}{-2\delta}c_1\right), \\ Z_{5+i} &= 1 + \frac{A}{2\alpha_{i+2}}\left(\frac{g_1^2}{g_2}\frac{a_{i+1}}{4\epsilon+2\delta} + \frac{g_1}{2\epsilon}b_{i+1} + \frac{g_2}{-2\delta}c_{i+1}\right), \\ i &= 1,2,3\,. \end{aligned} \tag{17}$$

From these expressions one can define corresponding anomalous dimensions $\gamma_i = \mu\partial_\mu \ln Z_i$ for all renormalization constants Z_i (logarithmic derivative $\mu\partial_\mu$ is taken when all bare parameters are fixed). The β-functions for all invariant charges (running coupling constants g_1, g_2, and parameters χ_i, α_{i+2}) where $i=1,2,3$ are given by the following relations: $\beta_{g_i} = \mu\partial_\mu g_i$ ($i=1,2$), $\beta_{\chi_i} = \mu\partial_\mu\chi_i$ and $\beta_{\alpha_{i+2}} = \mu\partial_\mu\alpha_{i+2}$ ($i=1,2,3$). Now using the equations (14) and definitions given above one can immediately write the β-functions in the form:

$$\begin{aligned} \beta_{g_1} &= -g_1(2\epsilon+\gamma_{g_1}) = g_1(-2\epsilon+3\gamma_1)\,, \\ \beta_{g_2} &= g_2(2\delta-\gamma_{g_2}) = g_2(2\delta+3\gamma_1-\gamma_5)\,, \\ \beta_{\chi_i} &= -\chi_i\gamma_{\chi_i} = -\chi_i(\gamma_{i+1}-\gamma_1)\,, \\ \beta_{\alpha_{i+2}} &= -\alpha_{i+2}\gamma_{\alpha_{i+2}} = -\alpha_{i+2}(\gamma_{i+5}-\gamma_5)\,, \quad i=1,2,3\,, \end{aligned} \tag{18}$$

where

$$\begin{aligned} \gamma_1 &= A(g_1 d_1 + g_2 e_1)\,, \\ \gamma_{i+1} &= \frac{A}{\chi_i}(g_1 d_{i+1} + g_2 e_{i+1})\,, \\ \gamma_5 &= -\frac{A}{2}\left(\frac{g_1^2}{g_2}a_1 + g_1 b_1 + g_2 c_1\right), \\ \gamma_{i+5} &= -\frac{A}{2\alpha_{i+2}}\left(\frac{g_1^2}{g_2}a_{i+1} + g_1 b_{i+1} + g_2 c_{i+1}\right), \quad i=1,2,3\,. \end{aligned} \tag{19}$$

By substitution of the functions γ_i (19) into the expressions for the β-functions one obtains:

$$
\begin{aligned}
\beta_{g_1} &= g_1(-2\epsilon + 3A(g_1 d_1 + g_2 e_1))\,, \\
\beta_{g_2} &= g_2\left[2\delta + 3A(g_1 d_1 + g_2 e_1) + \frac{A}{2}\left(\frac{g_1^2}{g_2}a_1 + g_1 b_1 + g_2 c_1\right)\right], \\
\beta_{\chi_i} &= -A\left[(g_1 d_{i+1} + g_2 e_{i+1}) - \chi_i(g_1 d_1 + g_2 e_1)\right], \\
\beta_{\alpha_{i+2}} &= -\frac{A}{2}\left[-\left(\frac{g_1^2}{g_2}a_{i+1} + g_1 b_{i+1} + g_2 c_{i+1}\right) + \alpha_{i+2}\left(\frac{g_1^2}{g_2}a_1 + g_1 b_1 + g_2 c_1\right)\right], \\
i &= 1,2,3\,.
\end{aligned}
\tag{20}
$$

The fixed point of the RG-equations is defined by the system of eight equations

$$\beta_C(C_*) = 0\,, \tag{21}$$

where we denote $C = \{g_1, g_2, \chi_i, \alpha_{i+2}\}, i = 1,2,3$ and C_* is corresponding value for fixed point. The IR stability of the fixed point is determined by the positive real parts of the eigenvalues of the matrix

$$\omega_{lm} = \left(\frac{\partial \beta_{C_l}}{\partial C_m}\right)_{C=C_*} \qquad l,\, m = 1, ...8. \tag{22}$$

Now we have all necessary tools at hand to investigate the fixed point and its stability. In *the isotropic case* all parameters which are connected to anisotropy equal zero and one can immediately find the Kolmogorov's fixed point, namely:

$$
\begin{aligned}
g_{1*} &= \frac{1}{A}\,\frac{8(2+d)\epsilon(2\epsilon - 3d(\delta+\epsilon) + d^2(3\delta + 2\epsilon))}{9(-1+d)^3 d(1+d)(\delta+\epsilon)}\,, \\
g_{2*} &= \frac{1}{A}\,\frac{8(-4 - 2d + 2d^2 + d^3)\epsilon^2}{9(-1+d)^3 d(1+d)(\delta+\epsilon)}\,,
\end{aligned}
\tag{23}
$$

and the corresponding ω_{ij} matrix have following eigenvalues

$$
\begin{aligned}
\lambda_{1,2} &= \frac{1}{6d(d-1)}\Big\{6d\delta(d-1) + 4\epsilon(2 - 3d + 2d^2) \\
&\pm \Big[(6d\delta(1-d) - 4\epsilon(2-3d+2d^2))^2 \\
&- 12d(d-1)\epsilon(12d\delta(d-1) + 4\epsilon(2-3d+2d^2))\Big]^{\frac{1}{2}}\Big\}.
\end{aligned}
\tag{24}
$$

By detail analysis of these eigenvalues we know that in the interesting region of parameters, namely $\epsilon > 0$ and $\delta \geq 0$ (it correspondence to $d \geq 2$) the above computed fixed point is stable. In the limit $d = 2$ this fixed point is in agreement with that given in [10, 15].

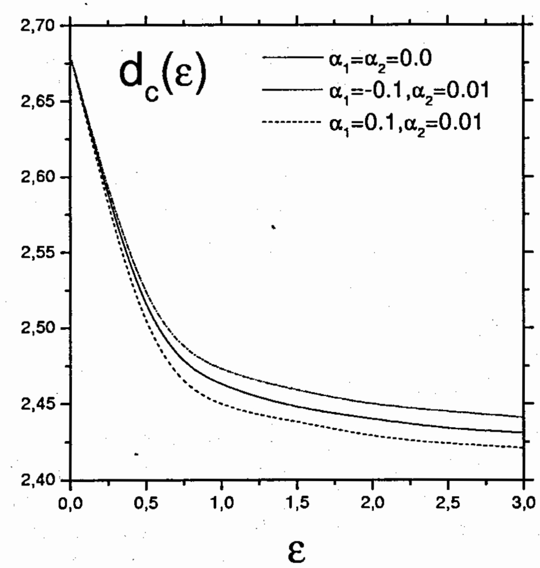


Figure 1: Dependence of the critical dimension d_c on the parameter ϵ.

When one consider *the weak anisotropy case* situation becomes more complicated because of necessity to use all system of β-functions if one wants to analyze the stability of the fixed point. It is also possible to find analytical expressions for fixed point in this more complicated case because in the weak anisotropy limit it is enough to calculate linear corrections of α_1 and α_2 to all quantities (see in Appendix II).

To investigate the stability of the fixed point it is necessary to apply it in the stability matrix. Analysis of this matrix shows us that it can be written in the block-diagonal form: $(6 \times 6)(2 \times 2)$). The (2×2) part is given by the β-functions of the parameters α_5 and χ_3 and, namely, this block is responsible for existence of the critical dimension d_c because one of its eigenvalue, say $\lambda_1(\epsilon, d, \alpha_1, \alpha_2)$, has solution $d_c \in \langle 2, 3\rangle$ of the equation $\lambda_1(\epsilon, d_c, \alpha_1, \alpha_2) = 0$ for defined value of $\epsilon, \alpha_1, \alpha_2$. The following procedure was used for finding the fixed point [15]: First we use the isotropic solution to g_1 and g_2 to compute expressions for α_{i+2} and χ_i, $i = 1,2,3$. From equations $\beta_{\alpha_5} = 0$ and $\beta_{\chi_3} = 0$ one can immediately find that $\alpha_{5*} = 0$ and $\chi_{3*} = 0$. After this we can calculate expressions for fixed point of the parameters α_{i+2} and χ_i, $i = 1,2$. In the end we come back to equations for g_1 and g_2, namely $\beta_{g_1} = 0$ and $\beta_{g_2} = 0$ and find linear corrections of α_1 and α_2 to the fixed point. Corresponding expressions for the fixed point and corresponding eigenvalue of the stability matrix responsible for instability are given in the Appendix II.

From numerical analysis of the stability matrix one can find that in some region of space dimensions d the stability is lost by influence of weak anisotropy. On the other hand, the critical dimension d_c arises when going from dimension $d = 2$ to $d = 3$. For energy pumping regime ($\epsilon = 2$) we found the critical dimension $d_c = 2.44$. This value corresponds to the $\alpha_1 = \alpha_2 = 0$. This is the case when one supposes only the fact of anisotropy. Using nonzero values of α_1 and α_2 one can also estimate the influence of these parameters on the critical dimension d_c. It is interesting to calculate the dependence of the d_c on the parameter ϵ too. In Fig. 1 is presented this dependence and dependence on small values of α_1 and α_2. As one can see from this figure d_c increases when $\epsilon \to 0$ and also parameters α_1 and α_2 give small corrections to d_c.

4 Conclusion

We have investigated the influence of the weak anisotropy on the fully developed turbulence using quantum field RG double expansion method and introduced modified minimal substraction scheme which differs from traditional one by tensor part in which we left dimensional dependence. We affirm that such modified approach is correct when one needs compute dimensional dependence of the important quantities and is necessary for restoration of the stability regimes when one makes transition from dimension $d = 2$ to $d = 3$. We have derived analytical expressions for the fixed point in the limit of weak anisotropy and found the equation which manage the stability of this point as function of the parameters ϵ, α_1 and α_2 and allows one to calculate the critical dimension d_c. Below this dimension the fixed point is unstable. In the limit case of infinitesimally small anisotropy ($\alpha_1 \to 0$ and $\alpha_2 \to 0$) and in the energy pumping regime ($\epsilon = 2$) we found the critical dimension $d_c = 2.44$. We have investigated also the ϵ-dependence of the d_c for different values of the anisotropy parameters α_1, α_2. The concrete results one can see in the Fig. 1.

Appendix I

The explicit form of the parameter A and functions a_i, b_i, c_i, d_i and e_i ($i = 1...4$) from the divergent parts of the diagrams (15):

$$
\begin{aligned}
a_1 &= \frac{1}{2d(2+d)(4+d)(6+d)}\\
&\times [-48 - 20d + 70d^2 + 30d^3 - 21d^4 - 10d^5 - d^6\\
&+ \alpha_2(24 + 16d - 22d^2 - 16d^3 - 2d^4) + \alpha_1(24 + 52d - 4d^2 - 50d^3 - 20d^4 - 2d^5)\\
&+ \chi_1(-36 - 78d + 6d^2 + 75d^3 + 30d^4 + 3d^5)\\
&+ \chi_2(-36 - 24d + 33d^2 + 24d^3 + 3d^4) + \chi_3(-36 - 9d + 36d^2 + 9d^3)],\\
a_2 &= \frac{1}{4d(2+d)(4+d)(6+d)}\\
&\times [\alpha_1(-96 - 64d + 88d^2 + 64d^3 + 8d^4)\\
&+ \alpha_2(-96 - 64d + 124d^2 + 82d^3 - 26d^4 - 18d^5 - 2d^6)
\end{aligned}
$$

$$
\begin{aligned}
&+ \chi_1(144 + 96d - 132d^2 - 96d^3 - 12d^4)\\
&+ \chi_2(144 + 96d - 186d^2 - 123d^3 + 39d^4 + 27d^5 + 3d^6)\\
&+ \chi_3(72 + 6d - 87d^2 - 9d^3 + 15d^4 + 3d^5)],\\
a_3 &= a_2,\\
a_4 &= \frac{6\chi_3(1-d^2)}{(2+d)(6+d)},\\
b_1 &= \frac{1}{d(2+d)(4+d)(6+d)}\\
&\times [-48 - 20d + 70d^2 + 30d^3 - 21d^4 - 10d^5 - d^6 + \alpha_5(12 + 3d - 12d^2 - 3d^3)\\
&+ (\alpha_2 + \alpha_4)(12 + 8d - 11d^2 - 8d^3 - d^4)\\
&+ (\alpha_1 + \alpha_3)(12 + 26d - 2d^2 - 25d^3 - 10d^4 - d^5)\\
&+ \chi_1(-36 - 78d + 6d^2 + 75d^3 + 30d^4 + 3d^5)\\
&+ \chi_2(-36 - 24d + 33d^2 + 24d^3 + 3d^4) + \chi_3(-36 - 9d + 36d^2 + 9d^3)],\\
b_2 &= \frac{1}{2d(2+d)(4+d)(6+d)}\\
&\times [(\alpha_1 + \alpha_3)(-48 - 32d + 44d^2 + 32d^3 + 4d^4)\\
&+ \alpha_5(-24 - 2d + 29d^2 + 3d^3 - 5d^4 - d^5)\\
&+ (\alpha_2 + \alpha_4)(-48 - 32d + 62d^2 + 41d^3 - 13d^4 - 9d^5 - d^6)\\
&+ \chi_1(144 + 96d - 132d^2 - 96d^3 - 12d^4)\\
&+ \chi_2(144 + 96d - 186d^2 - 123d^3 + 39d^4 + 27d^5 + 3d^6)\\
&+ \chi_3(72 + 6d - 87d^2 - 9d^3 + 15d^4 + 3d^5)],\\
b_3 &= b_2,\\
b_4 &= \frac{4(d^2-1)(\alpha_5 - 3\chi_3)}{(2+d)(6+d)},\\
c_1 &= \frac{1}{2d(2+d)(4+d)(6+d)}\\
&\times [-48 - 20d + 70d^2 + 30d^3 - 21d^4 - 10d^5 - d^6\\
&+ \alpha_5(24 + 6d - 24d^2 - 6d^3) + \alpha_4(24 + 16d - 22d^2 - 16d^3 - 2d^4)\\
&+ \alpha_3(24 + 52d - 4d^2 - 50d^3 - 20d^4 - 2d^5)\\
&+ \chi_1(-36 - 78d + 6d^2 + 75d^3 + 30d^4 + 3d^5)\\
&+ \chi_2(-36 - 24d + 33d^2 + 24d^3 + 3d^4) + \chi_3(-36 - 9d + 36d^2 + 9d^3)],\\
c_2 &= \frac{1}{4d(2+d)(4+d)(6+d)}\\
&\times [\alpha_3(-96 - 64d + 88d^2 + 64d^3 + 8d^4)\\
&+ \alpha_5(-48 - 4d + 58d^2 + 6d^3 - 10d^4 - 2d^5)\\
&+ \alpha_4(-96 - 64d + 124d^2 + 82d^3 - 26d^4 - 18d^5 - 2d^6)\\
&+ \chi_1(144 + 96d - 132d^2 - 96d^3 - 12d^4)
\end{aligned}
$$

$$
\begin{aligned}
&+ \ \chi_2(144+96d-186d^2-123d^3+39d^4+27d^5+3d^6) \\
&+ \ \chi_3(72+6d-87d^2-9d^3+15d^4+3d^5)]\,, \\
c_3 &= c_2\,, \\
c_4 &= \frac{(d^2-1)(4\alpha_5-6\chi_3)}{(2+d)(6+d)}\,, \\
d_1 &= \frac{1}{4d(2+d)(4+d)(6+d)} \\
&\times \ [24d-14d^2-33d^3+13d^4+9d^5+d^6+\alpha_2(12-4d-13d^2+4d^3+d^4) \\
&+ \ \alpha_1(-12-20d+3d^2+19d^3+9d^4+d^5) \\
&+ \ \chi_1(36+42d-18d^2-40d^3-18d^4-2d^5) \\
&+ \ \chi_2(-12+16d+15d^2-16d^3-3d^4)+\chi_3(6+9d-6d^2-9d^3)]\,, \\
d_2 &= \frac{1}{8d(2+d)(4+d)(6+d)} \\
&\times \ [\alpha_1(-48+16d+52d^2-16d^3-4d^4) \\
&+ \ \alpha_2(48+80d-60d^2-96d^3+10d^4+16d^5+2d^6) \\
&+ \ \chi_1(48-64d-60d^2+64d^3+12d^4) \\
&+ \ \chi_2(-48-104d+62d^2+127d^3-11d^4-23d^5-3d^6) \\
&+ \ \chi_3(-2d+7d^2+5d^3-7d^4-3d^5)]\,, \\
d_3 &= \frac{1}{8d(2+d)(4+d)(6+d)} \\
&\times \ [\alpha_1(48+56d-40d^2-56d^3-8d^4)+\alpha_2(-48-56d+40d^2+56d^3+8d^4) \\
&+ \ \chi_1(-48-104d+32d^2+104d^3+16d^4) \\
&+ \ \chi_2(48+32d-38d^2-25d^3-9d^4-7d^5-d^6) \\
&+ \ \chi_3(22d-d^2-21d^3+d^4-d^5)]\,, \\
d_4 &= \frac{\chi_3(-10+d+10d^2-d^3)}{2(2+d)(6+d)}\,, \\
e_1 &= \frac{1}{4d(2+d)(4+d)(6+d)} \\
&\times \ [24d-14d^2-33d^3+13d^4+9d^5+d^6+3d\alpha_5(-1+d^2) \\
&+ \ \alpha_4(12-4d-13d^2+4d^3+d^4) \\
&+ \ \alpha_3(-12-20d+3d^2+19d^3+9d^4+d^5) \\
&+ \ \chi_1(36+42d-18d^2-40d^3-18d^4-2d^5) \\
&+ \ \chi_2(-12+16d+15d^2-16d^3-3d^4)+\chi_3(6+9d-6d^2-9d^3)]\,, \\
e_2 &= \frac{1}{8d(2+d)(4+d)(6+d)} \\
&\times \ [\alpha_3(-48+16d+52d^2-16d^3-4d^4)+\alpha_5(-8d^2-2d^3+8d^4+2d^5) \\
&+ \ \alpha_4(48+80d-60d^2-96d^3+10d^4+16d^5+2d^6)
\end{aligned}
$$

$$
\begin{aligned}
&+ \ \chi_1(48-64d-60d^2+64d^3+12d^4) \\
&+ \ \chi_2(-48-104d+62d^2+127d^3-11d^4-23d^5-3d^6) \\
&+ \ \chi_3(-2d+7d^2+5d^3-7d^4-3d^5)]\,, \\
e_3 &= \frac{1}{8d(2+d)(4+d)(6+d)} \\
&\times \ [24d\alpha_5(-1+d^2)+\alpha_3(48+56d-40d^2-56d^3-8d^4) \\
&+ \ \alpha_4(-48-56d+40d^2+56d^3+8d^4) \\
&+ \ \chi_1(-48-104d+32d^2+104d^3+16d^4) \\
&+ \ \chi_2(48+32d-38d^2-25d^3-9d^4-7d^5-d^6) \\
&+ \ \chi_3(22d-d^2-21d^3+d^4-d^5)]\,, \\
e_4 &= \frac{6\alpha_5(1-d^2)+\chi_3(-10+d+10d^2-d^3)}{2(2+d)(6+d)}\,, \\
A &= \frac{S_d}{(2\pi)^d(d^2-1)}\,,
\end{aligned}
$$

where S_d is d-dimensional sphere given by the following relation:

$$S_d = \frac{2\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})}\,.$$

Appendix II

We present here explicit analytical expressions for the fixed point in the weak anisotropy limit and also equation which drives its stability.

The basic form of the fixed point is:

$$
\begin{aligned}
g_{1*} &= g_{10*}+g_{11*}\alpha_1+g_{12*}\alpha_2\,, \\
g_{2*} &= g_{20*}+g_{21*}\alpha_1+g_{22*}\alpha_2\,, \\
\alpha_{3*} &= e_{11}\alpha_1+e_{12}\alpha_2\,, \\
\alpha_{4*} &= e_{21}\alpha_1+e_{22}\alpha_2\,, \\
\chi_{1*} &= e_{31}\alpha_1+e_{32}\alpha_2\,, \\
\chi_{2*} &= e_{41}\alpha_1+e_{42}\alpha_2\,, \\
\alpha_{5*} &= 0\,, \\
\chi_{3*} &= 0\,,
\end{aligned}
$$

where g_{10*} and g_{20*} are defined in eq.(23) and $g_{11*}, g_{12*}, g_{21*}, g_{22*}$ and $e_{ij}, i=1,2,3,4$, $j=1,2$ are functions only of the dimension d and parameters ϵ and δ. They have the following form

$$g_{11*} = \frac{g_{11n}}{g_{11d}}\,, g_{12*} = \frac{g_{12n}}{g_{12d}}\,, g_{21*} = \frac{g_{21n}}{g_{22d}}\,, g_{22*} = \frac{g_{22n}}{g_{22d}}\,,$$

$$
\begin{aligned}
e_{11} &= \frac{e_{11n}}{e_d}, e_{12} = \frac{e_{12n}}{e_d}, e_{21} = \frac{e_{21n}}{e_d}, e_{22} = \frac{e_{22n}}{e_d}, \\
e_{31} &= \frac{e_{31n}}{g_s e_d}, e_{32} = \frac{e_{32n}}{g_s e_d}, e_{41} = \frac{e_{41n}}{g_s e_d}, e_{42} = \frac{e_{42n}}{g_s e_d},
\end{aligned}
$$

where

$$
\begin{aligned}
g_{11n} &= 3(d^2-1)g_{10*}(d^6(g_{10*}+g_{20*})((5e_{31}-3)g_{10*}-3e_{11}g_{20*}+5e_{31}g_{20*}) \\
&+ 3d^5(g_{10*}+g_{20*})((-2+3e_{31}+2e_{41})g_{10*}-(2e_{11}+e_{21}-3e_{31}-2e_{41})g_{20*}) \\
&- 8(g_{10*}+g_{20*})((-1+3e_{31}-e_{41})g_{10*}-(e_{11}-e_{21}-3e_{31}+e_{41})g_{20*}) \\
&+ d^3(-((g_{10*}+g_{20*})((-4+9e_{31}-6e_{41})g_{10*} \\
&+ (-4e_{11}+3(e_{21}+3e_{31}-2e_{41}))g_{20*})) \\
&+ 8\delta((-5+10e_{31}+3e_{41})g_{10*}-(5e_{11}+e_{21}-10e_{31}-3e_{41})g_{20*})) \\
&+ 2d(-((g_{10*}+g_{20*})((-1+6e_{41})g_{10*}-(e_{11}+3e_{21}-6e_{41})g_{20})) \\
&+ 16\delta((-1+3e_{31}-e_{41})g_{10*}-(e_{11}-e_{21}-3e_{31}+e_{41})g_{20*})) \\
&+ d^2((g_{10*}+g_{20*})((-15+34e_{31})g_{10*}+(-15e_{11}+5e_{21}+34e_{31})g_{20*}) \\
&+ 16\delta((-4+9e_{31}+2e_{41})g_{10*} \\
&+ (-4e_{11}+9e_{31}+2e_{41})g_{20*}))+d^4(8\delta(-g_{10*}+2e_{31}g_{10*}-e_{11}g_{20*}+2e_{31}g_{20*}) \\
&- (g_{10*}+g_{20*})((-10+15e_{31}+8e_{41})g_{10*} \\
&+ (-10e_{11}-3e21+15e_{31}+8e_{41})g_{20}))), \\
g_{11d} &= 2d(4+d)(-15d^6(g_{10*}+g_{20*})^2+6d^7(g_{10*}+g_{20*})^2 \\
&+ 2(g_{10*}+g_{20*})(16\epsilon-3(g_{10*}+g_{20*}))+4d^4(\epsilon(g_{10*}-2g_{20*}) \\
&+ 6(g_{10*}+g_{20*})^2+3\delta(2g_{10*}+g_{20*}))+d^5(3(g_{10*}+g_{20*})^2+12\delta(2g_{10*}+g_{20*}) \\
&- 4\epsilon(g_{10*}+4g_{20*}))-4d^3(6(g_{10*}+g_{20*})^2-3\epsilon(g_{10*}+4g_{20*}) \\
&+ \delta(8\epsilon+9(2g_{10*}+g_{20*})))+d(15(g_{10*}+g_{20*})^2-8\epsilon(g_{10*}+4g_{20*}) \\
&+ \delta(-128\epsilon+24(2g_{10*}+g_{20*})))-d^2(4\delta(32\epsilon+6g_{10*}+3g_{20*}) \\
&+ 3((g_{10*}+g_{20*})^2+4\epsilon(3g_{10*}+2g_{20*})))), \\
g_{21n} &= -((-1+d^2)(-3(-4-d+4d^2+d^3)g_{10*}(2(-2+d^2)g_{10*} \\
&- (4-3d+d^2)g_{20*})\times \\
&\times ((6e_{31}+d^2(-1+2e_{31})-2(1+e_{41})+3d(-1+2e_{31}+e_{41}))g_{10*} \\
&- ((2+3d+d^2)e_{11}+(-2+d)e_{21}-6e_{31}-6de_{31}-2d^2e_{31}+2e_{41}-3de_{41})g_{20*}) \\
&+ d(4+d)((-4+6e_{31}+d^2(-2+3e_{31})+6e_{41}+d(-8+12e_{31}+3e_{41}))g_{10*}^2 \\
&+ (2+d^2(1-2e_{11})-4e_{11}-4e_{21}-6e_{31}+d(1-8e_{11}-2e_{21}+6e_{31}-3e_{41}) \\
&+ 18e_{41})g_{10*}g_{20*}+((2+d+d^2)e_{11}+(-10+d)e_{21}-3(4e_{31}+2de_{31}+d^2e_{31} \\
&- 4e_{41}+2de_{41}))g_{20*}^2)(-8(2+d)\epsilon+3(-1+d)^2(1+d)(2g_{10*}+g_{20*})))), \\
g_{21d} &= 3(-1+d)^2d(4+5d+d^2)(-4-d+4d^2+d^3)g_{10*}(2(-2+d^2)g_{10*} \\
&- (4-3d+d^2)g_{20*})+d(4+d)^2(-8(2+d)\epsilon \\
&+ 3(-1+d)^2(1+d)(2g_{10*}+g_{20*}))(d^2(8\delta+3g_{10*})-4(g_{10*}+g_{20*})
\end{aligned}
$$

$$
\begin{aligned}
&- 3d^3(g_{10*}+2g_{20*})+d^4(g_{10*}+4g_{20*})+d(16\delta+3g_{10*}+6g_{20*})), \\
g_{12n} &= 3(-1+d^2)g_{10*}(3d^5(g_{10*}+g_{20*})((-1+3e_{32}+2e_{42})g_{10*} \\
&- (2e_{12}+e_{22}-3e_{32}-2e_{42})g_{20*})-8(g_{10*}+g_{20*})((1+3e_{32}-e_{42})g_{10*} \\
&- (e_{12}-e_{22}-3e_{32}+e_{42})g_{20*})+d^6(g_{10*}+g_{20*})(-3e_{12}g_{20*}+5e_{32}(g_{10*}+g_{20*})) \\
&+ d^3(-((g_{10*}+g_{20*})(3(1+3e_{32}-2e_{42})g_{10*} \\
&+ (-4e_{12}+3(e_{22}+3e_{32}-2e_{42}))g_{20*})) \\
&+ 8\delta((-1+10e_{32}+3e_{42})g_{10*}-(5e_{12}+e_{22}-10e_{32}-3e_{42})g_{20*})) \\
&+ 2d(-((g_{10*}+g_{20*})((-3+6e_{42})g_{10*}-(e_{12}+3e_{22}-6e_{42})g_{20*})) \\
&+ 16\delta((1+3e_{32}-e_{42})g_{10*}-(e_{12}-e_{22}-3e_{32}+e_{42})g_{20*})) \\
&+ d^4(-((g_{10*}+g_{20*})((-3+15e_{32}+8e_{42})g_{10*} \\
&+ (-10e_{12}-3e_{22}+15e_{32}+8e_{42})g_{20*})) \\
&+ 8\delta(-(e_{12}g_{20*})+2e_{32}(g_{10*}+g_{20*})))+d^2((g_{10*}+g_{20*})((5+34e_{32})g_{10*} \\
&+ (-15e_{12}+5e_{22}+34e_{32})g_{20*})+16\delta(9e_{32}(g_{10*}+g_{20*}) \\
&+ 2(-2e_{12}g_{20*}+e_{42}(g_{10*}+g_{20*}))))), \\
g_{12d} &= g_{11d}, \\
g_{22n} &= -((-1+d^2)(-3(-4-d+4d^2+d^3)g_{10*}(2(-2+d^2)g_{10*} \\
&- (4-3d+d^2)g_{20*})((2+6e_{32}+2d^2e_{32}-2e_{42}+d(-1+6e_{32}+3e_{42}))g_{10*} \\
&- ((2+3d+d^2)e_{12}+(-2+d)e_{22}-6e_{32}-6de_{32}-2d^2e_{32}+2e_{42}-3de_{42})g_{20*}) \\
&+ d(4+d)((-4+6e_{32}+3d^2e_{32}+6e_{42}+d(-2+12e_{32}+3e_{42}))g_{10*}^2 \\
&- (10+4e_{12}+2d^2e_{12}+4e_{22}+6e_{32}-18e_{42} \\
&+ d(-1+8e_{12}+2e_{22}-6e_{32}+3e_{42}))g_{10*}g_{20*} \\
&+ ((2+d+d^2)e_{12}+(-10+d)e_{22}-3(4e_{32}+2de_{32}+d^2e_{32} \\
&- 4e_{42}+2de_{42}))g_{20*}^2)(-8(2+d)\epsilon+3(-1+d)^2(1+d)(2g_{10*}+g_{20*})))), \\
g_{22d} &= g_{21d}, \\
e_{11n} &= (g_q-g_sp_2)(g_pg_s(m_4n_2-m_3n_3)p_1 \\
&+ +g_{10*}g_o((m_4n_1+m_1n_3)p_4-(m_3n_1+m_1n_2)p_5)), \\
e_d &= g_s^3(m_4n_2-m_3n_3)p_1p_2+g_{20*}g_og_q(-(m_4n_1+m_1n_3)p_4+(m_3n_1+m_1n_2)p_5) \\
&+ g_{20*}g_og_s((m_1n_3p_2+m_4n_1(-p_1+p_2))p_4+m_3n_1p_1p_5-m_3n_1p_2p_5 \\
&- m_1n_2p_2p_5+m_2p_1(n_3p_4-n_2p_5))+g_k(g_qg_s(-(m_4n_2)+m_3n_3) \\
&+ g_{20*}g_o(m_4n_1p_4-m_2n_3p_4-m_3n_1p_5+m_2n_2p_5)), \\
e_{12n} &= (g_q-g_sp_2)(g_pg_s(m_4n_2-m_3n_3)p_2 \\
&+ +g_{10*}g_o(-m_4n_1p_4+m_2n_3p_4+m_3n_1p_5-m_2n_2p_5)), \\
e_{21n} &= (g_k-g_sp_1)(g_pg_s(m_4n_2-m_3n_3)p_1 \\
&+ +g_{10*}g_o(m_4n_1p_4+m_1n_3p_4-m_3n_1p_5-m_1n_2p_5)), \\
e_{22n} &= (g_k-g_sp_1)(g_pg_s(m_4n_2-m_3n_3)p_2 \\
&+ g_{10*}g_o(-m_4n_1p_4+m_2n_3p_4+m_3n_1p_5-m_2n_2p_5)),
\end{aligned}
$$

$$
\begin{aligned}
e_{31n} &= g_{20*}g_p g_s p_1(-g_k m_4 n_1 + g_q m_4 n_1 + g_q m_1 n_3 + g_k m_2 n_3 + g_s m_4 n_1 p_1 \\
&- g_s m_2 n_3 p_1 - g_s m_4 n_1 p_2 - g_s m_1 n_3 p_2) + g_{10*}(g_s p_1(g_s^2(m_4 n_1 + m_1 n_3)p_2 \\
&- g_{20*}g_o(m_1+m_2)n_1 p_5) + g_k(-(g_q g_s(m_4 n_1 + m_1 n_3)) \\
&+ g_{20*}g_o(m_1+m_2)n_1 p_5))\,, \\
e_{32n} &= g_{20*}g_p g_s p_2(-g_k m_4 n_1 + g_q m_4 n_1 + g_q m_1 n_3 + g_k m_2 n_3 + g_s m_4 n_1 p_1 \\
&- g_s m_2 n_3 p_1 - g_s m_4 n_1 p_2 - g_s m_1 n_3 p_2) + g_{10*}(g_k g_q g_s(m_4 n_1 - m_2 n_3) \\
&+ g_s^3(-m_4 n_1 + m_2 n_3)p_1 p_2 - g_{20*}g_o g_q(m_1+m_2)n_1 p_5 \\
&+ g_{20*}g_o g_s(m_1+m_2)n_1 p_2 p_5)\,, \\
e_{41n} &= g_{20*}g_p g_s p_1(-g_k m_3 n_1 + g_q m_3 n_1 + g_q m_1 n_2 + g_k m_2 n_2 + g_s m_3 n_1 p_1 \\
&- g_s m_2 n_2 p_1 - g_s m_3 n_1 p_2 - g_s m_1 n_2 p_2) + g_{10*}(g_s p_1(g_s^2(m_3 n_1 + m_1 n_2)p_2 \\
&- g_{20*}g_o(m_1+m_2)n_1 p_4) + g_k(-(g_q g_s(m_3 n_1 + m_1 n_2)) \\
&+ g_{20*}g_o(m_1+m_2)n_1 p_4))\,, \\
e_{42n} &= g_{20*}g_p g_s p_2(-g_k m_3 n_1 + g_q m_3 n_1 + g_q m_1 n_2 + g_k m_2 n_2 + g_s m_3 n_1 p_1 \\
&- g_s m_2 n_2 p_1 - g_s m_3 n_1 p_2 - g_s m_1 n_2 p_2) + g_{10*}(g_k g_q g_s(m_3 n_1 - m_2 n_2) \\
&+ g_s^3(-m_3 n_1 + m_2 n_2)p_1 p_2 - g_{20*}g_o g_q(m_1+m_2)n_1 p_4 \\
&+ g_{20*}g_o g_s(m_1+m_2)n_1 p_2 p_4)\,,
\end{aligned}
$$

where

$$
\begin{aligned}
l_1 &= 24 + 16d - 22d^2 - 16d^3 - 2d^4, \\
m_1 &= 48 - 16d - 52d^2 + 16d^3 + 4d^4, \\
m_2 &= -48 - 80d + 60d^2 + 96d^3 - 10d^4 - 16d^5 - 2d^6, \\
m_3 &= -48 + 112d + 32d^2 - 130d^3 + 14d^4 + 18d^5 + 2d^6, \\
m_4 &= 48 + 104d - 62d^2 - 127d^3 + 11d^4 + 23d^5 + 3d^6, \\
n_1 &= 48 + 56d - 40d^2 - 56d^3 - 8d^4, \\
n_2 &= 48 + 104d - 32d^2 - 104d^3 - 16d^4, \\
n_3 &= -48 + 16d + 10d^2 - 41d^3 + 35d^4 + 25d^5 + 3d^6, \\
o_1 &= 26 - 7d - 27d^2 + 7d^3 + d^4, \\
o_2 &= -12 + 12d^2, \\
p_1 &= -96 - 64d + 88d^2 + 64d^3 + 8d^4, \\
p_2 &= -96 - 64d + 124d^2 + 82d^3 - 26d^4 - 18d^5 - 2d^6, \\
p_3 &= 96 + 40d - 140d^2 - 60d^3 + 42d^4 + 20d^5 + 2d^6, \\
p_4 &= 144 + 96d - 132d^2 - 96d^3 - 12d^4, \\
p_5 &= 144 + 96d - 186d^2 - 123d^3 + 39d^4 + 27d^5 + 3d^6, \\
p_6 &= -24d - 52d^2 + 4d^3 + 50d^4 + 20d^5 + 2d^6, \\
p_7 &= 96 + 16d - 192d^2 - 56d^3 + 92d^4 + 40d^5 + 4d^6, \\
q_1 &= 96 + 16d - 156d^2 - 38d^3 + 58d^4 + 22d^5 + 2d^6,
\end{aligned}
$$

$$
\begin{aligned}
r_1 &= 24 - 4d - 36d^2 + 2d^3 + 12d^4 + 2d^5, \\
r_2 &= 12 + 2d - 18d^2 - 3d^3 + 6d^4 + d^5, \\
r_3 &= 12 - 6d - 18d^2 + 5d^3 + 6d^4 + d^5, \\
g_s &= g_{10*} + g_{20*}, \\
g_p &= g_{10*} + g_{10*}^2/g_{20*}, \\
g_k &= (g_{10*}^2 p_3)/g_{20*} + g_{20*}p_6 + g_{10*}p_7, \\
g_q &= (-(d g_{20*}^2 l_1) + g_{10*}(g_{10*}p_3 + g_{20*}q_1))/g_{20*}, \\
g_o &= g_s^2/g_{20*}.
\end{aligned}
$$

Stability of the fixed point is determined by the (2×2) block of the stability matrix which corresponds to β functions of α_5 and χ_3. The eigenvalue which responds for instability has form:

$$\lambda = \lambda_0 + \lambda_1\alpha_1 + \lambda_2\alpha_2\,,$$

where

$$
\begin{aligned}
\lambda_0 &= \frac{d g_{20*}(g_{10*}+g_{20*})o_1 - \sqrt{t_1} + g_{10*}g_{20*}r_1 + g_{10*}^2 r_2 + g_{20*}^2 r_3}{8d(12+8d^2+d^2)g_{20*}}\,, \\
\lambda_1 &= \frac{\lambda_{1n}}{\lambda_d}\,, \\
\lambda_{1n} &= d g_{20*}^2(g_{11*}+g_{21*})o_1\sqrt{t_1} + g_{20*}\left(-t_2 + g_{11*}\sqrt{t_1}\,(g_{20*}r_1 + 2g_{10*}r_2)\right) \\
&+ g_{21*}\sqrt{t_1}\left(\sqrt{t_1} - g_{10*}^2 r_2 + g_{20*}^2 r_3\right)\,, \\
\lambda_d &= 8d(12+8d+d^2)g_{20*}^2\sqrt{t_1}\,, \\
\lambda_2 &= \frac{\lambda_{2n}}{\lambda_d}\,, \\
\lambda_{2n} &= d g_{20*}^2(g_{12*}+g_{22*})o_1\sqrt{t_1} + g_{20*}\left(-t_3 + g_{12*}\sqrt{t_1}\,(g_{20*}r_1 + 2g_{10*}r_2)\right) \\
&+ g_{22*}\sqrt{t_1}\left(\sqrt{t_1} - g_{10*}^2 r_2 + g_{20*}^2 r_3\right)\,,
\end{aligned}
$$

with

$$
\begin{aligned}
t_1 &= d^2 g_{20*}^2(g_{10*}+g_{20*})^2(o_1^2 - 4o_2^2) - 2d g_{20*}o_1(g_{10*}+g_{20*}) \times \\
&\times (g_{10*}g_{20*}r_1 + g_{10*}^2 r_2 + g_{20*}^2 r_3) + (g_{10*}g_{20*}r_1 + g_{10*}^2 r_2 + g_{20*}^2 r_3)^2\,, \\
t_2 &= 2(d^2 g_{20*}(g_{10*}+g_{20*})(g_{11*}g_{20*} + (g_{10*}+2g_{20*})g_{21*})(o_1^2 - 4o_2^2) \\
&+ (g_{10*}g_{20*}r_1 + g_{10*}^2 r_2 + g_{20*}^2 r_3)(g_{11*}g_{20*}r_1 + g_{10*}g_{21*}r_1 + 2g_{10*}g_{11*}r_2 \\
&+ 2g_{20*}g_{21*}r_3) - do_1(g_{10*}^3 g_{21*}r_2 + g_{10*}^2 g_{20*}(3g_{11*}r_2 + 2g_{21*}(r_1+r_2)) \\
&+ g_{20*}^3(4g_{21*}r_3 + g_{11*}(r_1+r_3)) + g_{10*}g_{20*}^2(2g_{11*}(r_1+r_2) + 3g_{21*}(r_1+r_3))))\,, \\
t_3 &= 2(d^2 g_{20*}(g_{10*}+g_{20*})(g_{12*}g_{20*} + (g_{10*}+2g_{20*})g_{22*})(o_1^2 - 4o_2^2) \\
&+ (g_{10*}g_{20*}r_1 + g_{10*}^2 r_2 + g_{20*}^2 r_3)(g_{12*}g_{20*}r_1 + g_{10*}g_{22*}r_1 + 2g_{10*}g_{12*}r_2 \\
&+ 2g_{20*}g_{22*}r_3) - do_1(g_{10*}^3 g_{22*}r_2 + g_{10*}^2 g_{20*}(3g_{12*}r_2 + 2g_{22*}(r_1+r_2)) \\
&+ g_{20*}^3(4g_{22*}r_3 + g_{12*}(r_1+r_3)) + g_{10*}g_{20*}^2(2g_{12*}(r_1+r_2) + 3g_{22*}(r_1+r_3))))\,.
\end{aligned}
$$

Critical dimension d_c is defined as solution of the equation $\lambda(d_c, \epsilon, \alpha_1, \alpha_2) = 0$.

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