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# SOLITARY BOUND STATES BETWEEN MAGNONS AND PHONONS IN A LINEAR X-Y MAGNETIC CHAIN

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# SOLITARY BOUND STATES BETWEEN MAGNONS AND PHONONS IN A LINEAR X-Y MAGNETIC CHAIN



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Солитонные магнон-фононные состояния в линейной магнитной цепочке X-Y

Работа посвящена исследованию магнитных возбуждений в линейной цепочке частиц со спином 1/2 с учетом деформации. Цель работы - выяснить влияние деформации цепочки на свойства магнитных возбуждений. Рассматривается простейшая точно решаемая магнитная модель - "модель X-Y". По отношению к колебаниям цепочки используется квазиклассический подход. Показано, что в такой модели могут существавать возбуждения солитонного типа, представляющие собой самосогласованные связанные состояния магнонов и фононов. Эти возбуждения распространяются вдоль цепочки с постоянной скоростью и без размытия. Их энергия может лежать ниже энергии свободных магнонов.

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Solitary Bound States Between Magnons and Phonons in a Linear X-Y Magnetic Chain

It is shown that in a linear magnetic chain solitary excitations can exist as selfconsistent bound states between the spin deviation and the lattice deformation. Such excitations can move along the chain with constant velocity without smearing. The energy of the excitation can lie lower than the free magnon energy.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1978



It was shown in /1/ and /2/ (in the quasiclassical and quantum-mechanical consideration, respectively) that in a one-dimensional ferromagnetic chain of the Heisenberg type not only ordinary but also solitary magnons (or briefly "solitons") can exist and propagate along the chain with constant velocity without smearing. The present paper is devoted to the investigation of the analogous problem in the case of the so-called X-Y model /3/ in the presence of a constant magnetic field $\mathcal{H}$ and a variable magnetic field $\mathcal{H}_1 \cos \Omega t$ in the direction of axis Z. We shall follow the quasiclassical way of consideration /1/; by a straightforward examination one can see that the consistent quantum treatment /2/ leads to the same results.

Let us write the Hamiltonian of the chain in the nearest-neighbour approximation in the form:

$$H = T + U - \mu \mathcal{H} \sum_j S_j^z - \mu \mathcal{H}_1 \cos \Omega t \sum_j S_j^z - \frac{1}{2} \sum_j J(x_{j+1} - x_j) \left[ S_j^+ S_{j+1}^- + S_j^- S_{j+1}^+ \right], \qquad (1)$$

where $\mu$ is the magnetic moment, and $J(x_{j+1} - x_j)$ is the exchange integral whose value is determined by positions of the atoms at the nearest neighbouring sites in the chain (j and j+1). T and U are the kinetic and potential energy of the chain, respectively:

$$T = \frac{1}{2m}\sum_j P_j^2, \qquad U = \frac{m v_0^2}{2}\sum_j (x_{j+1} - x_j)^2 . \qquad (2)$$

Here m is the mass of the atom, $v_0$ is the sound velocity; for U the harmonic approximation is taken, and the lattice constant $a$ is assumed to be equal to 1 for simplicity.

The cyclic components of the spin vector $S_j^{\pm} = S_j^x \pm i S_j^y$, $S_j^z = \frac{1}{2} - S_j^- S_j^+$ $(S = \frac{1}{2})$ can be expressed by the Fermi creation $a_j^+$ and annihilation $a_j$ operators as follows /4/:

$$S_j^- = a_j^+ \prod_{m<j}(1 - 2a_m^+ a_m), \quad S_j^+ = \prod_{m<j}(1 - 2a_m^+ a_m)\, a_j . \qquad (3)$$

For the exchange integral we can use the linear approximation with respect to the atom displacements:

$$J(x_{j+1} - x_j) \simeq J_0 - J_1 (x_{j+1} - x_j), \qquad (4)$$

where $J_1 = -\frac{\partial J}{\partial (x_{j+1} - x_j)} > 0$ ( J decreases when the distance between the atoms increases).

By means of (3) and (4) the Hamiltonian (1) turns into:

$$H = T + U - \mu(\mathcal{H} + \mathcal{H}_1 \cos\Omega t)\frac{N}{2} +$$
$$+ \mu(\mathcal{H} + \mathcal{H}_1 \cos\Omega t)\sum_j a_j^+ a_j - \frac{J_0}{2}\sum_j (a_j^+ a_{j+1} + a_{j+1}^+ a_j) + \qquad (5)$$
$$+ \frac{J_1}{2}\sum_j (x_{j+1} - x_j)(a_j^+ a_{j+1} + a_{j+1}^+ a_j),$$



where N is the number of the atoms in the chain.

We shall seek for the solution of the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}|\Psi(t)\rangle = H|\Psi(t)\rangle \qquad (6)$$

in the following structure:

$$|\Psi(t)\rangle = \sum_j C_j(t)\, a_j^+ |0\rangle \qquad (7)$$

and normalized by the condition:

$$\langle\Psi(t)|\Psi(t)\rangle = \sum_j |C_j(t)|^2 = 1 . \qquad (8)$$

Substituting of the Hamiltonian (5) and the wave function (7) into the Schrödinger equation (6) leads to the following set of equations for the coefficients $C_j(t)$ :

$$i\hbar\frac{\partial C_j}{\partial t} = \left[T + U - \mu\left(\frac{N}{2} - 1\right)(\mathcal{H} + \mathcal{H}_1 \cos\Omega t)\right]C_j - \frac{J_0}{2}(C_{j+1} + C_{j-1}) +$$
$$+ \frac{J_1}{2}\left[(C_{j+1} x_{j+1} - C_{j-1} x_{j-1}) - (C_{j+1} - C_{j-1}) x_j\right]. \qquad (9)$$

Following /1/ we construct the functional:

$$\tilde{H} = \langle\Psi(t)|H|\Psi(t)\rangle =$$
$$= T + U - \mu(\mathcal{H} + \mathcal{H}_1 \cos\Omega t)\frac{N}{2} + \mu(\mathcal{H} + \mathcal{H}_1 \cos\Omega t)\sum_j C_j^* C_j -$$
$$- \frac{J_0}{2}\sum_j (C_j^* C_{j+1} + C_{j+1}^* C_j) + \frac{J_1}{2}\sum_j (x_{j+1} - x_j)(C_j^* C_{j+1} + C_{j+1}^* C_j)$$

which plays the role of the classical Hamilton function in terms of the canonically conjugate variables $x_j$ and $P_j$. The corresponding Hamilton equations have the form:

$$\dot{X}_j = \frac{\partial \tilde{H}}{\partial P_j} = \frac{P_j}{m}$$

$$\dot{P}_j = -\frac{\partial \tilde{H}}{\partial X_j} = -m v_0^2 \left(2X_j - X_{j+1} - X_{j-1}\right) - \tag{10}$$

$$-\frac{J_1}{2}\left[\left(C^*_{j-1}C_j + C^*_j C_{j-1}\right) - \left(C^*_{j+1}C_j - C^*_j C_{j+1}\right)\right].$$

Eliminating $\dot{P}_j$ from the system (10) we obtain the equations:

$$m\frac{\partial^2 X_j}{\partial t^2} = m v_0^2\left(X_{j+1} + X_{j-1} - 2X_j\right) + \tag{11}$$

$$+\frac{J_1}{2}\left[C^*_j\left(C_{j+1} - C_{j-1}\right) + C_j\left(C^*_{j+1} - C^*_{j-1}\right)\right].$$

The two terms in the right-hand side of (11) describe the actions of different elastic forces upon the j'th atom from the deformed lattice. The first one is connected with the kinetic degrees of freedom in the undistorted lattice and the second originates from the deformation due to the spin deviation.

The equations (9) and (11) allow one to find the atom displacements $X_j$ and the amplitudes $C_j$ which determine the distribution of the excitation along the chain. We shall seek for this set of equations in the most interesting case when the size $\mathcal{L}$ of the lattice deformation is much larger than the lattice constant ($\mathcal{L} >> 1$). Then we can go over to a continuum approximation, i.e. we can consider $X_j$ and $C_j$ as smooth functions of the continuous variable $\xi$. So $X_j \longrightarrow X(\xi, t)$, $C_j \longrightarrow C(\xi, t)$. Then the set of equations (9) and (11) can be replaced by two differential equations:



$$i\hbar\frac{\partial C}{\partial t} = \left[T + U - \mu\left(\frac{N}{2} - 1\right)\left(\mathcal{H} + \mathcal{H}_1\cos\Omega t\right)\right]C -$$

$$-\frac{J_0}{2}\left(2C + \frac{\partial^2 C}{\partial \xi^2}\right) + J_1 C\frac{\partial X}{\partial \xi} \tag{12}$$

$$m\frac{\partial^2 X}{\partial t^2} = m v_0^2\frac{\partial^2 X}{\partial \xi^2} + J_1\frac{\partial}{\partial \xi}|C|^2. \tag{13}$$

Looking for a solution of the latter equation in the form of excitations which propagate along the chain with constant velocity $v$, we assume $X = X(\xi - vt)$ and get from (13) that

$$\frac{\partial X}{\partial \xi} = -\frac{J_1|C|^2}{m v_0^2(1-\beta^2)}, \quad \beta = \frac{v}{v_0}. \tag{14}$$

We see that under this assumption $|C|^2$ is a function of one argument ($\xi - vt$) only. The equation (14) has a plain physical meaning. The deformation $u = \frac{\partial X}{\partial \xi}$ is proportional to the probability distribution $|C|^2$ of the spin excitation along the chain. The coefficient for $|C|^2$ represents the total change of the length of the chain. One can see it easily taking into account that $\int|C|^2 d\xi = 1$ according to (8). Inserting (14) into (12) we get the following nonlinear equation:

$$i\hbar\frac{\partial C}{\partial t} = \left[T + U - \mu\left(\frac{N}{2} - 1\right)\left(\mathcal{H} + \mathcal{H}_1\cos\Omega t\right) - J_0\right]C -$$

$$-\frac{J_0}{2}\frac{\partial^2 C}{\partial \xi^2} - \frac{J_1^2}{m v_0^2(1-\beta^2)}|C|^2 C, \tag{15}$$

where $T + U$ is given by

$$T+U=\frac{J_1^2}{2mV_0^2}\frac{1+\beta^2}{(1-\beta^2)^2}\int_{-\infty}^{\infty}|C(\xi,t)|^4 d\xi . \qquad (16)$$

The exact solution of equation (15) which vanishes at infinity has the form:

$$C(\xi,t)=\frac{1}{\sqrt{2\mathcal{L}}}e^{i(\kappa\xi-\omega t-\alpha\sin\Omega t-\varphi_0)}\operatorname{sech}\frac{\xi-vt-\xi_0}{\mathcal{L}} , \qquad (17)$$

where $\kappa=\frac{\hbar v}{J_0}$ coincides with the wave vector of the free magnon with velocity $v$ ($\frac{\hbar^2}{J_0 a^2}=m^*$ being the effective mass of the magnon), $\mathcal{L}=2mV_0^2(1-\beta^2)J_0J_1^{-2}$ is the size of the region where the probability distribution of the excitation differs essentially from zero,

$$\hbar\omega=T+U-\left(\frac{N}{2}-1\right)\mu\mathcal{H}-J_0+\frac{\hbar^2v^2}{2J_0}-\frac{J_1^4}{8J_0(mV_0^2)^2(1-\beta^2)^2},$$

$\alpha=-\left(\frac{N}{2}-1\right)\frac{\mu\mathcal{H}_1}{\hbar\Omega}$, and $\varphi_0$ and $\xi_0$ are arbitrary constants which can be determined by the initial conditions.

For $T+U$ from (16) by means of (17) we obtain:

$$T+U=\frac{J_1^4}{12(mV_0^2)^2J_0}\frac{1+\beta^2}{(1-\beta^2)^3} \qquad (16a)$$

and therefore for the energy $E=\hbar\omega$ we get finally:

$$E=-\left(\frac{N}{2}-1\right)\mu\mathcal{H}-J_0+\frac{\hbar^2v^2}{2J_0}-\frac{J_1^4}{24J_0(mV_0^2)^2}\frac{1-5\beta^2}{(1-\beta^2)^3} \qquad (18)$$

If $v$ is small enough, the energy $E$ lies lower than the free



magnon energy. In particular, the soliton rest energy ($v=0$) is separated by a gap

$$\Delta E=\frac{J_1^4}{24J_0(mV_0^2)^2} .$$

So, the solitary state considered is more favourable. If $J_1=0$ the energy given by (18) coincides with the low-lying energy spectrum, found in /5/.

It is interesting to note that because of the identity

$$e^{\pm i\alpha\sin\Omega t}=\sum_{-\infty}^{\infty}J_m(\alpha)e^{\pm im\Omega t}$$

($J_m(\alpha)$ being the Bessel functions) the solution (17) can be presented as a linear one-soliton superposition:

$$C(\xi,t)=\sum_{-\infty}^{\infty}\frac{J_m(\alpha)}{\sqrt{2\mathcal{L}}}e^{i(\kappa\xi-(\omega+m\Omega)t-\varphi_0)}\operatorname{sech}\frac{\xi-vt-\xi_0}{\mathcal{L}} . \qquad (17a)$$

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