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Критические экспоненты z и η для двухкомпонентных квантовых систем при нулевой температуре

При использовании метода ренормгруппы Вильсона вычисляется динамический критический индекс z и аномальная часть размерности для двухкомпонентных квантовых систем при $T=0$. Экспоненты z и η находятся в приближении ϵ^2, где $\epsilon=2-d$, d – пространственная размерность системы. Получается также критическое асимптотическое поведение динамической восприимчивости.

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Lukierska-Walasek K., Walasek K. **E17 - 11961**

The Critical Exponents z and η for Two-Component Quantum System at Zero Temperature

The dynamical critical exponent z and anomalous part of the dimension η are calculated for two-component quantum systems at $T=0$ by using Wilson's renormalization group method. The exponents z and η are found in the ϵ^2 approximation, where $\epsilon=2-d$ and d is the space dimensionality of the system. The critical asymptotic behaviour of the dynamic susceptibility is also obtained.

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1. INTRODUCTION

Quantum effects in critical phenomena have been studied recently in a number of systems [1a,b,3,4,5,9,11,12a-e,13,18]. At the present stage it is well known that at $T\neq 0$ the critical behaviour of a quantum system is the same as the classical one since the quantum effects became irrelevant. Changes in the critical behaviour were found at $T=0$ when a transition is induced by changing an external parameter (e.g., the transverse field in an Ising or quantum X-Y model, etc.). A straightforward application of Wilson's renormalization group (RG) method [17] encounters the difficulty of non-commuting operators. This difficulty can be circumvented by applying the functional representation for the partition function leading to the classical free-energy functional which the conventional RG method can be applied to. Whenever the Matsubara frequencies enter into the problem in a form nonequivalent to wavevector components, a dimensional cross over from d to $d+z_{MF}$ arises when the temperature tends to zero. Here d is the space dimensionality of the system and z_{MF} is the value of the dynamical critical exponent as applied to the $T=0$ case in the "mean field" regime.

Our interest is two-component quantum system such as an X-Y model or a Bose system. In this case $z_{MF}=2$ and the expansion parameter for the fixed point "Hamiltonian" and critical exponents is $\epsilon=2-d$. It has been found [1a,b,3] that at $T=0$ for $d\geq 2$ the Gaussian model "Hamiltonian" should describe the leading critical behaviour.

For $d < 2$ the system shows a nonstandard critical behaviour. It is interesting that although for $d < 2$ there exists the non-trivial stable fixed point, the critical exponents γ and ν have Gaussian values ($\gamma = 1$, and $\nu = 1/2$) in the first order in ϵ /1 a,b, 3/. In the present article we calculate in the lowest non-trivial order in ϵ two other important exponents, the dynamical critical exponent z and anomalous part of the dimension η /2,8,17/.

2. *THE FREE-ENERGY FUNCTIONAL*

The free-energy F of a quantum system can be written as follows

$$-\beta F = \ln \mathrm{Tr} e^{-\beta H} = C + \ln \int d(\phi)\, e^{-S[\phi]} \quad , \qquad (2.1)$$

where H is the microscopic Hamiltonian of the system, C is some constant non-essential in our consideration, $S[\phi]$ is the "action" functional of the system for the imaginary "time", $d(\phi)$ denotes the functional integral over classical fields $\phi_\lambda(\vec{k}, \omega_\nu)$ entering into the problem ($\vec{k}$ and $\omega_\nu = 2\pi\nu T$ are the wave vector and Matsubara frequency, respectively). For the two-component system we have two real fields or, equivalently, one complex field $\phi(\vec{k}, \omega_\nu)$. The functional $S[\phi]$ can be written in the last case as follows

$$S[\phi] = \sum_{n=1}^{\infty} \frac{1}{(n!)^2} \frac{1}{\beta^{2n-1}} \sum_{\nu_1 \dots \nu_{2n}} \int_{\vec{k}_1 \dots \vec{k}_{2n}} u_{2n}(\vec{k}_1, \dots, \vec{k}_{2n}, \omega_{\nu_1}, \dots, \omega_{\nu_{2n}})$$

$$\delta(\vec{k}_1 + \dots + \vec{k}_n - \vec{k}_{n+1} - \dots - \vec{k}_{2n})\, \beta\, \delta_{\nu_1 + \dots + \nu_n, \nu_{n+1} + \dots + \nu_{2n}} \times$$

$$\times \phi^*(\vec{k}_1, \omega_{\nu_1}) \dots \phi^*(\vec{k}_n, \omega_{\nu_n})\, \phi(\vec{k}_{n+1}, \omega_{\nu_{n+1}}) \dots \phi(\vec{k}_{2n}, \omega_{\nu_{2n}}), \qquad (2.2)$$

where

$$\int_{\vec{k}} = \frac{1}{(2\pi)^d} \int_{|\vec{k}|<\Lambda} d^d\vec{k} \qquad (2.3)$$

with Λ a cut-off on wave vectors.

The correspondence between the microscopic Hamiltonian H and $S[\phi]$ can be easily established transforming to the functional integral the expression for a partition function which has been obtained in terms of functional derivatives by Walasek /16/. For example for the X-Y model in the transverse field Γ we have

$$H = -\Gamma \sum_i S_i^z - \frac{1}{2} \sum_{i,j} I_{ij} S_i^+ S_j^- , \qquad (2.4)$$

where I_{ij} is the exchange integral and $S_i^{\pm} = S_i^x \pm i S_i^y$. The functions u_{2n} are then defined as follows

$$u_2(\vec{k}, \omega_\nu) = [\, I^{-1}(\vec{k}) + \langle S^+ S^- \rangle_{\omega_\nu}]^{-1} \qquad (2.5)$$

$$u_{2n}(\vec{k}_1, \dots, \vec{k}_{2n}, \omega_{\nu_1}, \dots, \omega_{\nu_{2n}}) = u_{2n}(\omega_{\nu_1}, \dots, \omega_{\nu_{2n}}) =$$

$$= \int_0^\beta d\tau_1 \dots d\tau_{2n} \langle T_\tau S_i^+(\tau_1) \dots S_i^+(\tau_n)\, S^-(\tau_{n+1}) \dots S^-(\tau_{2n}) \rangle_{H_0} \times$$

$$\times \exp\{ i [\omega_{\nu_1}\tau_1 + \dots + \omega_{\nu_n}\tau_n - \omega_{\nu_{n+1}}\tau_{n+1} - \dots - \omega_{\nu_{2n}}\tau_{2n}]\} \quad (n \geq 2),$$

$$(n \geq 2), \qquad (2.6)$$

where

$$I(\vec{k}) = \sum_{\vec{r}_{ij}} I_{ij}\, e^{i\vec{k}\vec{r}_{ij}} \qquad (2.7)$$

$$\langle S^+ S^- \rangle_{\omega_\nu} = \int_0^\beta d\tau\, e^{i\omega_\nu \tau} \langle S_i^+(\tau) S_i^- \rangle_{H_0} \qquad (2.8)$$

$$\langle \dots \rangle_{H_0} = \frac{\mathrm{Tr} e^{-\beta H_0} \dots}{\mathrm{Tr} e^{-\beta H_0}} \qquad (2.9)$$

$$S_i^{\pm}(\tau) = e^{\tau H_0} S_i^{\pm} e^{-\tau H_0} \tag{2.10}$$

with

$$H_0 = -\Gamma \sum_i S_i^z . \tag{2.11}$$

The transformation from H to $S[\phi]$ based on the formalism developed by Walasek[16] can be performed for each spin and Bose system, and seems to be somewhat simpler than that which has been realized for some quantum systems[3,4,5,9,11,18], using the Hubbard-Stratonovich formula[6,14].

It is easy to see that the field ϕ and inverse temperature β can be always transformed in such a way that for $u_2(\vec{k}, \omega_\nu)$ we have

$$u_2(\vec{k}, \omega_\nu) = r + \vec{k}^2 - i\omega_\nu + O(\vec{k}^4) . \tag{2.12}$$

At $T = 0$ $\omega_\nu \to \omega$,

$$\frac{1}{\beta} \sum_\nu \longrightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \tag{2.13}$$

and

$$\beta \delta_{\nu_1 + \ldots \nu_n, \nu_{n+1} + \ldots + \nu_{2n}} \to \delta(\omega_1 + \ldots + \omega_n - \omega_{n+1} - \ldots - \omega_{2n}). \tag{2.14}$$

Thus the functional $S[\phi]$ can be written in the following form

$$S[\phi] = S_0[\phi] + S_1[\phi] , \tag{2.15}$$

where

$$S[\phi] = \sum_{n=1}^{\infty} \frac{1}{(n!)^2} \int_{\vec{k}_1,\omega_1} \ldots \int_{\vec{k}_{2n},\omega_{2n}} u_{2n}(\vec{k}_1, \ldots, \vec{k}_{2n}, \omega_1, \ldots, \omega_{2n})$$

$$\delta(\vec{k}_1 + \ldots + \vec{k}_n - \vec{k}_{n+1} - \ldots - \vec{k}_{2n})\, \delta(\omega_1 + \ldots + \omega_n - \omega_{n+1} - \ldots - \omega_{2n})$$

$$\phi^*(\vec{k}_1,\omega_1) \ldots \phi^*(\vec{k}_n,\omega_n)\, \phi(\vec{k}_{n+1},\omega_{n+1}) \ldots \phi(\vec{k}_{2n},\omega_{2n}) \tag{2.16}$$

$$S_0[\phi] = \int_{\vec{k},\omega} (\vec{k}^2 - i\omega)\, \phi^*(\vec{k},\omega)\, \phi(\vec{k}, \omega) \tag{2.17}$$

$$S_1[\phi] = S[\phi] - S_0[\phi] \tag{2.18}$$

and

$$\int_{\vec{k},\omega} = \frac{1}{(2\pi)^{d+1}} \int_{|\vec{k}|<\Lambda} d^d\vec{k} \int_{-\infty}^{\infty} d\omega . \tag{2.19}$$

3. *THE RG TRANSFORMATION FOR QUANTUM SYSTEMS*

The RG transformation consists of two steps (see for example ref.[1a]). The first step is to integrate $\exp\{-S[\phi]\}$ over the fields $\phi(\vec{k}, \omega)$ with $\vec{k}$ in the external shell $\frac{\Lambda}{s} < |k| < \Lambda$, where $s > 1$. The second step is to combine the scale transformation of the wave vectors and frequencies and the renormalization transformation of the fields which survive the smoothing process, e.g.,

$$\phi(\vec{k}, \omega) \longrightarrow \zeta_s \phi(s\vec{k}, s^z \omega) , \tag{3.1}$$

where ζ_s and the dynamical exponent z are chosen so that S_0' in the renormalized functional S' remains of the same form as the corresponding S_0 in the original functional

For $d < 2$ the non-trivial fixed point functional $S^*[\phi]$ can be found by using the perturbation diagrammatic method with respect to S_1, which is equivalent to the expansion into a power series with respect to the parameter $\epsilon = 2 - d$.

In the first order in ϵ we have[1a, b]

$$S^*[\phi] = \int_{\vec{k},\omega} (r^* + \vec{k}^2 - i\omega)\, \phi^*(\vec{k}, \omega)\, \phi(\vec{k}, \omega) +$$

$$+ \frac{u^*}{4} \int_{\vec{k}_1,\omega_1} \ldots \int_{\vec{k}_4,\omega_4} \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4)\, \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) \times$$

$$\times\ \phi^*(\vec{k}_1, \omega_1)\, \phi^*(\vec{k}_2, \omega_2)\, \phi(\vec{k}_3, \omega_3)\, \phi(\vec{k}_4, \omega_4) \tag{3.2}$$

with

$$r^* = -\Lambda^2 \epsilon \tag{3.3}$$

and

$$u^* = 8\pi\epsilon \ . \tag{3.4}$$

The coefficient ζ_s used in the renormalization transformation defined with respect to the fixed point is the following

$$\zeta_s = s^{\frac{d}{2} + 2 - \eta - \frac{a}{2}} , \tag{3.5}$$

where

$$s^{\eta} = 1 + \frac{\partial \Sigma^*_s}{\partial k^2} \Big|_{\vec{k} = 0,\ \omega = 0} \tag{3.6}$$

$$s^{-a} = 1 + \frac{\partial \Sigma_s}{\partial (i\omega)} \Big|_{\vec{k} = 0,\ \omega = 0} \tag{3.7}$$

and $\Sigma^*_s(\vec{k}, i\omega)$ is the self-energy; i.e., the sum of all connected diagrams with two external lines. All internal lines of these diagrams have wave vectors $\vec{q}$ in the shell $\frac{\Lambda}{s} < |\vec{q}| < \Lambda$ while two external lines have wave vectors $\vec{k}$ restricted to $|\vec{k}| < \frac{\Lambda}{s}$. The dynamical critical exponent z is defined as

$$z = 2 + \eta + a = 2 + c\eta , \tag{3.8}$$

where

$$c = 1 + \frac{a}{\eta} . \tag{3.9}$$

4. *THE EXPONENTS* η *AND* z .

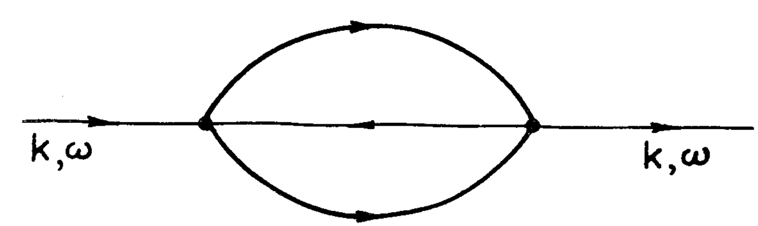
It is easy to see that if $\Sigma^*_s(\vec{k}, i\omega)$ is independent of $\vec{k}$ and ω , $\eta = 0$, $a = 0$ and $z = 2$, i.e., these exponents have the values corresponding to the Gaussian fixed point. Therefore for $d < 2$ the non-Gaussian values for η and z appear in the ϵ^2 approximation. To calculate η and z in this order we must consider the diagram shown in the *figure*. The four-point vertex in this diagram denotes the "interaction constant" $u^* = 8\pi\epsilon$, which is connected with the non-trivial fixed point and an internal line corresponds to the propagator $G_0(\vec{q}, i\omega) = (q^2 - i\omega)^{-1}$. It is easy to see that to the second order in ϵ we have

$$\frac{\partial \Sigma^*_s}{\partial k^2} \Big|_{\vec{k}=0,\ \omega=0} = \frac{\partial I_s}{\partial k^2} \Big|_{\vec{k}=0,\ \omega=0} \tag{4.1}$$

and

$$\frac{\partial \Sigma^*_s}{\partial (i\omega)} \Big|_{\vec{k}=0,\ \omega=0} = \frac{\partial I_s}{\partial (i\omega)} \Big|_{\vec{k}=0,\ \omega=0} , \tag{4.2}$$

where $I_s(\vec{k}, i\omega)$ is the expression corresponding to the diagram shown in the *figure*.



The diagram giving a contribution to η *and* z *in the* ϵ^2 *approximation.*

After simple transformations we obtain

$$\eta = \frac{1}{\ln s} \frac{\partial I_s}{\partial k^2} \Big|_{\vec{k}=0,\ \omega=0} \tag{4.3}$$

and

$$a = -\frac{1}{\ln s} \frac{\partial I_s}{\partial (i\omega)} \Big|_{\vec{k}=0,\ \omega=0} \ . \tag{4.4}$$

The diagram for $I_s(\vec{k}, i\omega)$ can be explicitly evaluated after transformation from the $\vec{q}, \omega$ representation to the $\vec{r}, \tau$ representation, where $\vec{r}$ and τ are the position vector in the real space and imaginary "time" parameter, respectively. Note that for $T = 0$, $0 < \tau < \infty$.

Thus $I_s(\vec{k}, i\omega)$ can be written as follows

$$I_s(\vec{k}, i\omega) = -\frac{1}{2}\frac{u^{*2}}{(2\pi)^6}\int d^2\vec{r}\int_0^\infty d\tau \times$$

$$\times\ e^{i(\vec{k}\vec{r}+\omega\tau)}\Big[\int_{\frac{\Lambda}{s}<|\vec{q}|<\Lambda} d^2\vec{q}\, e^{i\vec{q}\vec{r}} G_0(\tau, \vec{q})\Big]^3, \qquad (4.5)$$

where

$$G_0(\tau, \vec{q}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega\tau}}{q^2 - i\omega} = \theta(\tau)\, e^{-\tau q^2}. \qquad (4.6)$$

and $\theta(\tau)$ is the Heaviside step function.

In order to calculate (4.5), we note that the integration of $G_0(\vec{q}; \tau)$ over large wave-vectors lying in the shell $\frac{\Lambda}{s} < |\vec{q}| < \Lambda$ with s close to 1 corresponds to the integration over the shell in the real space with $\frac{1}{\Lambda} < |\vec{r}| < \frac{s}{\Lambda}$.

Therefore we replace equation (4.5) by following one

$$I_s(\vec{k}, i\omega) = -\frac{1}{2}\frac{u^{*2}}{(2\pi)^2}\int_{\frac{1}{\Lambda}<|\vec{r}|<\frac{s}{\Lambda}} d^2\vec{r}\int_0^\infty d\tau\, e^{i(\vec{k}\vec{r}+\omega\tau)} \times G_0^3(\vec{r}, \tau) \qquad (4.7)$$

with

$$G_0(\vec{r}, \tau) = \frac{1}{(2\pi)^2}\int_{0<|\vec{q}|<\infty} d^2\vec{q}\, G_0(\vec{q}, \tau)\, e^{i\vec{q}\vec{r}} = \tau^{-3} e^{\frac{-3r^2}{4\tau}}. \qquad (4.8)$$

Note that the similar method of evaluating such a diagram was used in the classical case /10/. From equation (3.7) it is easy to calculate

$$\frac{\partial I_s}{\partial k^2}\Big|_{\vec{k}=0,\omega=0} = \frac{u^{*2}}{72\pi^2}\ln s, \qquad (4.9)$$

$$\frac{\partial I_s}{\partial(i\omega)}\Big|_{\vec{k}=0,\omega=0} = -\frac{u^{*2}}{48\pi^2}\ln s. \qquad (4.10)$$

Taking into account equations (4.3), (4.4), (4.9), (4.10), (3.9) and putting $u^* = 8\pi\epsilon$ we have

$$\eta = \frac{8}{9}\epsilon^2 \qquad (4.11)$$

$$a = \frac{1}{12}\epsilon^2 \qquad (4.12)$$

and

$$c = \frac{29}{32}. \qquad (4.13)$$

5. *THE ASYMPTOTIC CRITICAL BEHAVIOUR OF THE DYNAMIC SUSCEPTIBILITY*

Consider the functional average $\langle \phi^*_{\vec{k},\omega}\phi_{\vec{k}',\omega'}\rangle_{S[\phi]}$ defined as follows

$$\langle \phi^*_{\vec{k},\omega}\phi_{\vec{k}',\omega'}\rangle_{S[\phi]} = \frac{\int d(\phi)\, e^{-S[\phi]}\phi^*_{\vec{k},\omega}\phi_{\vec{k}',\omega'}}{\int d(\phi)\, e^{-S[\phi]}}, \qquad (5.1)$$

Due to the laws of conservation of the wave-number and frequency we have

$$\langle \phi^*_{\vec{k},\omega}\phi_{\vec{k}',\omega'}\rangle_{S[\phi]} = \delta(\vec{k}-\vec{k}')\,\delta(\omega-\omega')\,G(\vec{k},\omega, S[\phi]). \qquad (5.2)$$

The function $G(k, \omega; S[\phi])$ is related to the dynamic susceptibility $\chi(\vec{k}, \omega)$ of the system at $T=0$. For example for the $X-Y$ model we have

$$G(\vec{k}, \omega, S[\phi]) = I(\vec{k}) - 2\pi i\, I^2(\vec{k})\,\chi(\vec{k}, -i\omega). \qquad (5.3)$$

We obtain equation (5.3) taking into acount that $G(\vec{k},\omega;S[\phi])$ can be reduced to the effective interaction (cf.[7]) and

$$2\pi i\chi(\vec{k},-i\omega) = \int_0^\infty d\tau\, e^{i\omega\tau} \langle \tilde{S}^+_{\vec{k}}(\tau)\, S^-_{-\vec{k}} \rangle_H =$$

$$= 2\pi i \langle\langle S^+_{\vec{k}} | S^-_{-\vec{k}} \rangle\rangle_{E=-i\omega} \quad , \tag{5.4}$$

where

$$\tilde{S}^+_{\vec{k}}(\tau) = e^{\tau H} S^+_{\vec{k}} e^{-\tau H} \quad , \tag{5.5}$$

$$\langle \dots \rangle_H = \frac{Tr e^{-\beta H} \dots}{Tr e^{-\beta H}} \tag{5.6}$$

with H given by equation (3.4). Here $\langle\langle S^+_{\vec{k}} | S^-_{-\vec{k}} \rangle\rangle_E$ is the Fourier transform of the retarded Green function (cf.[15]). Proceeding similarly as in the classical case (see, for example, ref.[8]), we find the following scaling relation

$$G(\vec{k},\omega;S[\phi]) = s^{2-\eta} G(s\vec{k}, s^z\omega; R_s S[\phi]). \tag{5.7}$$

If $S_c[\phi]$ is taken from the critical surface and $s\to\infty$ we have

$$G(\vec{k},\omega;S_c[\phi]] = s^{2-\eta} G(s\vec{k}, s^z\omega; S^*[\phi]). \tag{5.8}$$

Putting $s = \frac{1}{|\vec{k}|}$ $(k\to 0)$ we obtain

$$G(\vec{k},\omega, S_s[\phi]) = |\vec{k}|^{-2+\eta} f\left(\frac{\omega}{|\vec{k}|^z}\right], \tag{5.9}$$

where

$$f\left(\frac{\omega}{|\vec{k}|^z}\right) = \Lambda^{2-\eta} G\left(\Lambda, \frac{\Lambda^z\omega}{|\vec{k}|^z}; S^*[\phi]\right). \tag{5.10}$$

If we take $s = \left(\frac{|\omega|}{\Lambda^2}\right)^{-1/z}$ $(\omega\to 0)$ the following relation holds

$$G(\vec{k},\omega;S_c[\phi]) = |\omega|^{-\frac{2-\eta}{z}} f'_\pm(\omega^{-\frac{1}{z}}\vec{k}) \quad , \tag{5.11}$$

where

$$f'_\pm(\omega^{-\frac{1}{z}}\vec{k}) = \Lambda^{\frac{2-\eta}{z}} G[\left(\frac{|\omega|}{\Lambda^2}\right)^{-\frac{1}{z}}\vec{k}, \pm\Lambda^2; S^*[\phi]] . \tag{5.12}$$

Therefore for the critical susceptibility $\chi_c(\vec{k},\omega)$ we have the following asymptotic behaviour

$$\chi_c(\vec{k},\omega=0) \sim |\vec{k}|^{-2+\eta} \quad \text{for} \quad \vec{k}\to 0 \tag{5.13}$$

$$\chi_c(\vec{k}=0,\omega) \sim c_+ |\omega|^{\frac{-2-\eta}{z}} \quad , \tag{5.14}$$

where the constants c_+ and c_- are related to $\omega\to 0^+$ and $\omega\to 0^-$, respectively.

Taking into account the formulas (3.8), (4.11), (4.13), (5.13) and (5.14) for the one-dimensional quantum system $(\epsilon = 1)$ at $T=0$ for small $\vec{k}$ and ω we have

$$\chi_c(\vec{k},\omega=0) \sim |\vec{k}|^{-1.11} \tag{5.15}$$

and

$$\chi_c(\vec{k}=0,\omega) \sim c_\pm |\omega|^{-0.40} . \tag{5.16}$$

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