2757/2-79

сообщения
объединенного
института
ядерных
исследований
дубна

С326
В-24

E17 - 12300

Yu.N.Barabanenkov, V.D.Ozrin, A.V.Shelest

COLLECTIVE EFFECTS INFLUENCE ON THE TIME ASYMPTOTICS OF THE HYDRODYNAMIC MODES IN THE NEUTRAL GASES.

I. Analytical Singularities of the Hydrodynamic Modes at Small k and z

1979

E17 - 12300

Yu.N.Barabanenkov, V.D.Ozrin, A.V.Shelest

COLLECTIVE EFFECTS INFLUENCE ON THE TIME ASYMPTOTICS OF THE HYDRODYNAMIC MODES IN THE NEUTRAL GASES.

I. Analytical Singularities of the Hydrodynamic Modes at Small k and z

Объединенный институт
ядерных исследований
БИБЛИОТЕКА

Барабаненков Ю.Н., Озрин В.Д., Шелест А.В. E17 - 12300

Влияние коллективных эффектов на временную асимптотику гидродинамических мод в нейтральных газах. 1. Аналитические особенности гидродинамических мод при малых k и $|z|$

Рассматривается поведение на больших пространственно-временных интервалах нормальных мод гидродинамики, построенной на основе кинетического уравнения для газа твердых сфер, которое учитывает эффекты, обусловленные взаимодействием частиц с коллективными возбуждениями в среде. Исследованы аналитические особенности нормальных мод в области малых k и $|z|$ (z - переменная Лапласа).

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1979

Barabanenkov Yu.N., Ozrin V.D., Shelest A.V. E17 - 12300

Collective Effects Influence on the Time Asymptotics of the Hydrodynamic Modes in the Neutral Gases. I. Analytical Singularities of the Hydrodynamic Modes at Small k and $|z|$

The behaviour of the hydrodynamics normal modes in large time-space intervals is considered. The hydrodynamics is constructed on the basis of the kinetic equation for the hard-sphere gas, which takes into account the effects, caused by interaction of the particles with collective excitations in medium. The analytical singularities of the normal modes are investigated in the region of small k and $|z|$ (z is the Laplace variable).

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1979

© 1979 Объединенный институт ядерных исследований Дубна

In 1968 B.I.Alder and T.E.Wainwright [1] made the numerical calculations of the velocity autocorrelation function in the systems of hard disks and hard spheres using the molecular dynamics method. They found out that the damping of this function was obviously nonexponential. After the animated discussion [2,3] of these results it turned out that the interaction between the particles and collective motions should necessarily be taken into account for the more precise definition of the kinetic equations. These collective motions are caused by the viscosity, heat conductivity and sound wave. Such a corrected system of the kinetic equations was formulated by M.H.Ernst, I.R.Dorfman [4] and N.N.Bogolubov [5] (the EDB equations). It should be noted that this system is similar in character to the system of equations for plasma, from which the Lennard-Balescu kinetic equation [6,7] is derived.

We use the EDB system of equations for the investigation of the behaviour of the hydrodynamic normal modes at the time intervals large compared to the mean free path time and with small wave number in the units of inverse free path. In other words, in the Laplace z-representation in time and the Fourier k-representation in coordinates the case $k \to 0, |z| \to 0$ is considered.

In section 1 the EDB equations for the low density system are linearized near the equilibrium solution. The linearized nonlocal hydrodynamic equations generalizing the Navier-Stokes equations are written down. The solutions of these equations (in the limit of small k and $|z|$), which are hydrodynamic normal modes $\chi^j_{kz} = \chi^j_k(0)/D_j(k, z)$, are given in section 2. Section 3 is devoted to the general investigation of the character of the asymptotical expansion of the denominators $D_j(k, z)$ at $|z|, k \to 0$. Here it is shown that the main terms of the expansion are irregular in z and k. As a consequence there exists not only the nonanalytical dependence of the poles $D^{-1}_j(k, z)$ on the wave number k, demonstrated in /4/, but also appear the cut-lines in z-plane, where $D_j(k, z)$ loses its continuity. The existence of such cuts leads finally to not purely exponential dependence of the normal modes $\chi^j_k(t)$ at sufficiently large t.

1. KINETIC EQUATION WITH REGARD FOR THE COLLECTIVE MOTIONS AND NONLOCAL HYDRODYNAMICS

The kinetic equation for the one-particle distribution function $F_t(1)$ is obtained from hierarchy of equations /8/ for the s-particle distribution functions $F_t(1, 2, \dots, s)$ or correlation functions $G_s(1,\dots, s)$. According to /4,5,9/ the first two equations of the hierarchy for the hard-sphere system are of the form

$$\{ \frac{\partial}{\partial t} - \Lambda_1 \} F_t(1) = n \int d2\, T(1,2) \{ F_t(1) F_t(2) + G_t(1,2) \}, \quad (1.1)$$

$$\{ \frac{\partial}{\partial t} - \Lambda_1 - \Lambda_2 \} G_t(1,2) = T(1,2) \{ F_t(1) F_t(2) + G_t(1,2) \} + n(1 + P_{12}) \int d3\, T(1,3) \{ F_t(1) G_t(2,3) + F_t(3) G_t(1,2) + G_t(1,2,3) \}, \quad (1.2)$$

where n is the number density, the operator $\Lambda_s = -\vec{v}_s \frac{\partial}{\partial \vec{r}_s}$ and operator P_{12} permutes the particle indices. $T(1.2)$ is the retarded collision operator for hard spheres, defined as

$$T(1,2) = a_0^2 \int d\vec{\sigma} (\vec{v}_{12}\vec{\sigma}) \Theta(\vec{v}_{12}\vec{\sigma}) \{ \delta(\vec{r}_{12} - a_0\vec{\sigma}) B_{12}(\vec{\sigma}) - \delta(\vec{r}_{12} + a_0\vec{\sigma}) \}, \quad (1.3)$$

where a_0 is sphere diameter, $\vec{\sigma}$ is unit vector, $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$ and $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$ are relative velocity and distance, and the operator $B_{12}(\vec{\sigma})$ changes the velocities $\vec{v}_1, \vec{v}_2$ into the velocities "before the collision",

$$B_{12} f(\vec{v}_1, \vec{v}_2) = f(\vec{v}_1^*, \vec{v}_2^*); \quad \vec{v}_1^* = \vec{v}_1 - \vec{\sigma}(\vec{v}_{12}\vec{\sigma}),$$

$$\vec{v}_2^* = \vec{v}_2 + \vec{\sigma}(\vec{v}_{12}\vec{\sigma}).$$

Let us write down the one-particle function $F_t(1)$ and the two-particle correlation function $G_t(1,2)$ in the form

$$F_t(1) = \phi(1)\{1 + \chi_t(1)\},$$

$$G_t(1,2) = \phi(1)\phi(2)\{g(|\vec{r}_1 - \vec{r}_2|) + \chi_t(1,2)\}, \qquad (1.4)$$

where $\phi(1)$ is the normalized Maxwellian

$$\phi(1) = \phi(v_1) = (2\pi u^2)^{-3/2} \exp\{-v_1^2/2u^2\}, \quad u = \sqrt{\theta/m},$$

and $g(|\vec{r}_{12}|)$ is the equilibrium binary correlation function. We insert (1.4) into the first equations of the hierarchy (1.1), (1.2), and, assuming that the system weakly deviates from the equilibrium state and has sufficiently small density, neglect the nonlinear terms of the type $\chi \cdot \chi$ as well as the tripple correlations $G_t(1,2,3)$. As a result we obtain the linear system of equations for $\chi_t(1)$ and $\chi_t(1,2)$, which is used for the investigation of the hard-sphere gas state evolution. Suppose that at the initial moment $t=0$ the system is already in a local equilibrium state and put $\chi_{t=0}(1,2)=0$. Then passing on to the Laplace time representation and the Fourier space representation,

$$f_z = \int_0^\infty dt\, e^{-zt} f(t), \quad f_{\vec{k}} = \int d\vec{r}\, e^{-i\vec{k}\vec{r}} f(\vec{r})$$

and solving the equation for $\chi_t(1,2)$ we get the following equation for the function $\chi_t(1)$ /4/:

$$\{z + i\vec{k}\vec{v} - nL_{\vec{k}}(\vec{v}) - nR_{\vec{k}z}(\vec{v})\}\chi_{\vec{k}z}(\vec{v}) = \chi_{\vec{k}}(\vec{v}, 0). \qquad (1.5)$$

Here* $\chi_{\vec{k}}(\vec{v}, 0)$ is the space Fourier-transform of the function $\chi_{t=0}(1)$. The operator $L_{\vec{k}}(\vec{v})$ is the Fourier-transform of the operator $L(1)$ and has the form:

$$L_{\vec{k}}(\vec{v}_1) = \int d\vec{v}_2\, \phi(\vec{v}_2)\{T_0(\vec{v}_1, \vec{v}_2) + T_{\vec{k}}(\vec{v}_1, \vec{v}_2)P_{12}\}, \qquad (1.6)$$

where $T_{\vec{k}}(\vec{v}_1, \vec{v}_2)$ is the Fourier-transform with respect to $\vec{r}_{12}$ of the collision operator (1.3)

$$T_{\vec{k}}(\vec{v}_1, \vec{v}_2) = \int d\vec{r}_{12}\, e^{-i\vec{k}\vec{r}_{12}}\, T(1,2). \qquad (1.7)$$

*Note that in deriving (1.5) the terms, including the equilibrium correlations $g(|\vec{r}_{12}|)$, were neglected as in /4/. The consideration of these terms will lead apparently to the appearance of density corrections to the collision operator $T(1,2)$. Point out also that in the expression for $R_z(1)$ (1.8) it is convenient, as it has been shown in /4/, to conserve the operator $T(1,2)$ in braces. This term is absent in the system of equations /5/, but in the considered approximation it is not essential.

Finally $R_{\vec{k}z}(\vec{v})$ is the Fourier-transform of the operator

$$R_z(1) = \int d2\,\phi(2)\,T(1,2)\{z - \Lambda_1 - \Lambda_2 - T(1,2) - nL(1) - nL(2)\}^{-1}\,T(1,2)\,(1 + P_{12}) \tag{1.8}$$

The explicit form of the Fourier representation for $R_z(1)$ will be obtained later on.

On the basis of the kinetic equation (1.5) we shall build the linearized hydrodynamic equations, i.e., the system of equations for deviations of the average values of the functions $1, \vec{v}$ and v^2 from the corresponding equilibrium values. It is convenient to consider the linear combinations of $1, \vec{v}, v^2$ which are eigenfunctions of the linearized Boltzmann operator $(nL_0 - i\vec{k}\vec{v})$ calculated in the lowest in $k \to 0$ approximation,

$$a^{\sigma}_{\vec{k}}(\vec{v}) = \frac{v^2}{\sqrt{30}\,u^2} + \sigma \vec{g}^{(3)}\frac{\vec{v}}{u\sqrt{2}}, \quad \sigma = \pm 1;$$

$$a^{T}_{\vec{k}}(\vec{v}) = \frac{1}{\sqrt{10}}\left(\frac{v^2}{u^2} - 5\right); \tag{1.9}$$

$$a^{\eta_a}_{\vec{k}}(\vec{v}) = \vec{g}^{(a)}\,\vec{v}/u, \quad a = 1,2,$$

where $\vec{g}^{(1)}, \vec{g}^{(2)}, \vec{g}^{(3)}$ are unit vectors,

$$\vec{g}^{(3)}k = \vec{k}, \quad \vec{g}^{(i)}\vec{g}^{(j)} = \delta_{ij}.$$

The functions $a^{j}_{\vec{k}}(\vec{v})$ are orthonormalized,

$$\langle a^{r}_{\vec{k}}, a^{s}_{\vec{k}} \rangle = \delta_{rs},$$

with respect to the scalar product

$$\langle f_1, f_2 \rangle = \int d\vec{v}\,\phi(v)\,f_1(\vec{v})\,f_2(\vec{v}). \tag{1.10}$$

The deviations from the equilibrium averages we are interested in are defined by

$$\chi^{r}_{kz} = \langle a^{r}_{\vec{k}}, \chi_{\vec{k}z} \rangle; \quad r = \sigma, T, \eta_1, \eta_2. \tag{1.11}$$

The hydrodynamics equations can be obtained from (1.5) by using the projection-operator method of Zwanzig [10] and represented in the form [4],

$$\sum_{j=\sigma,T,\eta_1,\eta_2} \{(z + ik\omega_i)\delta_{ij} + k^2\bar{U}_{ij}(k,z)\}\chi^{j}_{kz} = \chi^{i}_{k}(0) - ikI^{i}_{kz}, \tag{1.12}$$

where

$$\omega_T = \omega_{\eta_a} = 0, \quad \omega_\sigma = \sigma c, \quad c = u\sqrt{\frac{5}{3}}, \tag{1.13}$$

c is the low-density sound velocity. The matrix elements $\bar{U}_{ij}(k,z)$ are of the form

$$\bar{U}_{rs}(k,z) = \langle a^{r}_{\vec{k}}, (v_3 - \omega_r)\{z + iP_\perp(\vec{k}\vec{v}) - nL_0 - nR_{\vec{k}z}\}^{-1}(v_3 - \omega_s)a^{s}_{\vec{k}} \rangle, \tag{1.14}$$

where $P_\perp$ is the projection-operator of the velocity functions onto the subspace orthonormal to the hydrodynamic basis (1.9). Here for the sake of definiteness $\vec{k}$ is taken to be parallel to the z axis, $\vec{k}\vec{v} = kv_3$. Both functions in the right-hand side of (1.12) depend on the initial condition $\chi_{\vec{k}}(\vec{v}, 0)$. The first of them is the projection $\chi_{\vec{k}}(\vec{v},0)$ onto the corresponding basis function, $\chi^i_{\vec{k}}(0) = \langle a^i_{\vec{k}}, \chi_{\vec{k}}(0)\rangle$. The second one can be obtained if in the right-hand side (1.14) under the sign of the scalar product, one replaces $(v_3 - \omega_s)a^s_{\vec{k}}$ by $P_\perp \chi_{\vec{k}}(0)$. When deriving (1.12) and (1.14) we, as in /4/, have replaced in (1.5) the operator $L_{\vec{k}}(v)$ by the linearized Boltzmann operator $L_0(v)$. Here we suppose that the matrix elements $\langle a^i_{\vec{k}}, L_{\vec{k}}, a^j_{\vec{k}}\rangle$, as well as $\langle a^i_{\vec{k}}, R_{\vec{k}z}\, a^j_{\vec{k}}\rangle$, give disappearing in the low density limit corrections to the coefficients of the regular part of expansion in k for the functions in the braces (1.12), to the sound velocity, for instance.

2. HYDRODYNAMIC NORMAL MODES

Finally we shall be interested in the character of the decrease of the hydrodynamical perturbations at $t \to \infty$

$$\chi^j_k(t) = \int_{-i\infty+\sigma_0}^{i\infty+\sigma_0} \frac{dz}{2\pi i} e^{zt} \chi^j_{k,z}, \qquad (2.1)$$

slowly changing at the distances of the order of free path length λ_0, calculated in the lowest order in the density $(k \ll k_0 \sim \lambda_0^{-1})$. Therefore, it is sufficient to consider equation (1.12) at small k and $|z|$ $(|z| \lesssim ck)$.

If the nondiagonal elements (of the order of k^2) of the coefficient matrix are neglected in (1.12), the system of equations "splits" and its solutions, which are the normal hydrodynamic modes at small k, can be written as

$$\chi^j_{k,z} = \frac{\chi^j_k(0)}{D_j(k,z)}; \quad D_j(k,z) = z + ik\omega_j + k^2 \bar{U}_{jj}(k,z). \qquad (2.2)$$

The consideration of the nondiagonal elements $\bar{U}_{ij}(k,z)$ and the $I^j_{k,z}$ quantities from the right-hand side of (1.12) gives the corrections of $O(k^3)$ to the function $D_j(k; z)$ and the corrections of $O(k)\chi^j_k(0)$ to the function $\chi^j_k(0)$. (Note, that if the linearization of the system of equations (1.1), (1.2) is performed correctly, the initial value $\chi^j_k(0) \to 0$ at $k \to 0$). Within the same approximation we may also neglect the explicit dependence on k and z in formulae (1.14) and use for $\bar{U}_{jj}(k, z)$ the following expression:

$$\bar{U}_{jj}(k, z) \equiv \bar{U}_j(k, z) = -\frac{1}{n}\langle a^j_k, (v_3 - \omega_j)\{L_0 + R_{\vec{k},z}\}^{-1}(v_3 - \omega_j) a^j_{\vec{k}}\rangle. \qquad (2.3)$$

Here $\bar{U}_j(k,z)$ (2.3) depends on k and z only through the k and z dependence of the operator $R_{\vec{k},z}$. The last can be justified by the fact that the asymptotical expansion of the functions $[\bar{U}_j(k,z) - \bar{U}_j(0,0)]$ begins from the nonregular terms decreasing at $k, |z| \to 0$ slower than z or k.

From this point of view consider the operator $R_{\vec{k},z}(\vec{v})$. Let us expand the braces in (1.8) in the series with respect to the collision operator $T(1,2)$ and perform the Fourier transformation. We get

$$R_{\vec{k},z}(\vec{v}) = \sum_{n=1}^{\infty} R^{(n)}_{\vec{k},z}(\vec{v}), \qquad (2.4)$$

where

$$R^{(n)}_{\vec{k},z}(\vec{v}_1) = \int \frac{d\vec{q}_1 \dots d\vec{q}_n}{(2\pi)^{3n}} \, d\vec{v}_2 \phi(v_2) T_{\vec{q}_1}(\vec{v}_1, \vec{v}_2) \times$$

$$\times G(\vec{k}-\vec{q}_1, \vec{v}_1; \vec{q}_1, \vec{v}_2|z) T_{\vec{q}_2-\vec{q}_1}(\vec{v}_1,\vec{v}_2) G(\vec{k}-\vec{q}_2, \vec{v}_1; \vec{q}_2, \vec{v}_2|z) \dots$$

$$\dots T_{\vec{q}_n-\vec{q}_{n-1}}(\vec{v}_1,\vec{v}_2) G(\vec{k}-\vec{q}_n, \vec{v}_1; \vec{q}_n, \vec{v}_2|z) \times \qquad (2.5)$$

$$\times [T_{-\vec{q}_n}(\vec{v}_1, \vec{v}_2) + T_{\vec{k}-\vec{q}_n}(\vec{v}_1, \vec{v}_2) P_{12}].$$

We have introduced here the operator $G(\vec{k}_1,\vec{v}_1; \vec{k}_2,\vec{v}_2|z)$ directly connected with the Green function for the linearized Enskog-Boltzmann equation,

$$G(\vec{k}-\vec{q}, \vec{v}_1; \vec{q}, \vec{v}_2|z) =$$

$$= [z + i\vec{q}(\vec{v}_2 - \vec{v}_1) + i\vec{k}\vec{v}_1 - nL_{\vec{k}-\vec{q}}(\vec{v}_1) - nL_{\vec{q}}(\vec{v}_2)]^{-1} =$$

$$= \sum_{\alpha,\beta} \frac{||\bar{a}^{\alpha}_{\vec{k}-\vec{q}}(\vec{v}_1)\bar{a}^{\beta}_{\vec{q}}(\vec{v}_2)\rangle\rangle\langle\langle\bar{a}^{\beta}_{\vec{q}}(\vec{v}_2)\bar{a}^{\alpha}_{\vec{k}-\vec{q}}(\vec{v}_1)||}{z - \bar{z}_{\alpha}(\vec{k}-\vec{q}) - \bar{z}_{\beta}(q)} . \qquad (2.6)$$

The last equality corresponds (in Dirac notations and with the scalar product (1.10) to the expansion of the kernel of the operator $G(\vec{\ell}, \vec{v}_1; \vec{q}, \vec{v}_2|z)$ in the system of the exact eigenfunctions $\bar{a}^{\alpha}_{\vec{k}}(\vec{v})$ of the operator $(nL_{\vec{k}} - i\vec{k}\vec{v})$, corresponding to the eigenvalues $\bar{z}_{\alpha}(k)$. We shall omit here the detailed investigation of the expressions (2.5), (2.6), performed in [4]. Note, however, that any analytical singularities of the integrals (2.5) at $k \to 0$, $z \to 0$ can be connected only with the integration over the region $|\vec{q}_j| \sim 0$. Moreover, the contribution to these singularities is given only by the terms of the sum (2.6) which correspond to the intermediate ("virtual") hydrodynamic modes with the frequencies $\bar{z}_{\alpha}(q) \to 0$ at $q \to 0$. Therefore, in order to guarantee the accuracy only of the first terms of the asymptotic expansion $R_{\vec{k}z}(\vec{v})$ at $k, |z| \to 0$ we limit ourselves in (2.6) only to the contributions from the "virtual" hydrodynamic modes to all operators $G(\vec{\ell}, \vec{v}_1; \vec{q}, \vec{v}_2|z)$ in (2.5). Cut off all integrals over $\vec{q}_1, \vec{q}_2, \dots, \vec{q}_n$ at $|\vec{q}| \le k_0$, where k_0^{-1} is of the order of free path λ_0. Furthermore, due to $a_0 \ll \lambda_0$ we can neglect also the dependence of the collision operator $T_{\vec{q}}(\vec{v}_1, \vec{v}_2)$ on the wave number, as this dependence is essential at $q \sim a_0^{-1}$ according to (1.7). Besides, for the values $\bar{a}^{\alpha}_{\vec{k}}(\vec{v})$ and $\bar{z}_{\alpha}(k)$ in (2.6) we use the eigenfunctions $a^{i}_{\vec{k}}(\vec{v})$ (1.9) of the perturbed linearized Boltzmann operator $(nL_0 - i\vec{k}\vec{v})$. These eigenfunctions are calculated to the zero approximation in k, and the corresponding eigenfrequences are calculated in k^2-approximation,

$$z_\sigma(k) = -i\sigma ck - \frac{1}{2}\Gamma k^2 , \quad \sigma = \pm 1;$$

$$z_T(k) = -k^2 D_T ;$$

$$z_{\eta_1}(k) = z_{\eta_2}(k) = z_\eta(k) = -k^2 D_\eta ; \qquad (2.7)$$

$$\frac{1}{2}\Gamma = \frac{1}{3} D_T + \frac{2}{3} D_\eta .$$

Here $D_\eta = \eta/nm$ is the kinematic viscosity, $D_T = 2\kappa/5n$ is the thermodiffusion coefficient, Γ is the sound-wave damping constant. Coefficients of the shear viscosity η and the heat conductivity κ are given by their low-density values,

$$\eta = -\frac{m}{u^2} < v_x v_y , L_0^{-1} v_x v_y > \approx \frac{5um}{16\sqrt{\pi}\, a_0^2} ,$$

$$\kappa = -\frac{1}{4} <(\frac{v^2}{u^2} - 5)v_x , L_0^{-1}(\frac{v^2}{u^2} - 5)v_x > \approx \frac{15}{4}\frac{\eta}{m} . \qquad (2.8)$$

Due to these approximations the series (2.4), (2.5) can be summed. The insertion of the result of summation into (2.3) gives after using the definition of the scalar product and hydrodynamic frequencies (1.13), (2.7), (2.8), the following expression for the functions $\bar{U}_j(k, z)$:

$$k^2 \bar{U}_j(k, z) = -z_j(k) - ik\omega_j + k^2 U_j(k, z). \qquad (2.9)$$

Thus the denominators in the expressions (2.2) for the hydrodynamic normal modes can be written down now as

$$D_j(k, z) = z - z_j(k) + k^2 U_j(k, z). \qquad (2.10)$$

For the functions $U_j(k, z)$ we get

$$U_j(k, z) = \sum_{r,\ell} U_j^{(r,\ell)}(k, z), \qquad (2.11)$$

$$U_j^{(r,\ell)}(k, z) = \frac{u^2}{2n} \int_{q \le k_0} \frac{d\vec{q}}{(2\pi)^3} \frac{|A_j^{r,\ell}(\vec{k} - \vec{q}, \vec{q})|^2}{z - z_r(\vec{k} - \vec{q}) - z_\ell(\vec{q})} , \qquad (2.12)$$

where j, r, ℓ are the "hydrodynamic" indices, $j, r, \ell = \sigma, T, \eta_1, \eta_2$. The coefficients $A_j^{r,\ell}(\vec{q}, \vec{q}')$ depend only on the unit vectors $\vec{q}/q$, $\vec{q}'/q'$ and represent the following integrals:

$$A_j^{r,\ell}(\vec{q}, \vec{q}') .=$$

$$= u^{-1} \int d\vec{v} \phi(v)\, a^j_{\vec{k}}(\vec{v})\, a^r_{\vec{q}}(\vec{v}) a^\ell_{\vec{q}'}(\vec{v}) \{ v_3 - \omega_j \}. \qquad (2.13)$$

Note that some of the coefficients $A_j^{r,\ell}(\vec{q}, \vec{q}')$ turn into zero due to integrand (2.13) being odd relative to the vector $\vec{v}$ components. Namely:

$$A_{\eta_\alpha}^{\eta_\beta, T}(\vec{q}, \vec{q}') = A_{\eta_\alpha}^{\sigma, T}(\vec{q}, \vec{q}') = A_{\eta_\alpha}^{T,T}(\vec{q}, \vec{q}') = 0;$$

$$A_T^{T,T}(\vec{q}, \vec{q}) = A_T^{\eta_\alpha, \eta_\beta}(\vec{q}, \vec{q}') = 0 \quad A_\sigma^{T,T}(\vec{q}, \vec{q}') = 0. \qquad (2.14)$$

Therefore, the corresponding terms $U_j^{(r,\ell)}$ drop out of the sums (2.11). Besides, using the invariance of the functions $a_{\vec{k}}^{j}(\vec{v})$ (1.9) and the frequencies $z_j(k)$ (2.7) relative to the rotations of the vectors $\vec{k}$ and $\vec{v}$, it may be shown that the functions $U_j(\vec{k},z)$ depend only on $|\vec{k}|$ and the equality $U_{\eta_1}=U_{\eta_2}\equiv U_\eta$ is true.

3. THE BEHAVIOUR OF THE FUNCTIONS $U_j(k, z)$ AT SMALL $k, |z|$

The relations (2.10) and (2.11), obtained as a result of the above-mentioned approximations, permit us to guarantee only the main terms of the asymptotical expansion of the functions $D_j(k, z)$ and $U_j(k, z)$ with coefficients, defined in the low density limit. Show that in this approximation we should neglect some terms in sums (2.11). Just from the form of integrals (2.12) it is easy to see that any analytical singularities of the functions $U_j(k, z)$ at $k, |z|\to 0$ can be connected only with integration over the region $q\sim 0$ (say $0\le q\le q_0<k_0$, q_0 is arbitraty), where all the hydrodynamic frequencies $z_j(q)$ turn into zero. For example, all the derivatives from $U_j(k, z)$ with respect to z, beginning from the second one, diverge at $k, |z|\to 0$, namely, near the lowest limit $q=0$. At the same time the integrals of the type (2.12) over the region $q_0\le q\le k_0$ are the regular functions z at sufficiently small k and $|z|$. Thus, if one expresses the first terms of the asymptotical expansion $U_j(k, z)$ as a sum (in the region of the small k and $|z|$) of the regular and nonanalytic functions,

$$U_j(k, z) = U_j(0, 0) + U_j^{(reg)}(k, z) + U_j^{(sing)}(k, z),$$

$$U_j^{(reg)}(k, z) = c_1 z + c_2 k + \dots ,$$

then for the existence of $U_j^{(sing)}(k, z)$ only the integral (2.12) over the vicinity of q=0 is responsible, while the contribution to the regular part $\{U_j(0, 0) + U_j^{(reg)}(k, z)\}$ is given mainly by the region of large $q\approx k_0$. The coefficients $U_j(0, 0), c_1, c_2$ depend, surely, on the parameter k_0. If the expansion of the function $\{U_j(k, z) - U_j(0, 0)\}$ begins from the regular part, and $U_j^{(sing)}(k, z)$ decreases more quickly at $k, |z|\to 0$, then, in the framework of the accepted approximation, one must neglect not only $U_j^{(sing)}$, but also $U_j^{(reg)}$ (as it was done in the course of the derivation of (2.3)). Then the consideration of $U_j(k, z)$ in $D_j(k, z)$ (2.10) will lead to the appearance of the small addition (in the low - density limit) to the corresponding kinetic coefficient of the form

$$U_j(0,0) = -\sum_{\ell,r} \frac{u^2}{2n} \int_{q<k_0} \frac{d\vec{q}}{(2\pi)^3} \frac{|A_j^{\ell,r}(-\vec{q},\vec{q})|^2}{z_\ell(q) + z_r(q)} \sim \frac{u^2 k_0}{nD_j} \sim (na_0^3)^2 D_j, \quad (3.1)$$

where D_j is one of the coefficients: D_η, D_T or Γ.

Thus, we are interested only in the case, when the main term in the expansion of the difference $\{U_j(k, z) - U_j(0, 0)\}$ is $U_j^{(sing)}(k, z)$. It is easy to see that we deal with just this situation. To this end it is sufficient to show that in the sum (2.11) there exist the terms $U_j^{(r,\ell)}(k,z)$ with even first derivatives with respect to z divergent at $k, |z| \to 0$. Really, equations (2.12) yield:

$$\frac{\partial}{\partial z} U_j^{(r,\ell)}(k,z)\Big|_{\substack{k\to 0\\ z\to 0}} = -\frac{u^2}{2n} \int_{q<k_0} \frac{d\vec{q}}{(2\pi)^3} \left| \frac{|A_j^{r,\ell}(-\vec{q}, \vec{q})|^2}{[z_r(\vec{k}-\vec{q}) + z_\ell(q) - z]^2} \right|_{\substack{k\to 0\\ z\to 0}} . \qquad (3.2)$$

The quantities $A_j^{r,\ell}(-\vec{q},\vec{q})$ are the functions of the unit vector $\vec{q}/q$, and so convergence of the integral depends only on the behaviour of the integral denominator at $q \to 0$. From the formulae (2.7) for hydrodynamic frequencies it follows that when

$$r, \ell = \eta_\alpha, \eta_\beta;\ r, \ell = \eta_\alpha, T;\ r = \ell = T;\ r = \sigma,\ \ \ell = -\sigma, \qquad (3.3)$$

the sum of frequencies $[z_r(q) + z_\ell(q)] \sim q^2$ at $q \to 0$ and integral (3.2) diverges at $q=0$ if $k, |z| \to 0$. For the rest couples of indices r, ℓ integral (3.2) converges* at $k=0, z=0$. It is possible also to show that the first derivative of $U_j^{(r,\ell)}(k,z)$ with respect to k for the indices r, ℓ in (3.3) even if diverges at $k, |z| \to 0$ then not faster than integral (3.2). In the rest cases $\partial U^{(r,\ell)}/\partial k$ exists. It means that the series expansion of $U_j^{(\ell,r)}(k,z)$ for the indices ℓ, r out of set (3.3) begins either from the regular terms of the type $c_1 z + c_2 k$ or from the nonregular terms decreasing at $k, |z| \to 0$ faster than $c_1 z + c_2 k$. In both

*For some couples of indices r, ℓ the functions $U_j^{(r,\ell)}(k,z)$ are nondifferentiable near $z=0$ for finite k. In this case the reasoning made above needs the correction which, however, doesn't change the obtained conclusions.

the cases one must neglect these terms with respect to the main asymptotics of the functions $U_j^{(\ell,r)}(k,z)$ with the indices ℓ, r from the set (3.3), which only must be retained in the sum (2.11).

Thus, taking into account (2.14) we get the following expression for the main terms of asymptotical expansion of the functions

$$U_\eta(k,z) = U_\eta^{(s)} + U_\eta^{(\eta)}\ ;\ \ U_T(k,z) = U_T^{(s)} + U_T^{(\eta T)}\ ;$$

$$U_\sigma(k,z) = U_\sigma^{(s)} + U_\sigma^{(\eta)} + U_\sigma^{(\eta T)}\ ;$$

$$U_j^{(\eta)} = \sum_{\alpha,\beta=1,2} U_j^{(\eta_\alpha,\eta_\beta)};\ \ U_j^{(s)} = \sum_{\sigma'=\pm 1} U_j^{(\sigma',-\sigma')}; \qquad (3.4)$$

$$U_j^{(\eta T)} = \sum_{\alpha=1,2} \{U_j^{(\eta_\alpha,T)} + U_j^{(T,\eta_\alpha)}\}.$$

From the rest terms of sum (2.11) the constants $U_j^{(r,\ell)}(0,0)$ retained in (3.4), which are of the same order of magnitude as $U_\eta(0,0)$, $U_\sigma(0,0)$, $U_T(0,0)$. Finally, however, we shall neglect the quantities $U_j(0,0)$ giving, according to estimation (3.1), nonessential corrections to the kinetic coefficients and don't affecting the character of the time asymptotics of the hydrodynamic modes. For the same reason the functions $U_j^{(r,\ell)}(k,z)$ in (3.4) may be defined by relations (2.12), where the arguments $(\vec{k}-\vec{q})$ and $\vec{q}$ in the integrand are replaced by $(a\vec{k}-\vec{q})$ and $(b\vec{k}+\vec{q})$ with sufficiently arbitrary constants a, b satisfying the condition $a + b = 1$, $|a| \sim 1$, $|b| \sim 1$. In fact, substitution of the variable $\vec{q}=\vec{q}'+b\vec{k}$ into (2.12) shifts the whole integration region of the vector $b\vec{k}$, $|\vec{q}'+b\vec{k}|\le k_0$. If $bk\sim k\ll k_0$ we can consider, however, that as before $|\vec{q}'|\le k_0$, neglecting at the same time the contribution in (2.12), which results from the integration over the region, adjoining to the boundary, $q'\sim k_0$. As is shown above, this circumstance doesn't affect analytic singularities of the functions $U_j^{(r,\ell)}(k,z)$ from (3.4) at small k and $|z|$ and counts only on the constants $U_j^{(r,\ell)}(0,0)$ depending on the cutting parameter.

In conclusion we make some general remarks concerning the character of time dependence of the hydrodynamic modes represented by integral (2.1). The function χ^j_{kz} (2.2) has the poles on z-plane at the points where the denominator $D_j(k,z)$ (2.10) turns into zero. As according to (3.1) $U_j(k,z)$ is proportional to the small parameter of the problem, the solution of the equation $D_j(k,z)=0$ in the low density approximation is of the form

$$\bar{z}_j(k)\approx z_j(k)-k^2U_j(k,z_j(k)). \qquad (3.5)$$

For the reason of nonregularity of the main terms of the functions $U_j(k,z)$ (3.4) expansion in k and z it is clear that the "renormalized" hydrodynamic frequencies $\bar{z}_j(k)$ should depend nonanalytically on the wave number k /4/.

Besides, for each of the functions $U_j(k,z)$ as well as for $D_j(k,z)$ there is the line on the z-plane crossing which the function changes by jump. As is shown in Appendix A, just the functions $U_j^{(\eta)}$, $U_j^{(\eta T)}$ from (3.4), containing the contributions from the intermediate viscous modes or viscous and heat ones, possess these singularities. (Remark that, according to Appendix A results, all the functions $U_j^{(r,\ell)}(k,z)$ not included in (3.4) are continuous in z).

Thus, the integral (2.1) breaks down into two parts

$$\chi^j_k(t)/\chi^j_k(0)=E^j(t)=E^j_1(t)+E^j_2(t), \qquad (3.6)$$

where the first term contains the contribution from the poles $D_j^{-1}(k,z)$ and is of the form

$$E^j_1(t)\sim\exp\{t\bar{z}_j(k)\}. \qquad (3.7)$$

The second term represents the integral over the cut-line and is written down as

$$E^j_2(t)=\int_{x_1}^{x_2}\frac{dx}{\pi}e^{xt}\,\mathrm{Im}\{D_j^{-1}(k,x)\}, \qquad (3.8)$$

where the position of the cut-line and the value of the jump of the function $D_j^{-1}(k,z)$ on it, $\mathrm{Im}\{D_j^{-1}(k,z)\}$, are determined with the help of (A.8), (A.7) - type relations. The asymptotics of the integral (3.8) is computed, as it is known, by means of expansion of $\mathrm{Im}\{D_j^{-1}\}$ near the right end of the cut. This asymptotics in contradistinction to (3.7) is not of purely exponential character. In our next work, where we intend to make the concrete calculations for each of the hydrodynamic modes, it will be shown that at sufficiently large times namely $E^j_2(t)$ plays the main role at the time asymptotics of the mode $\chi^j_k(t)$.

The authors sincerely thank Academician N.N.Bogolubov for his interest to this work and useful remarks.

APPENDIX A

Consider some analytical properties of the functions $U_j^{(r,\ell)}(k, z)$ in the complex variable z. Considering the dependence of all hydrodynamic frequencies $z_r(q)$ only on $|\vec{q}|$ we pass on in the integrals (2.12) to the spherical coordinates, representing these integrals in the forms:

$$U(z) = \int_0^{k_0} dq \int_{-1}^{1} d\xi \frac{\Psi(\xi, q)}{z - \phi(\xi, q)}, \qquad (A.1)$$

Here $\xi = \cos(\widehat{\vec{k},\vec{q}})$ and $\phi(\xi, q)$ is the sum of frequencies,

$$\phi(\xi, q) = z_r(|\vec{k} - \vec{q}|) + z_\ell(|\vec{q}|) = \phi_1(\xi, q) + i\phi_2(\xi, q). \qquad (A.2)$$

All analytical singularities of the function $U(z)$ are determined by the existence and the nature of the integrand (A.1) denominator zeros. Evidently, $U(z)$ is regular in that region of the z-plane, for the points of which the equations

$$z = x + iy = \phi(\xi, q) \quad \text{or} \quad \begin{cases} x = \phi_1(\xi, q) \\ y = \phi_2(\xi, q) \end{cases} \qquad (A.3)$$

have no solution.

Suppose that for the region Φ points of the z-plane the solution of (A.3) exists. Introduce the Jacobian

$$J(\xi, q) = \frac{\partial\phi_1}{\partial\xi}\frac{\partial\phi_2}{\partial q} - \frac{\partial\phi_1}{\partial q}\frac{\partial\phi_2}{\partial\xi} \qquad (A.4)$$

and consider two cases: a) the Jacobian $J(\xi, q)=0$ for all ξ, q ($|\xi| \le 1$, $0 \le q$), b) the Jacobian $J(\xi, q) \ne 0$ for all ξ, q.

The Jacobian (A.4) may be equal to zero for all ξ, q only when ϕ_1 and ϕ_2 are functionally connected, i.e., are bound, for instance, by the relation

$$\phi_2(\xi, q) = F(\phi_1(\xi, q)). \qquad (A.5)$$

Then the region Φ is some curve in the z-plane, defined by the equation

$$y = F(x). \qquad (A.6)$$

We show that when passing over the curve $z = x + iF(x)$ on the z-plane the function $U(z)$ changes by jump. Make to this end the replacement of variables ξ and q in the integral (A.1) by $\xi, \zeta = \phi_1(\xi, q)$ and consider the difference of the function $U(z)$ values "above" and "below" the curve in the points $z_{1,2} = x + iF(x) \pm i0[1 + iF'(x)]$. The contribution to this difference is given, evidently, only by the integration over the vicinity of the point $\zeta = x$, where the denominator of the integrand (A.1) in the variables ξ, ζ may be written, taking account of (A.5), as

$$x + iF(x) \pm i0[1 + iF'(x)] - \zeta - iF(\zeta) \approx [1 + iF'(x)](x - \zeta \pm i0).$$

Now use the relation $[x - i0]^{-1} - [x + i0]^{-1} = 2\pi i\delta(x)$, and, returning to the variables ξ, q, obtain the expression for the jump

$$U(z(x) - i0z'(x)) - U(z(x) + i0z'(x)) = \qquad (A.7)$$

$$= 2\pi i[1 + iF'(x)]^{-1}\int_0^{k_0}\int_{-1}^{1} dq\, d\xi\, \Psi(\xi, q)\delta(x - \phi_1(\xi, q)).$$

A simple example of the considered situation is the case of both frequencies in (A.2) being real quantities, such as $r, \ell = \eta_\alpha, \eta_\beta$ or $r, \ell = \eta_\alpha, T$. Then $\phi_2(\xi, q) = 0$ and $y = F(x) = 0$ and the formula (A.7) determines the jump of the function $U(z)$ imaginary part when crossing the real axis $y = 0$ on the section

$$\min_{\xi,q} \phi_1(\xi, q) \leq x \leq \max_{\xi,q} \phi_1(\xi, q). \qquad (A.8)$$

Consider now the case b). As $J(\xi, q) \neq 0$ for all ξ, q, we can pass on in the integral (A.1) to new variables $\zeta = \phi_1(\xi, q)$, $\kappa = \phi_2(\xi, q)$ and write it down in the form

$$U(z) = \iint_{\Phi} d\zeta\, d\kappa \frac{\tilde{\Psi}(\zeta, \kappa)}{x + iy - \zeta - i\kappa}. \qquad (A.9)$$

It is easy to make sure that the integral (A.9) doesn't diverge if $z \in \Phi$. For this purpose it is sufficient to single out from (A.9) the integral $\delta U(z)$ over the small vicinity $|\zeta + i\kappa - z| \leq \delta$ of the "singular" point $\zeta + i\kappa = z$ and in this vicinity to pass to the polar coordinates $\zeta - x = r\cos\varphi$, $\kappa - y = r\sin\varphi$. Then we get

$$\delta U(x) = -\int_0^{\delta} dr \int_0^{2\pi} d\varphi\, e^{-i\varphi}\tilde{\Psi}(x + r\cos\varphi, y + r\sin\varphi) = o(\delta). \qquad (A.10)$$

If the point $\zeta + i\kappa = z$ is situated at the distance $r_0 < \delta$ from the region Φ boundary, the U estimation δU changes somewhat, $\delta U = O(\delta)$, but in both cases the integral over the vicinity $\zeta + i\kappa = z$ remains finite and disappears at $\delta \to 0$. Using the obtained estimations it's easy to check that $U(z)$ is the continuous function of z in the region Φ. More detailed analysis shows the function $U(z)$ being non-differentiable in the region Φ (the Cauchy-Riemann conditions are not satisfied).

So, if the conditions $J(\xi, q) = 0$ or $J(\xi, q) \neq 0$ are satisfied for all $|\xi| \leq 1$, $0 \leq q$, then in the first case the function $U(z)$ is regular on the z-plane with the cut along the curve (A.6), $z = x + iF(x)$, near which its value changes by jump. In the second case the function $U(z)$ is regular outside the region Φ of the existence of the equations system (A.3) solutions, and it is continuous but non-differentiable inside Φ. It is easy to spread the last conclusing to the case when $J(\xi, q) \neq 0$ for ξ, q excepting some curve on ξ, q -plane. Namely such situation arises if one or both of the "virtual" modes are sound ones, e.g., in (A.2) the indices $r = \sigma$; $\ell = \sigma$ or $r = \sigma$, $\ell = \sigma'$.

REFERENCE

1. Alber B.J., Wainwright T.E. Journ.Phys, Soc. Japan (Suppl.), 1968, v.26, p. 267; Phys.Rev.Lett., 1969, v.18, p. 988.
2. Pomeau Y., Resibois P. Time Dependent Correlation Functions and Mode-Mode Coupling Theories, Physics reports, 1975, v. 19c, No. 2.
3. Bogolubov N.N. JINR, E17-10514, Dubna, 1977.
4. Ernst M.N., Dorfman J.P. Physica, 1972, v. 61, p. 157.
5. Боголюбов Н.Н. Кинетические уравнения и функции Грина в статистической механике. Препринт № 57 АН Азерб.ССР, Институт физики Баку, 1977.
6. Lenard A. Ann.Phys., 1960, v. 10, p. 390.
7. Balescu R. Phys.Fluids, 1960, v. 3, p.52.
8. Боголюбов Н.Н. Проблемы динамической теории в статистической физике. ОГИЗ, Гостехиздат, М.-Л., 1946.
9. Ernst M.N. et al. Physica, 1969, v. 45, p. 127.
10. Zwanzig R. Lectures in Theoretical Physics, III Britten W.E., Downs B.W., Downs J. eds. Interscience Publ., New York, 1961.

Reseived by Publishing Department on March 16 1979.