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Параметрическое возбуждение в полупроводниках со сверхрешетками

Исследуется параметрическое воздействие сильного поля излучения на электронную плазму полупроводника со сверхрешеткой. На основе формализма, предложенного в [1], получены аналитические выражения для инкрементов неустойчивости волн в разных случаях, показывающие явную зависимость от параметров сверхрешетки. Численные оценки даны для образца InSb -кристалла.

Работа выполнена в Лаборатории теоретической физики, ОИЯИ.

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Parametric Excitation in Semiconductors with Superlattices

The parametric action of a strong radiation field on the electron plasma of a semiconductor with a superlattice (SL) is studied in the framework of the quantum formalism proposed in [1].

Analytical expressions for the wave instability growth rates are obtained in a number of cases which show an explicit dependence on SL-parameters. Numerical estimates are presented for a sample of InSb -crystal.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

Manylayered periodic semiconductor structures - superlattices (SL) have been a subject that attracts much attention of physicists in recent years as perspective materials for applications in semiconductor electronics (see, for example, [2]).

The inhomogeneity of the media due to the introducing of superstructures and the nonparabolicity of the electron energy dispersion arising from this effect lead to essential nonlinearity in optical properties that can be strong enough to be observed already in rather weak external fields [3-11].

Investigation of the process of parametric excitation of electromagnetic waves in semiconductors with superlattices represents certain interest since the changes in the symmetry properties of the crystal lattice due to the presence of a SL as well as the nonparabolicity of the electron energy dispersion will considerably influence the parametric interactions in such structures. The cases of parametric excitation in semiconductors with anisotropic phonon spectra and nonparabolic energy dispersion (in the Kane's model) were studied separately in [12,13].

In this paper we solve the same problem for semiconductors with SL using the formalism developed in [1]. The instability analysis is given for the cases of the external field incidence parallel and perpendicular to the SL-axis, and the analytical expressions for the instability growth rates in different cases are obtained which show an explicit dependence on the SL-parameters. Some numerical estimations of the instability threshold fields are performed for a sample of InSb -crystal.

2. EQUATIONS OF MOTION FOR PERTURBATION FIELDS

We consider an electron plasma of a semiconductor with a one-dimensional SL directed along Z-axis. As is well known, the energy dispersion law for an electron of momentum $\vec{p}$ and effective mass m in the S-miniband in such a plasma has the form [14-16]:

$$\epsilon^{(S)}(\vec{p}) = \epsilon_S + \frac{p_\perp^2}{2m} - \Delta_S \cos(p_\parallel d), \tag{1}$$

where $\epsilon_S = \mathrm{const}$ is determined by SL-potential parameters, Δ_S the half-width of the S-th energy miniband, d the SL period, the indices $\parallel, \perp$ mean the directions parallel and perpendicular to the SL-axis, respectively.

Let our system be subjected to the action of an external radiation field presented in the dipole approximation as

$$\vec{E}_0(t) = \vec{E}_0 \sin \Omega t. \tag{2}$$

(Conditions in which this approximation is valid were discussed in detail in [1,12-13]).

We shall assume that

$$\Omega < (\epsilon_S - \epsilon_0) \pm (\Delta_S + \Delta_0) \quad \text{for} \quad S \neq 0. \tag{3}$$

This means that transitions between the minibands are ignored and we can confine ourselves to the consideration of the first miniband. The Hamiltonian of such a plasma system may be written in the form:

$$H = H_e + H_{ee} + H_\gamma + H_{e\gamma}, \tag{4}$$

$$H_e = \sum_{\vec{p}} \epsilon\left(\vec{p} - \frac{e}{c}\vec{A}_0(t)\right) a^+_p a_p, \tag{4a}$$

$$H_{ee} = \frac{1}{2} \sum_{\substack{\vec{p},\vec{p}',\vec{q} \\ n,n'}} \phi_{\vec{q}} \cdot M(-q_\parallel) M(q_\parallel) a^+_{p+q+ng} a^+_{p'-q+n'g} a_{p'} a_p, \tag{4b}$$

$$H_\gamma = \sum_{\nu,\vec{k}} \omega_{\nu k} C^+_{\nu k} C_{\nu k}, \tag{4c}$$

$$H_{e\gamma} = -\frac{e}{mc} \sum_{\vec{p},\vec{k},n} M(-k_\parallel)\left[m\vec{V}(\vec{p},\vec{k}) + \frac{\vec{k}}{2} - \frac{e}{c}\vec{A}_0(t)\right]\vec{A}(\vec{k}) a^+_{p+k+ng} a_p. \tag{4d}$$

Here H_e and H_γ are the Hamiltonians of non-interacting electron and "self-consistent" photon system, respectively; a^+_p (a_p) is the creation (annihilation) operator for electrons, $C^+_{\nu k}$($C_{\nu k}$) the corresponding operator for a photon with energy $\hbar\omega_{\nu k}$, momentum $\hbar\vec{k}$ and polarization ν, which is connected with the self-consistent electromagnetic field vector potential $\vec{A}$ by the formula

$$\vec{A}(\vec{k}) = \frac{1}{L(Nd)^{1/2}} \sum_\nu \left(\frac{2\pi c}{k}\right)^{1/2} \vec{e}_{\nu k}\left(C_{\nu k} + C^+_{\nu -k}\right) \tag{5}$$

($\vec{e}_{\nu k}$ is the polarization vector, L the dimension of the system in the directions perpendicular to SL-axis, N is the number of SL periods); $\vec{A}_0(t)$ is the vector potential of the external radiation fields; H_{ee} represents the Coulomb interaction between electrons, where $\phi_q = \frac{4\pi e^2}{L^2 (Nd) q^2}$; $H_{e\gamma}$ the interaction between electrons and self-consistent photons in the presence of the external field;

$$M(q_\parallel) = \int_0^{Nd} e^{-iq_\parallel z} \phi_0^*(z)\phi_0(z)dz, \tag{6}$$

where $\phi_S(z)$ is the electron wave function of the S-th state in one of the one-dimensional potential wells forming the SL-potential;

$$\vec{V}(\vec{p},\vec{k}) = \left(\frac{p_x}{m}, \frac{p_y}{m}, V_z = \frac{1}{M(-k_\parallel)} \sum_S e^{ip_\parallel z} \int_0^{Nd} e^{ik_\parallel z} \phi_0^*(z)(-i\nabla_z)\phi_0(z-Sd)dz\right); \tag{7}$$

$\vec{g} = \frac{2\pi}{d}(0,0,1)$ is the reciprocal SL-vector.

We note here that, since the self-consistent field is a perturbation one, H_γ is small and then its further contribution to the equations of motion may be neglected. By the procedure described in [1] the Hamiltonian (4) leads to the linearized equations of motion for the electron distribution function $\langle a^+_p a_{p+q+ng}\rangle$ and the photon operator $\langle \vec{A}(\vec{k})\rangle$. Thus we have:

$$\langle a^+_p a_{p+q+ng}\rangle = \langle a^+_p a_p\rangle \delta_{q,0} + f(\vec{p}, \vec{p}+\vec{q}+n\vec{g}, t),$$

where $\langle a_p^+ a_p^+ \rangle \equiv n_p$ in the linear approximation is considered as an equilibrium Fermi distribution and

$$i\frac{\partial}{\partial t}f(\vec{p}, \vec{p}+\vec{q}+n\vec{g}, t) = [\epsilon(\vec{p}+\vec{q}-\frac{e}{c}\vec{A}_0(t)) - \epsilon(\vec{p}-\frac{e}{c}\vec{A}_0(t))]\, f(\vec{p}, \vec{p}+\vec{q}+n\vec{g}, t) -$$

$$-(n_{p+q+ng} - n_p)\{S(\vec{q})\delta n(\vec{q}, t) - \tag{8}$$

$$-\frac{e}{mc}\sum_{n'} M^*(q_\parallel + n'g)[mV(\vec{p}, \vec{q}+n'\vec{g}) - \frac{e}{c}\vec{A}_0(t)]\langle \vec{A}(\vec{q}+n'\vec{g})\rangle\},$$

where

$$\delta n(\vec{q}, t) \equiv \sum_{\vec{p}, n} f(\vec{p}, \vec{p}+\vec{q}+n\vec{g}, t),$$

$$S(\vec{q}) \equiv \sum_n \phi_{q+ng}|M(q_\parallel + \frac{2\pi}{d}n)|^2 . \tag{8a}$$

The equation for $\vec{A}(\vec{k})$ (the bracket $\langle \ldots \rangle$ is omitted for brevity) is as follows:

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + q^2)\vec{A}(\vec{q}) = \frac{4\pi}{c}\,\frac{1}{L^2 Nd}\{\frac{e}{m}M(q_\parallel)\times$$

$$\times \sum_{\vec{p}, n}[m\vec{V}(\vec{p}+\vec{q}+n\vec{g}, -\vec{q}) - \frac{\vec{q}}{2} - \frac{e}{c}\vec{A}_0(t)]\, f(\vec{p}, \vec{p}+\vec{q}+n\vec{g}, t) -$$

$$-\frac{e^2 N_0}{mc}\sum_n M^*(n\vec{g})\vec{A}(\vec{q}+n\vec{g})\},$$

where $N_0 = \sum_{\vec{p}} n_p$.

Now, following the works [1,12-13], we introduce the transformation

$$\tilde{X} = X e^{-i\lambda \sin\Omega t}$$

$$\lambda \equiv \frac{e(\vec{E}_0\vec{q})}{m\Omega^2}\sin\alpha\cos\beta\sin\theta, \tag{10}$$

where $\alpha = \sphericalangle(\vec{q}, \vec{z})$, $\beta = \sphericalangle(\vec{q}_\perp, \vec{E}_{0\perp})$, $\theta = \sphericalangle(\vec{E}_0, \vec{z})$.

We also neglect the umklapp-processes between minibands in further treatment since the long-wavelength case is considered. The quantities N_0, $\delta n(\vec{q})$, n_p are related to an unit volume and then the factor $(L^2 Nd)^{-1}$ is omitted in our equations. In these conditions (8)-(9) yield the following equations for Fourier components:

$$\tilde{\phi}(\vec{q}, \omega) = \sum_{\ell\ell'}\{|M(q_\parallel)|^2 P_{\ell\ell'}(\vec{q}, \omega)\tilde{\phi}(\vec{q}, \omega + (\ell-\ell')\Omega) -$$

$$- M^*(q_\parallel)\vec{R}_{\ell\ell'}(\vec{q}, \omega)\tilde{\vec{A}}(\vec{q}, \omega + (\ell-\ell')\Omega) +$$

$$+\mu M^*(q_\parallel)P_{\ell\ell'}(\vec{q}, \omega)\vec{e}_0\,[\tilde{\vec{A}}(\vec{q}, \omega + (\ell-\ell'-1)\Omega) + \tilde{\vec{A}}(\vec{q}, \omega + (\ell-\ell'+1)\Omega)]\}, \tag{11}$$

$$\tilde{\vec{A}}(\vec{q}, \omega)(1 + \frac{\omega_p^2}{q^2c^2}) - \sum_{\ell, n} J_\ell(\lambda) J_{\ell+n}(\lambda)\frac{(\omega-\ell\Omega)^2}{q^2c^2}\tilde{\vec{A}}(\vec{q}, \omega + n\Omega) =$$

$$= -M(q_\parallel)\{\mu\vec{e}_0\,[\tilde{\phi}(\vec{q}, \omega+\Omega) + \tilde{\phi}(\vec{q}, \omega-\Omega)] + \frac{\vec{q}}{2mc}\tilde{\phi}(\vec{q}, \omega)\} +$$

$$+|M(q_\parallel)|^2\sum_{\ell\ell'}\{M(q_\parallel)\vec{R}_{\ell\ell'}(\vec{q}, \omega)\tilde{\phi}(\vec{q}, \omega + (\ell-\ell')\Omega) - \tag{12}$$

$$-\hat{Q}_{\ell\ell'}(\vec{q}, \omega)\tilde{\vec{A}}(\vec{q}, \omega + (\ell-\ell')\Omega) +$$

$$+\mu\vec{R}_{\ell\ell'}(\vec{q}, \omega)(\vec{e}_0\cdot[\tilde{\vec{A}}(\vec{q}, \omega + (\ell-\ell'-1)\Omega) + \tilde{\vec{A}}(\vec{q}, \omega + (\ell-\ell'+1)\Omega)])\}.$$

The following notations have been introduced in (11)-(12):

$$\tilde{\phi}(\vec{q}, \omega) = \frac{4\pi e^2}{q^2}\, \delta\tilde{n}(\vec{q}, \omega), \tag{13a}$$

$$P_{\ell\ell'}(\vec{q}, \omega) = \sum_{\vec{p}} J_\ell(\tilde{\lambda}) J_{\ell'}(\tilde{\lambda}) \Pi_p(\vec{q}, \omega), \tag{13b}$$

$$\vec{R}_{\ell\ell'}(\vec{q}, \omega) = \frac{1}{c} \sum_{\vec{p}} J_\ell(\tilde{\lambda}) J_{\ell'}(\tilde{\lambda}) \Pi_p(\vec{q}, \omega) \vec{V}(\vec{p}, \vec{q}), \tag{13c}$$

$$\vec{\tilde{R}}_{\ell\ell'}(\vec{q}, \omega) = \frac{1}{c} \sum_{\vec{p}} J_\ell(\tilde{\lambda}) J_{\ell'}(\tilde{\lambda}) \Pi_p(\vec{q}, \omega) \vec{V}(\vec{p} + \vec{q}, -\vec{q}), \tag{13d}$$

$$\hat{Q}_{\ell\ell'} \equiv Q^{ij}_{\ell\ell'}(\vec{q}, \omega) = \frac{1}{c^2} \sum_{\vec{p}} J_\ell(\tilde{\lambda}) J_{\ell'}(\tilde{\lambda}) \Pi_p(\vec{q}, \omega) \vec{V}_i(\vec{p} + \vec{q}, -\vec{q}) \vec{V}_j(\vec{p}, \vec{q}), \tag{13e}$$

$$\Pi_p(\vec{q}, \omega) = \frac{4\pi e^2}{q^2} \frac{n_{p+q} - n_p}{(\epsilon(\vec{p} + \vec{q}) - \epsilon(\vec{p}) - \omega - \ell\Omega - i0)}, \tag{13f}$$

$$\omega_p^2 = \frac{4\pi e^2 N_0}{m}, \quad \mu = \frac{eE_0}{2mc\Omega}, \quad \vec{e}_0 = \frac{\vec{E}_0}{E_0}, \tag{13g}$$

$$\tilde{\lambda} = \frac{e(\vec{E}_0 \vec{q})}{\Omega^2} \Delta_0 d^2 \cos\alpha \cos\theta \cos p_{\parallel} d. \tag{13h}$$

(Here $\mu, \lambda, \tilde{\lambda} \ll 1$ as in/1,12-13/), $J_\ell(\tilde{\lambda})$ -Bessel function of the first kind.

In the next section equations (11)-(12) will be analyzed for some partial cases when relatively simple form of dispersion relations is available.

3. INSTABILITY CALCULATIONS

Further calculations will be performed in the high-frequency long-wavelength limit ($qV_F \ll \omega$, $qV_T \ll \omega$, $qd \ll 1$ and then $M(q_\parallel) \approx 1$, $V_z \approx \partial\epsilon(\vec{p})/\partial p_z$, where $V_T = \Delta_0 \cdot d$, $V_F = [\frac{2(\epsilon_F - \epsilon_0)}{m}]^{1/2}$ is the Fermi velocity) for a degenerate semiconductor electron gas.

We present here the detailed analysis for the cases when $\vec{E}_0 \parallel$ SL-axis and $\vec{E}_0 \perp$ SL-axis separately.

3.1. The case of $\vec{E}_0 \parallel$ SL-axis

In this case for perturbations propagating parallelly to the external electric field vector $\vec{E}_0(\vec{q} \parallel \vec{E}_0)$ we have: $\lambda = 0$, $\alpha = \theta = 0$ and

$$Q^{ij}_{\ell\ell'} = Q_{\ell\ell'} \delta_{ij},$$

$$\vec{R}_{\ell\ell'} = (0, 0, R_{\ell\ell'}), \quad \vec{\tilde{R}}_{\ell\ell'} = (0, 0, \tilde{R}_{\ell\ell'}).$$

Neglecting all terms of orders higher than $J_1(\lambda_0)$ $(\lambda_0 \equiv \frac{e(\vec{E}_0 \vec{q})}{m\Omega^2})$ and using equation $\operatorname{div}\vec{A} = 0$ one obtains from (11)-(12):

$$\mathcal{E}(\vec{q}, \omega + S\Omega)\tilde{\phi}(\vec{q}, \omega + S\Omega) = [P_{10}(\vec{q}, \omega + S\Omega) + P_{0-1}(\vec{q}, \omega + S\Omega)]\tilde{\phi}(\vec{q}, \omega + (S+1)\Omega) + \tag{14a}$$

$$+ [P_{-10}(\vec{q}, \omega + S\Omega) + P_{01}(\vec{q}, \omega + S\Omega)]\tilde{\phi}(\vec{q}, \omega + (S-1)\Omega),$$

$$\Delta(\vec{q}, \omega + S\Omega)\vec{\tilde{A}}(\vec{q}, \omega + S\Omega) = -[Q_{10}(\vec{q}, \omega + S\Omega) + Q_{0-1}(\vec{q}, \omega + S\Omega)]\vec{\tilde{A}}(\vec{q}, \omega + (S+1)\Omega) -$$

$$-[Q_{-10}(\vec{q}, \omega + S\Omega) + Q_{01}(\vec{q}, \omega + S\Omega)]\vec{\tilde{A}}(\vec{q}, \omega + (S-1)\Omega), \tag{14b}$$

where

$$\mathcal{E}(\vec{q}, \omega) = 1 - P_{00}(\omega), \tag{14c}$$

$$\Delta(\vec{q}, \omega) = 1 - \frac{\omega^2 - \omega_p^2}{q^2 c^2} + Q_{00}(\vec{q}, \omega).$$

From (14a) one finds the dispersion relation for the longitudinal plasma mode $\omega=\omega_L(\vec{q})$, where $\omega_L(\vec{q})$ is determined from $\mathcal{E}(\vec{q},\omega)=0$.

$$\mathcal{E}(\vec{q},\omega)\approx 1-\frac{\omega_p^2}{\omega^2}(\sin^2\alpha+\frac{V_T^2}{V_F^2}\cos^2\alpha),$$

and has the form

$$\omega_L(\vec{q})=\omega_p(\sin^2\alpha+\frac{V_T^2}{V_F^2}\cos^2\alpha)^{1/2}. \tag{15}$$

(In this case $\alpha=0$ and $\omega_L(\vec{q})=\omega_p\frac{V_T}{V_F}$).

In the resonant situation when $\Omega=2\omega_L(\vec{q})$ the dispersion equation is

$$\mathcal{E}(\vec{q},\omega)\mathcal{E}(\vec{q},\omega-\Omega)=[P_{10}(\vec{q},\omega-\Omega)-P_{10}(\vec{q},\omega-2\Omega)]^2 \tag{16}$$

that yields the following expressions for instability growth rate γ :

$$\gamma=\frac{1}{8}e(\vec{q}\vec{E}_0)d^2(\frac{qV_T}{\Omega})^{-2} \tag{17}$$

and for the threshold field value E_{oth}:

$$E_{oth}=\frac{4}{eqd^2\tau_e}(\frac{\Omega}{qV_T})^2 \tag{18}$$

(τ_e -single-electron lifetime).

For the transversal mode $\omega_T(\vec{q})=(\omega_p^2+q^2c^2)^{1/2}$ determined from $\Delta(\omega)=0$, , in the same resonant condition ($\Omega=2\omega_T(\vec{q})$) the dispersion equation has the form:

$$\Delta(\vec{q},\omega)\Delta(\vec{q},\omega-\Omega)=[Q_{10}(\vec{q},\omega-\Omega)-Q_{10}(\vec{q},\omega-2\Omega)]^2. \tag{19}$$

The growth rate γ is

$$\gamma=eE_0qd^2(\frac{\omega_p}{\Omega})^2(\frac{qV_T}{\Omega})^4. \tag{20}$$

As it is seen from (20), γ is of the order higher than that in the case of longitudinal waves (see (17)). It is easily to show that the damping of the linear transversal mode $\Gamma=\frac{2}{\tau_e}(\frac{\omega_p}{\Omega})^2$ and the threshold field E_{oth} is given by

$$E_{oth}=\frac{2}{eqd^2\tau_e}(\frac{\Omega}{qV_T})^4. \tag{21}$$

If the propagation direction of the waves is perpendicular to $\vec{E}_0(\vec{q}\perp\vec{E}_0)$, then we have:

$$\theta=0,\ \alpha=\frac{\pi}{2},\ \lambda=\tilde{\lambda}=0,$$

$$P_{\ell\ell'}=\vec{R}_{\ell\ell'}=Q^{ij}_{\ell\ell'}=0 \quad \text{for} \quad \ell\neq 0,\ \ell'\neq 0; \tag{22}$$

$$Q^{ij}_{00}=\begin{pmatrix} Q^{\perp}_{00} & 0 & \\ & Q^{\perp}_{00} & \\ 0 & & Q^{\parallel}_{00}\end{pmatrix},$$

and the equations for potential Fourier components in this case are of the form:

$$\mathcal{E}(\vec{q},\omega)\tilde{\phi}(\vec{q},\omega)-\mu P_{00}(\vec{q},\omega)[\tilde{A}_z(\vec{q},\omega-\Omega)+\tilde{A}_z(\vec{q},\omega+\Omega)]=0, \tag{23a}$$

$$\Delta_{\parallel}(\vec{q},\omega)\tilde{A}_z(\vec{q},\omega)+\mu[\tilde{\phi}(\vec{q},\omega-\Omega)+\tilde{\phi}(\vec{q},\omega+\Omega)]=0, \tag{23b}$$

$$\Delta_{\perp}(\vec{q},\omega)\tilde{A}_{\perp}(\vec{q},\omega)=0, \tag{23c}$$

where

$$\Delta_{\perp,\parallel}(\vec{q},\omega)=1-\frac{\omega^2-\omega_p^2}{q^2c^2}+Q^{\perp,\parallel}_{00}(\vec{q},\omega),$$

(since qV_F , $qV_T\ll\omega_p$, one can set $\Delta_{\perp}=\Delta_{\parallel}=1-\frac{\omega^2-\omega_p^2}{q^2c^2}$).

Equation (23c) shows the well-known result that the electromagnetic waves polarized perpendicularly to the pumping field direction are not affected by this field. For the modes with "mixed" polarization a usual analysis leads to the dispersion equation (the resonant condition now reads $\Omega = \omega_L + \omega_T$):

$$\mathcal{E}(\vec{q}, \omega)\Delta_{\parallel}(\vec{q}, \omega - \Omega) = -\mu^2 , \qquad (24)$$

from that one has

$$\gamma = \frac{1}{4}\frac{e(E_0 q)}{m\Omega}\left(\frac{\beta}{1-\beta}\right)^{1/2} \qquad (25)$$

and

$$E_{oth} = \frac{2m\Omega}{eq r_e}\left(\frac{\beta}{1-\beta}\right)^{1/2} , \qquad (26)$$

where the parameter $\beta \equiv \frac{\omega_p}{\Omega}$ must satisfy the condition $\beta \leq \frac{1}{2}$. This result is identical to that obtained in [1] and means that for the waves propagating across the SL-axis the dependence of the amplifying process on SL parameters is negligible in this accepted approximation.

3.2. The Case of $\vec{E}_0 \perp$ SL-Axis

Now, if $\vec{q} \parallel \vec{E}_0$, then $\theta = \alpha = \pi/2$, $\lambda = \lambda_0$, $Q^{ij}_{\ell\ell'} = Q^{ij}_{00}\delta_{\ell\ell'}\delta_{\ell 0}$ and takes the same diagonal form as in (22) with

$$Q^{\perp}_{00} = \frac{1}{4}\frac{V_F^2}{c^2}\frac{\omega_p^2}{\omega^2}\left(1 + \frac{\Delta_0^2}{2(\epsilon_F - \epsilon_0)^2}\right), \quad Q^{\parallel}_{00} = \frac{1}{2}\frac{V_T^2}{c^2}\frac{\omega_p^2}{\omega^2} .$$

Equations (11)-(12) for potential Fourier components become

$$\tilde{\phi}(\vec{q}, \omega) = P_{00}(\vec{q}, \omega)\tilde{\phi}(\vec{q}, \omega), \qquad (27a)$$

$$\left(1 + \frac{\omega_p^2}{q^2c^2}\right)\tilde{\vec{A}}(\vec{q}, \omega) = \qquad (27b)$$

$$= \sum_{\ell, n} J_\ell(\lambda_0)J_{\ell+n}(\lambda_0)\frac{(\omega - \ell\Omega)^2}{q^2c^2}\tilde{\vec{A}}(\vec{q}, \omega + n\Omega) - \hat{Q}_{00}\tilde{\vec{A}}(\vec{q}, \omega).$$

Calculations do not show any wave amplifying effect in this case. There is only a small shift $\Delta\omega \sim J_1^2(\lambda_0)\Omega$ in frequency for transversal modes. This result is clear in the framework of the general theory when the electron-phonon interaction is ignored (see [1]).

For the propagation direction $\vec{q} \perp \vec{E}_0$ one has $\theta = \pi/2$, $\alpha = 0$, $\lambda = 0$. The tensor Q^{ij}_{00} in this case has also the form (22). The system of equations is the same as in (23) with the only change in notations: $\perp \to \parallel$, $\parallel \to \perp$.

The modes undergoing amplification by the external field are the "mixed" ones with frequencies

$$\omega_L = \frac{V_T}{V_F}\omega_p , \quad \omega_T = [\omega_p^2 + q^2c^2]^{1/2} \qquad (\Omega = \omega_L + \omega_T).$$

(Note that the frequency of longitudinal modes depends on the propagation direction with respect to SL-axis). The instability growth rate and threshold field are

$$\gamma = \frac{1}{4}\frac{eqE_0}{m\Omega}\left(\frac{\tilde{\beta}}{1-\tilde{\beta}}\right)^{1/2} , \qquad (28)$$

$$\tilde{\beta} \equiv \frac{V_T}{V_F}\beta ,$$

$$E_{oth} = \frac{2m\Omega}{eq r_e}\frac{V_F}{V_T}\left(\frac{\tilde{\beta}}{1-\tilde{\beta}}\right)^{1/2} , \qquad (29)$$

where the parameter $\tilde{\beta}$ must satisfy the condition

$$\tilde{\beta} \leq \frac{V_T}{V_F}\left(1 + \frac{V_T}{V_F}\right)^{-1} . \qquad (30)$$

Finally, we present here in conclusion some numerical estimates for a sample of InSb -crystal. The following numerical data have been taken from [14]: $\Delta_0 = 0{,}15\,\omega_0$, $\epsilon_0 = 1{,}5\omega_0$, where $\hbar\omega_0 = 0{,}33$ eV, $d = 10^{-6}$ cm, $N_0 = 2\cdot 10^{17}$ cm^{-3}, $r_e = 1{,}7\cdot 10^{-10}$ s, $m = 0{,}016\, m_e$. With these data we have $\omega_p = 2{,}06\cdot 10^{14}$ s^{-1}, $\epsilon_F = 1{,}09\cdot 10^{-12}$ erg, $V_T = 7{,}5\cdot 10^7$ cm$\cdot$s^{-1}, $V_F = 3{,}8\cdot 10^8$ cm$\cdot$s^{-1}.

The threshold field values for the most "sensitive" to excitation modes are estimated as follows (the value $q = 3\cdot 10^4$ cm^{-1} was taken):

- for longitudinal modes with $\vec{q} \parallel \vec{E}_0 \parallel$ SL-axis (see (18));

$$E_{oth} = 7\cdot 10^3 \frac{V}{cm}, \qquad (31)$$

- for the coupled modes with $\vec{q} \perp \vec{E}_0$:

$$E_{oth} = 1{,}8 \cdot 10^3 \, \frac{V}{cm} \quad \text{for} \quad \vec{E}_0 \parallel \text{SL-axis (see (26))}, \qquad (32)$$

$$E_{oth} = 3{,}5 \cdot 10^3 \, \frac{V}{cm} \quad \text{for} \quad \vec{E}_0 \perp \text{SL-axis (see (29))}. \qquad (33)$$

These field values, as one can see, are practically available by current lasers,although somewhat higher (but still easily available) values must be expected when the electron-phonon interaction is taken into account.

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