ОБЪЕДИНЕННЫЙ
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ДУБНА

E17 - 11806
15/1-79

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1978

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Submitted to "Physica B"

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Шрайбер Ю. **E17 - 11806**

Коммутаторная и антикоммутаторная функция Грина в теории неупорядоченных спиновых систем

Обсуждено применение коммутаторной (КФГ) и антикоммутаторной функции Грина (АФГ) для неупорядоченной модели Гейзенберга с флуктуирующими обменными интегралами J_{ij} или локальными полями H_i. На основе КФГ или АФГи в одинаковом приближении для структурного усреднения получены результаты в приближении Тябликова, разница в которых обусловлена различием в алгебраической структуре уравнений движения. При этом точность сделанного приближения существенно зависит от характера спиновых состояний. В случае распределения Лоренца величин J_{ij} или H_i, в которых спиновые состояния при низких энергиях локализованы, КФГ не работает в данном приближении, но АФГ приводит к физическим разумным результатам, которые описывают новый вид аморфного ферромагнетизма (АФМ). Такой АФМ характеризуется ненасыщенной намагниченностью вследствие замороженных локализованных магнонов. Обсуждается возможность существования другого АФМ с ненасыщенной намагниченностью, но однородной спиновой структурой в силу коллективизированных нулевых колебаний. Для приближенного описания этого АФМ более подходит КФГ.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

Schreiber J. **E17 - 11806**

Commutator and Anticommutator Green Functions in the Theory of Disordered Spin Systems

For the Heisenberg model with random exchange integrals J_{ij} or local fields H_i the application of the commutator (CGF) and anticommutator Green function (AGF) is discussed. Because of the different algebraic structure of both Green functions the Tjablikov-decoupling procedure and an approximated structure averaging yield different results. For a Lorentzian distribution of J_{ij} and H_i, where the low energy excitations are localized, the density of states, the magnetizations, the susceptibility and the specific heat are investigated by the AGF. The CGF fails to work in this case which is adequate for amorphous unsaturated ferromagnets (FM) with an inhomogeneous spin structure. We have also discussed the opposite case of an amorphous unsaturated FM in which the spin structure is homogenized due to extended zero-point fluctuations. The CGF appropriatly describe this FM while the approximate results of the AGF describe inadequate.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1978

1. INTRODUCTION

Amorphous and disordered crystalline magnetic systems, in the last time intensively investigated by rather different experimental[1-4,6] and theoretical methods[2,5,6], have several unusual and interesting dynamic and thermodynamic features. The loss of translational symmetry, caused by microscopic potential fluctuations or topological disorder can lead to the existence of new magnetic structures, to complicated spin excitations, and hence to motivated thermodynamic behaviour of the magnetic system.

As we have learned from the investigation of electrons in a random one-particle potential a qualitatively new feature of disordered systems is the possible coexistence of collective extended and localized states[7-9] where the relation between them can be altered changing the disorder strength. In the language of Green functions (GF) localized states can be determined by isolated pole-singularities[9-11]. However, in the averaged GF the isolated pole becomes smeared out[11] so that, e.g., in the averaged density of one-particle states (DS) the localized states are not in general separated from the extended one by an energy gap but they come together at the mobility edge with a finite DS[8,9]. With the help of such a concept a lot of experimental data of amorphous semiconductors can be explained[8,9,12], although a quantitative theoretical approach to this problem is quite difficult.

The avidence of a large density of localized spin states, coexisting with propagating spin waves, was for the first time found by inelastic neutron scattering[13] in disordered magnetic systems $KCo_xMn_{1-x}F_3$ and

$Co_x Mn_{1-x} F_2$. Although some theoretical investigations along this line were carried out[14,15], the problem of localization of spin excitations is essentially open, since the many body effects can substantially influence the so far considered one-particle behaviour. On the other hand it is obvious that the final thermodynamical behaviour of the spin systems can essentially depend on the character of the spin excitations. For instance certain kind of low energy excitations causes characteristic low temperature behaviour, e.g., ferromagnetic spin waves yield the Bloch $T^{3/2}$-law while the localized excitations in the Ising model are connected with an exponential T-dependence. A qualitative description of the different possible thermodynamical behaviours can be got by an approximated treatment of the many body problem, e.g., in the Heisenberg model, incorporating immediately the different character of the excited states. Then by comparing with the real situation one can get indirectly some information about the kind of spin excitations. Furthermore, a breakdown of an approximate changing the disorder strength is perhaps connected with an essential change in the character of spin excitations rather than with a breakdown of the long-range order. It is the aim of this paper to discuss just in the above sense the theory of disordered Heisenberg systems based on the method of double-time Green functions[16-18]. In Section 2 it is pointed out that because of the different algebraic structures of the commutator GF(CGF) and anticommutator GF(AGF), approximated results of both GF may not coincide. For the Heisenberg model with fluctuating exchange integrals J_{ij} and local fields H_i this circumstance is analyzed (Section 3) on the basis of the random phase approximation (RPA)[17]. When approximating the structure averaged product of GF 's by the product of the averaged GF 's it is shown that: i) The CGF does not yield correct results if the low lying excitations about the ground state are localized in space. The AGF, however, works quite well just in this case. ii) The AGF can fail to describe long range effects as, e.g., extended zero-point fluctuations in a ferromagnetic phase. Such effects can be

incorporated using the CGF (cf. Sec . 5.2). The application of the AGF to the model case where either J_{ij} or H_i fluctuate randomly according to a Lorentzian distribution function is presented in Section 4. These results describe a new thermodynamical behaviour of amorphous ferromagnetic systems for which the low lying excitations are localized in space rather than spin waves. Finally (Sec. 5), in the light of the presented considerations we reexamine several existing theories of disordered spin systems and propose a qualitative phase diagram for amorphous magnets.

2. *PRELIMINARIES*

The method of double-time GF[16,17] is a suitable tool for studying the dynamical and thermodynamical properties of spin systems, where in general the CGF is used[17,18]. On the other hand, the application of the AGF would lead to the same results at least for exact calculations. It is, however, necessary to point out that the algebraic structure of the AGF differs from that one of the CGF which has consequences for an approximated solution of the GF"s. To illustrate this fact let us consider the correlation function of two operators A and B for which we write down the spectral intensity $I^{+}_{AB}(\omega)$ and $I^{-}_{AB}(\omega)$ obtained by the AGF and CGF, respectively, via the spectral representation[16,17]

$$I^{\pm}_{AB}(\omega) = \frac{1}{e^{\beta\omega} \pm 1} \frac{i}{2\pi} \{G^{\pm}_{BA}(\omega + i0^{+}) - G^{\pm}_{BA}(\omega - i0^{+})\},$$

$$I^{\pm}_{AB}(\omega) = \frac{1}{e^{\beta\omega} \pm 1} \sum_{n,m} \langle n|B|m\rangle\langle m|A|n\rangle (e^{-\beta E_m} \pm e^{-\beta E_n}) * \\ * \delta(\omega - E_m + E_n), \qquad (1)$$

$|n\rangle$ and E_n are the eigenfunctions and eigenvalues of the

considered Hamiltonian; the GF 's are defined as usual [16]:

$$G^{\pm}_{AB}(t-t') = -i\theta(t-t')\langle [A(t), B(t')]_{\pm} \rangle = \langle\langle A(t); B(t') \rangle\rangle_{\pm} .$$

Assuming that $I_{AB}(\omega)$ do not contain a $\delta(\omega)$-term, without any approximation $I^{+}_{AB} = I^{-}_{AB}$ follows. The same holds if disorder is present. However, in an approximate solution for $G^{\pm}_{AB}(\omega)$ this feature can be lost. For instance, due to the decoupling of the higher order GF or due to the approximate structure averaging of the sum in (1) it is possible that the factor $1/(e^{\beta\omega} \pm 1)$ will not be cancelled out as it is for the exact treatment. Then for $\omega \to 0$ $I^{-}_{AB}(\omega)$ would show a singular behaviour $(I^{-}_{AB}(\omega \to 0) \to \infty)$, while $I^{+}_{AB}(\omega)$ can behave more or less correctly at $\omega \simeq 0$. We note that this difference of the CGD and AGF is tightly related to the circumstance that because of the algebraic structure poles at $\omega = 0$ are allowed for the AGF, while for the CGF this is excluded doing exact derivations [19].

3. GREEN FUNCTIONS AND APPROXIMATIONS

Disordered crystalline and amorphous magnetic systems can commonly be described by a random Heizenberg lattice model [2,5]. The Hamiltonian is

$$\mathcal{H} = -1/2 \sum_{ij} J_{ij} \vec{S}_i \vec{S}_j - \sum_i H_i S_i^z , \qquad (2)$$

where the spins $(S=1/2)$ build a regular lattice, i.e., topological disorder is neglected, and the exchange integrals J_{ij} or local fields H_i fluctuate independently with definite probability distribution functions $P(J_{ij})$ and $P'(H_i)$.

The GF 's $G^{\pm}_{ij} = \langle\langle S^{+}_i ; S^{-}_j \rangle\rangle^{\pm}$ satisfy the following equation of motion

$$(\omega - H_i) G^{\pm}_{ij}(\omega) = A^{\pm}_i \delta_{ij} + (1 \pm 1) B_{ji} (1 - \delta_{ij}) + \sum_{\ell} J_{i\ell} \{ \langle\langle S^{+}_i S^{z}_{\ell} ; S^{-}_j \rangle\rangle_{\pm} - \langle\langle S^{z}_i S^{+}_{\ell} ; S^{-}_j \rangle\rangle_{\pm} \} , \qquad (3)$$

where

$$A^{+}_i = \langle [S^{+}_i, S^{-}_i]_{+} \rangle = 1 , \quad A^{-}_i = 2\langle S^{z}_i \rangle ,$$

and $B_{ji} = \langle S^{-}_j S^{+}_i \rangle$.

As an exact solution of this equation cannot be reached in general certain approximation must be done, where the difficulties to treat the disorder as well as the many body problem are of the same level.

3.1. *Tjablikov Decoupling*

The starting point for an approximation has to be an assumption about the ground state of the spin system. In the case $J_{ij} > 0$ and $H_i > 0$ a pure ferromagnetic state (all spins aligned) can be proposed. If, however, some J_{ij} , $H_i < 0$ are permitted while the averaged values $\langle J_{ij} \rangle_c = \int dJ_{ij} P(J_{ij}) J_{ij} > 0$ and $\langle H_i \rangle_c = \int dH_i P'(H_i) H_i \gtrless 0$, the magnetic ordering can be more complicated, e.g., an antiparallel spin alignment and even a noncollinear magnetic structure can occur [4].

As a first step to describe the complicated magnetic ordering, let us assume a collinear spin-structure, i.e., $\langle S^{z}_i \rangle \neq 0$ and $\langle S^{\pm}_i \rangle = 0$. Later we will see whether the proposed magnetic state is stable or not within the considered approximation. Then a powerful first order approximation is the Tjablikov decoupling or RPA [17,18], where the higher order GF 's in Eq. (3) are decoupled in the following manner

$$\langle\langle S^{+}_i S^{z}_{\ell} ; S^{-}_j \rangle\rangle_{\pm} \simeq \langle S^{z}_{\ell} \rangle \langle\langle S^{+}_i ; S^{-}_j \rangle\rangle_{\pm} . \qquad (4)$$

Eq. (3) may now be rewtitten in the form

$$G^{\pm}_{ij}(\omega) = \tilde{G}_{ij}(\omega) A^{\pm}_j + (1 \pm 1) \sum_{\ell \neq j} G_{i\ell}(\omega) B_{j\ell} , \qquad (5a)$$

$$(\omega - H_i - \sum_{\ell} J_{i\ell} \langle S^{z}_{\ell} \rangle) \tilde{G}_{ij}(\omega) = \delta_{ij} - \sum_{\ell} \langle S^{z}_i \rangle J_{i\ell} \tilde{G}_{\ell j}(\omega) . \qquad (5b)$$

$\langle S^{z}_i \rangle$ is self-consistency determined by the relation

$$\langle S_i^z \rangle = 1/2 - \langle S_i^- S_i^+ \rangle \qquad (6)$$

and the correlation function $B_{j\ell}$ is given by

$$B_{j\ell} = \langle S_j^- S_\ell^+ \rangle = \int \frac{d\omega}{\pi} \operatorname{Im} G^{\pm}_{\ell j} \; (\omega - i0^+)/(e^{\beta\omega} \pm 1). \qquad (7)$$

It is easy to check that in the ordered case the solutions of Eq. (5) for the CGF and the AGF yield the same self-consistent solution for $\langle S_i^z \rangle$ and $B_{j\ell}$. The elementary excitations are then collective spin waves and, consequently, the magnetization or the specific treat show the Bloch $T^{3/2}$-law [17]. The simplest kind of disorder is a single impurity in a ferromagnetic matrix. Besides spin waves also localized states outside the spin wave band can appear if the impurity potential exceeds a critical value [10]. The situation becomes more complicate when considering an antiferromagnetic coupled pair of impurity atoms in a ferromagnetic matrix [10]. In the general disordered case the character of spin excitations (or states) will be determined by the complicate interplay between the diagonal randomness (DR), appearing in the diagonal matrix element of Eq. (5b) $\tilde{H}_i = H_i + \sum_\ell J_{i\ell} \langle S_\ell^z \rangle$, and the off-diagonal randomness (ODR) in the off-diagonal matrix element $\langle S_i^z \rangle J_{ij}$. For fluctuating J_{ij} both kinds of disorder are present where the relation between ODR and DR depends on the interaction radius r_0. In the case of nearest-neighbour interactions the influence of ODR has a maximum while for $r_0 \to \infty$ the effect of ODR vanished.

As $\tilde{G}_{ij}$ is determined by an equation which is analogous to that one for the GF of electrons in a random one particle potential, some results of the theory of electron localization [7-9,11,20-23] can be applied for further considerations. According to this, the disorder is at first reflected at the band edges where localized states appear separated from the extended one by a mobility edge ω_c. For small randomness there are indications [22,23] that due to ODR ω_c may move initially inwards into the band while DR acts against this [20].

However, for a sufficient large strength of disorder the opposite picture is obtained, i.e., ω_c is the more shifted to the centre of the band the larger DR, and for critical strength of DR all states become localized. With increasing ODR ω_c eventually moves outwards leaving always a region of extended states around the middle of the band. The existing coupling of DR and ODR leads to an asymmetric shift of ω_c [21] where the effect of randomness is smaller at the lower band edge ($\omega \simeq 0$).

3.2. Approximate Structure Averaging

Macroscopic, i.e., structure averaged quantities like the magnetization $\sigma = 1/N \sum_i \langle S_i^z \rangle = \langle\langle S_i^z \rangle\rangle_c$, can be obtained from the structure averaged GF $\langle G^{\pm}_{ij} \rangle_c$. From Eqs. (6), (7) it can be seen that in Eq. (5a) the first and second term contain products of GF $\tilde{G}_{ij}$, that structure average is hard to calculate. The same problem in obtaining the conductivity of electrons in a random potential [8,9]. Let us here discuss the lowest order approximation (LOA) to break off averaged products of GF neglecting the so-called vertex corrections:

$$\langle \tilde{G}_{ij} A_j^{\pm} \rangle_c \simeq \langle \tilde{G}_{ij} \rangle_c \langle A_j^{\pm} \rangle_c, \langle \tilde{G}_{i\ell} B_{j\ell} \rangle_c \simeq \langle \tilde{G}_{i\ell} \rangle_c \langle B_{j\ell} \rangle_c \,. \qquad (8)$$

The LOA is frequently applied dealing with disordered spin systems [2,5,18,24-28], where so far nobody has used (8) for the AGF.

Although this approximation has a similar character for the CGF and AGF, the results can differ strongly from each other because of the different algebraic structure (see Eq. (5) and Sec. 2).

1) If the structure fluctuations are small the LOA as the first order of a perturbation theory works quite well for the CGF and for the AGF.

2) In the case where the excitations at $\omega \lesssim 0$ are localized, i.e., DR is strong enough and DR >ODR, the LOA for the CGF is an inadequate approximation. To il-

lustrate this fact we investigate the extreme case $J_{ij}=0$. From the CGF one obtaines

$$\langle G^{-}_{ij}\rangle_c = 2\sigma\langle\frac{\delta_{ij}}{\omega-H_i}\rangle_c,\quad \sigma=(2\int dH_i P'(H_i)\,\mathrm{cth}(\frac{\beta H_i}{2}))^{-1}. \qquad (9)$$

The right result follows from the AGF:

$$\langle G^{+}_{ij}\rangle_c = \langle\frac{\delta_{ij}}{\omega-H_i}\rangle_c,\quad \sigma=1/2\int dH_i P'(H_i)\,\mathrm{th}(\frac{\beta H_i}{2}). \qquad (10)$$

If all $H_i>0$ the error of Eq. (9) is only a quantitative one while for any admitted negative H_i the unphysical answer $\sigma>1/2$ is obtained. We note that, of course, the exact treatment of the CGF leads to Eq. (10) for σ, i.e., the local value of $\langle S^z_i\rangle$ must be considered to avoid a spurious singular behaviour of $G^{-}_{ij}(\omega)$ at $\omega=0$.

The same drawback of the CGF holds also for $J_{ij}\neq 0$ if yet localized states (isolated poles in $\tilde{G}_{ij}(\omega)$) at $\omega\leq 0$ exist. These states are connected with some local deviations of $\langle S^z_i\rangle$, e.g., due to some local values $\langle S^z_i\rangle=0$ a pole at $\omega=0$ would not appear. Furthermore, localized states at $\omega<0$ can even lead to some $\langle S^z_i\rangle<0$ so that the spectral intensity of $\langle S^-_i S^+_i\rangle$ always remains positive[17]. However, in the LOA this feature would be lost and a spurious pole at $\omega=0$ may occur.

The AGF, on the other hand, is free of such difficulties in the present case. From Eq. (10) it follows that the LOA for the AGF works just the better the more the states of $\tilde{G}_{ij}(\omega)$ are localized. In other words, the strong local potential fluctuations provide an exponential decay of $\tilde{G}_{ij}(\omega)$ increasing the distance $|i-j|$[29]. Consequently, the second term in Eq. (5a) becomes less important, i.e., of second order, in comparison with the first one. Furthermore, the sum in the second term of Eq. (5a) can be regarded as a kind of averaging over the environment of i and j which can support the applications of the LOA for the AGF as well in cases where besides localized states also the extended spin wave dynamics takes place or the interaction is long ranged.

In this sense the LOA for the AGF provides an interpolation scheme between the correctly treated limits: i) the ordered case - only spin wave dynamics - and ii) the case of strong local scattering - only strong localized states. Of course, in the same way as the RPA Curie-temperature, the LOA for the AGF yields only a crude picture for the middle of the two limits. As for the localized states, which spread to a finite extended region, the correlations between $\tilde{G}_{i\ell}$ and $B_{j\ell}$ are of importance, the LOA can eventually suppress the appearance of localized states in $G^{+}_{ij}(\omega)$ at certain ω. Consequently, some features of the system which straightforward depend on the character of the states (see Sec. 4.1 and 4.3) cannot be reproduced by the LOA.

3) In systems where the states at $\omega\gtrless 0$ are extended, i.e., for ODR strong enough, the LOA can be applied to the CGF but may fail for the AGF. Because of the large delocalization effect due to strong ODR one can expect that local spin deviations will extend to the whole system, i.e., $\langle S^z_i\rangle\simeq\sigma$. However, because of the strong fluctuations the pure ferromagnetic state will not be the ground state. First of all the zero-point reduction of the magnetization can occur, i.e., $\sigma<\sigma_S$ - the saturation magnetization. The mixed magnetic order will be essentially characterized by long range fluctuations and correlations. These long range effects have to be included especially in the second term of Eq. (5a) which can be of the same order as the first one. Using LOA for the AGF unphysical results, e.g., $\sigma>\sigma_S$, may then be obtained. Contrary to that the LOA for the CGF can correctly account for the main effect of certain strong ODR as long as $\sigma>0$ (see Sec. 5.2).

After the LOA in Eq. (5a) it remains to solve the effective one-particle equation (5b) to get $\langle\tilde{G}_{ij}\rangle_c$. This task is always yet very complicate as first of all the effective one particle potential contains the quantities $\langle S^z_\ell\rangle$. At the first step of solving Eq. (5b) one therefore neglects the fluctuations in $\langle S^z_\ell\rangle$ putting $\langle S^z_\ell\rangle\simeq\sigma$. Then one can investigate the stability of the ferromagnetic like along range order against J_{ij} and H_i fluctuations. If one finds a stable solution $\sigma\neq 0$ in a self-consistent manner, a further minimization of the free energy would yield a relaxation of $\langle S^z_i\rangle$ and may be a change of σ, which is

probably essential only in the critical region ($\sigma \to 0$). The undergoing relaxation of $\langle S_i^z \rangle$ can cover two processes: i) A stabilization of the ordered phase caused by softening of any sharp alterations of J_{ij} and H_i in the effective one particle potential of Eq. (5b) due to collective spin excitations in the ground state. ii) An increase of the localization tendency at places where localized states already exist for $\langle S_i^z \rangle \simeq \sigma$. Then the magnetic ordered phase can become more and more inhomogeneous.

It is worth while to emphasize at this place that besides $\langle S_i^z \rangle$ fluctuations also many-body effects can substantially influence local relaxations or fluctuations of the order parameter. Consequently, both effects would have to be estimated at the same time. However, the so far developed theory of disordered system [18] is yet far away to study these corrections. In this sense one has to discuss the results of the proposed RPA-LOA as results of a first order approximation. Maybe they can already describe some of the characteristic features of disordered magnets. Unfortunately it is not possible to estimate the errors more accurately [17,18]. These approximations will be investigated in detail for a simple model for which the remaining structure averaging $\langle \tilde{G}_{ij} \rangle_c$ can be performed exactly.

4. *A SIMPLE MODEL FOR AMORPHOUS FERROMAGNETS*

Here we consider two model cases where I : J_{ij} and II: H_i are randomly distributed according to the Lorentzian distribution function. In the first case (I) DR and ODR are present. Such a kind of randomness is typical for one component amorphous transition metals and amorphous transition metal-metalloids [3,4]. In the second case (II) only DR is considered which corresponds to systems where long range interactions or strong local varying anisotropy suppress ODR. Examples are dilute magnetic alloys and amorphous rare earth compounds

and alloys [3,4,6,32]. We note that in this case one can incorporate even $\langle S_i^z \rangle$ -fluctuations:

$$\tilde{H}_i = H_0 + \Delta H_i \,, \quad \Delta H_i = H_i + \sum_\ell J_{i\ell} \;\; (\langle S_\ell^z \rangle - \sigma), \quad H_0 = \sum_\ell J_{i\ell}\, \sigma .$$

Then the distribution function $P'(H_i)$ has a temperature dependent width $\Gamma(T)$.

As has been shown recently [30,31], the averaging of $\tilde{G}_{ij}$ in the above described approximation (Sec. 3.2) can be done analytically. After the integration $\langle \tilde{G}_{ij} \rangle_c$ may be obtained from Eq. (5b) by replacing in the case I: $J_{ij} \to \langle J_{ij} \rangle_c + i J_0 \gamma$ and in the case II: $\tilde{H}_i \to H_0 + i J_0 \Gamma$, where $J_0\gamma$ and $J_0\Gamma$ are the widths of the distributions and $J_0 = \sum_\ell \langle J_{i\ell} \rangle_c$. The Fourier transform of $\langle G_{ij} \rangle_c$ is then given by

$$\begin{aligned} \text{I:} \quad & \tilde{G}_{\vec{k}}(\omega) = (\omega - \sigma J_0 (1 - f_{\vec{k}})(1 + i\gamma))^{-1} , \\ \text{II:} \quad & \tilde{G}_{\vec{k}}(\omega) = (\omega - \sigma J_0 (1 - f_{\vec{k}}) - i J_0 \Gamma)^{-1} , \end{aligned} \tag{11}$$

where

$$f_{\vec{k}} = \sum_\ell \langle J_{i\ell} \rangle_c \exp(i\vec{k}\vec{R}_{i\ell})/J_0 .$$

Defining the quasi DS (QDS)

$$\tilde{\rho}_{\vec{k}}(\omega) = \frac{1}{\pi} \operatorname{Im} G_{\vec{k}} (\omega - i0^+) \tag{12}$$

the LOA yields the following Fourier transform of the correlation function

$$B_{\vec{k}} = 2\sigma \int d\omega \tilde{\rho}_{\vec{k}}(\omega)/(e^{\beta\omega} - 1) \tag{13}$$

based on the CGF, while with the AGF

$$B_{\vec{k}} = 2\sigma \tilde{B}_{\vec{k}}/(1 - 2\tilde{B}_{\vec{k}}), \tag{14}$$

$$\tilde{B}_{\vec{k}} = \int d\omega \tilde{\rho}_{\vec{k}}(\omega)/(e^{\beta\omega} + 1). \tag{15}$$

A typical feature of the QDS in both cases I and II is the infinite long tails as a consequence of the Lorentzian

distribution function. In application to more real situations one has to cut off $\tilde{\rho}_{\vec{k}}(\omega)$ at certain energies ω_t, otherwise diverging initial energy $\langle\langle \mathcal{H} \rangle\rangle_c$ occurs. It is of interest to note that in case I an asymmetric QDS results where even the state at $\omega = 0$ and $\vec{k} = 0$ is undamped [31].

For the considered model the localization of the states in $\tilde{G}_{ij}(\omega)$ was previously analysed within the analytical localization theory of Econoumon and Cohen [11,21]. Although exact data on the position of the mobility edges are missing there are strong indications that for $\gamma \neq 0$ and $\Gamma \neq 0$ the states at $\omega \leq 0$ (and also at $\omega \geq 2J_0$) become localized in both cases where for II the states even insides the band $(0 < \omega < 2J_0)$ can also localize increasing Γ. Therefore, according to the statements in the last section the results of the CGF should be incorrect. Indeed, the CGF fails to describe a stable (or physically reasonable) state with $\sigma \neq 0$ already for arbitrary small γ and Γ. Earlier [31], we have argued that in the case I the high part of negative J_{ij} destroys the ferromagnetic order, i.e., $\sigma \equiv 0$. However, keeping in mind that the mean field approximation (MFA) leads to $\sigma \neq 0$ for sufficiently small γ or Γ [32], it seems to be clear that this conclusion is wrong. Contrary to the CGF, the AGF yields results which are qualitatively expected and contain that one of the MFA putting as usual $f_{\vec{k}} = 0$ in Eq. (11). Using Eqs. (6), (11) and (14) the magnetization σ is obtained from the following self-consistent equation

$$\sigma = \frac{1}{2} \frac{1}{\frac{1}{N} \sum_{\vec{k}} (1 - 2\tilde{B}_{\vec{k}})^{-1}} . \qquad (16)$$

A few words about quasi spin wave excitations which energy and damping can be determined from the poles of $\tilde{G}_{\vec{k}}(\omega)$. In the case II all $\vec{k}$-states are damped in the same way and the larger γ the worse $\vec{k}$-states describe the elementary excitations in the spin system. In the case II the small k-states are particular strongly damped so that for $\vec{k} \to 0$ no spin waves can be observed while for k-vectors with $\sigma(1 - f_{\vec{k}}) > \Gamma$ quasi spin waves can propagate for some time. The fact that the $\vec{k} \to 0$ modes become damped is just the consequence of the existence of localized states at $\omega \simeq 0$ which scatter the $\vec{k} \to 0$ spin wave excitation to all other $\vec{k} \to 0$-states. It may happen, the observed peculiarities of the inelastic neutron scattering at rare-earth-transition metal alloys [33] can be explained qualitatively on the basis of our model.

4.1. *Zero-Temperature Magnetization and Curie-Temperature*

At zero-temperatures $T = 0$ the ω-integration in Eq. (14) can be made analytically and $\sigma_0 = \sigma(T = 0)$ is given by

$$\text{I:} \quad \sigma_0 = \frac{1}{\pi} \frac{1}{\frac{1}{N} \sum_{\vec{k}} \left(\operatorname{arctn} \left(\operatorname{sign}(\sigma_0)/\gamma + h_\ell / (|\sigma_0| (1 - f_{\vec{k}}) \gamma) \right) \right)^{-1}} , \qquad (17a)$$

$$\text{II:} \quad \sigma_0 = \frac{1}{\pi} \frac{1}{\frac{1}{N} \sum_{\vec{k}} \left(\operatorname{arctn} \left((h_\ell + \sigma_0 (1 - f_{\vec{k}}))/\Gamma \right) \right)^{-1}} , \qquad (17b)$$

where an external field $H_\ell = J_0 h_\ell$ was taken into account. With increasing γ or Γ, σ_0 decreases as more and more states at $\omega < 0$ appear. That means in localized regions spins are excited, and even antiparallel alignment of some spins is possible. We want to call this type of ground state inhomogeneous unsaturated ferromagnetism (IUFM).

The cases I and II differ qualitatively as the damping of the k-states (see Eq. (11)) in the case I depends directly on σ while in the case II it does not. In the case I the effect of increasing disorder is weakened by decreasing σ_0 and $\sigma_0 \to 0$ only for $\gamma \to \infty$. In contrast to that for case II a breakdown of the IUFM $(\sigma_0 = 0)$ is obtained if $\Gamma \geq \Gamma_c = 1/(\pi W)$, $W = 1/N \sum_{\vec{k}} (1 - f_{\vec{k}})^{-1}$

The field dependence of σ_0 is sketched in *Fig. 1*. In the ordered case ($\gamma = \Gamma = 0$) the saturation magnetization $\sigma_S = 1/2$ is reached always for $h_\ell \gtrsim 0$. Furthermore, no hysteresis is observed as an infinitesimal negative h_ℓ yields instability of the ferromagnetic phase with $\sigma_0 > 0$ against spin wave excitations with $\vec{k} \to 0$, i.e., the magnetization jumps to $-\sigma_S$.

For $\gamma, \Gamma > 0$ the excited states at $\omega < 0$ prevent the saturation and only for $h_\ell \to \infty$ $\sigma \to \sigma_S$. Also, in this case Eqs. (17a and b) do not show hysteresis. Obviously, as well in this case quasi spin waves will be excited for $h_\ell < 0$ ($\sigma_0 > 0$) and in the considered approximation this breaks the $\sigma_0 > 0$ solution.

If, however, the states at $\omega \gtrsim 0$ ($\sigma_0 > 0$) would be localized (no spin waves), a sufficient small negative h_ℓ would only lead to the excitation of localized states which do not destroy immediately the ferromagnetic phase $\sigma_0 > 0$. In such a case hysteresis can occur.

As we have mentioned above localized states at $\omega \gtrsim 0$ can just be expected in the case II. However, the LOA suppresses the appearance of such localized states in $G^+_{ij}(\omega)$, i.e, the LOA is unsufficient to describe hysteresis (cf. Sec. 3.2).

Let us here briefly discuss an opposite approximation. Supposing that in the case II for sufficient large Γ $\tilde{G}_{i\ell}(\omega)$ and $B_{\ell j}$ are nearly diagonal only with an exponential small off-diagonal perturbation, we propose to neglect the second term in Eq. (5a) as a second order effect *. Then σ_0 follows from the equation

$$\sigma_0 = \frac{1}{\pi}\,\frac{1}{N}\sum_{\vec{k}} \operatorname{arctn}\left((h_\ell + \sigma_0(1-f_{\vec{k}}))/\Gamma\right), \qquad (18)$$

which has a similar structure as the MFA-equation. The critical value Γ_c, for which $\sigma_0 = 0$, is here the same as

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* A similar approximation was done by Riess [34] developing a second order theory for the AGF. In this approach the DS contains only discrete δ-functions due to isolated localized states.

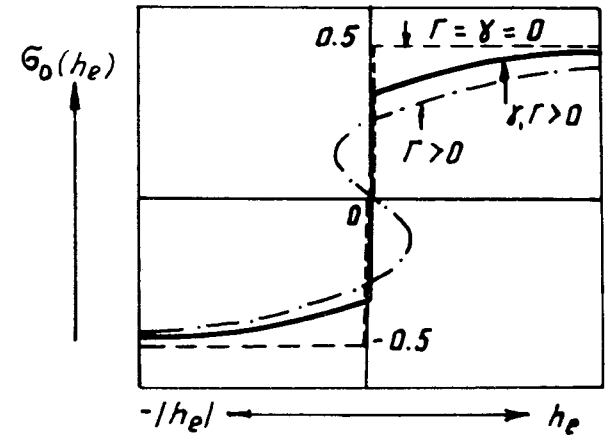

Fig. 1. Field dependence of the zero-temperature magnetization $\sigma_0(h_\ell)$ according to Eq. (17). (——) and (– – –) and to Eq. (18) (– · – · –). Calculations were done for an elliptic band $f_{\vec{k}}$.

in the MFA: $\Gamma_c = 1/\pi$, $\sigma_0(h_\ell)$ shows now the typical hysteresis loop (*Fig. 1*). Consequently, in spite of the crude approximation one can site that localized states at $\omega \gtrsim 0$ may essentially influence the hysteresis behaviour of amorphous ferromagnets [3,35].

The Curie-temperature T_c is determined from the condition $\sigma(T_c) \to 0$ when assuming a second order phase transition. Since in the case I $\tilde{\rho}_{\vec{k}}(\omega)$ becomes a δ-function for $\sigma \to 0$ (see Eqs. (11), (12)), T_c does not depend on γ and has the value: $k_B T_c / J_0 = 1/(4W)$. That means the fluctuations of J_{ij}, especially the presence of some negative J_{ij}, reduce only σ_0. This feature of the model system resembles the observations in amorphous Ni-samples (cf. [36]), where σ_0 is reduced stronger than T_c in comparison with the crystalline values. A theoretical explanation of this circumstance could also be given on the basis of a Stoner like theory [36].

The opposite behaviour is obtained in the case II, where

$$k_B T_c / J_0 = B(T_c)/W, \quad B(T) = \frac{\Gamma}{\pi}\int dx\, \frac{1}{x^2+\Gamma^2}\,\frac{e^{\beta x}}{(e^{\beta x}+1)^2}. \qquad (19)$$

It can be shown that $T_c \to 0$ if $\Gamma \to \Gamma_c$. The relative values $\sigma_r = 2\sigma_0$ and $T_{cr} = 4WT_c$ are plotted in *Fig. 2.*

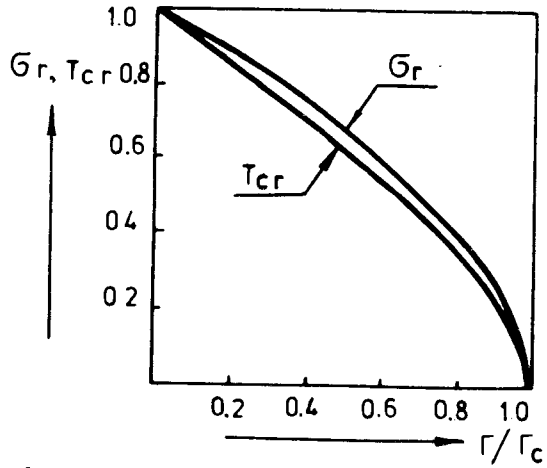

Fig. 2. Reduce d zero-temperature magnetization $\sigma_r = \sigma_0(\Gamma)/\sigma_0(0)$ *and reduced Curie-temperature* $T_{cr} = T_c(\Gamma)/T_c(0)$ *versus disorder strength* Γ *in the case II for an elliptic band* $f_{\vec{k}}$.

4.2 Low Temperature Properties

At first let us consider $\sigma(T)$. An expression of (15) at low temperatures $k_B T/J_0 \ll 1$ yields the relation

$$\sigma(T) = \sigma_0 (1 - A_2 (k_B T)^2) + 0(T^4), \tag{20}$$

where

$$A_2 = 4\sigma_0 D/N \sum_{\vec{k}} P_{\vec{k}} / (1 - 4\sigma_0^2/N \sum_{\vec{k}} R_{\vec{k}})$$

$$D = \int_0^\infty dx\, x^2 e^x/(e^x+1)^2, \quad P_{\vec{k}} = \frac{\partial \tilde{\rho}_{\vec{k}}(\omega)}{\partial \omega}\Big|_{\omega=0} / (1 - 2\tilde{B}^0_{\vec{k}})^2,$$

$$R_{\vec{k}} = -\int_{-\infty}^{0} d\omega \frac{\partial \tilde{\rho}_{\vec{k}}(\omega)}{\partial \sigma}\Big|_{\sigma=\sigma_0} / (1 - 2\tilde{B}^0_{\vec{k}})^2, \quad \tilde{B}^0_{\vec{k}} = \tilde{B}_{\vec{k}}(T=0). \tag{21}$$

In the case I $R_{\vec{k}} = 0$ while in the case II $A_2 \to \infty$ as $\Gamma \to \Gamma_c$. From the definition of the isothermal susceptibility $\chi(T) = \partial\sigma/\partial H$ and Eq. (20) it follows that

$$\chi(T) = \chi_0 + \Delta\chi (k_B T)^2 + 0(T^4), \tag{22}$$

where $\chi_0 = \partial\sigma_0/\partial H_\ell$ and $\Delta\chi = \partial(\sigma_0 A_2)/\partial H_\ell$.

The so far obtained T^2-behaviour of $\sigma(T)$ and $\chi(T)$ in disordered FM [4,37,38] was attributed to Stoner excitations and is characteristic for weak band FM. According to the results of our model the same T^2-dependence should be observed for IUFM. The comparison with real systems requires, however,a more carefull analysis of the T-dependence in some experiments.

A characteristic deviation from the T-dependence in crystalline systems is obtained as well for the specific heat C(T), which is defined by

$$\frac{C(T)}{k_B} = \frac{\partial E(T)}{\partial T}, \quad E(T) = \langle\langle \mathcal{H} \rangle\rangle_c. \tag{23}$$

The initial energy E(T) can be calculated by the exact formula [17]

$$E(T) = \frac{1}{2}\sum_i \langle\langle S_i^- \, i\frac{d}{dt} S_i^+ \rangle\rangle_c - \frac{1}{4}\sum_{ij} J_{ij} \langle\langle S_i^- S_j^+ \rangle\rangle_c -$$

$$- \frac{1}{4}\sum_{ij} J_{ij} \langle S_j^z \rangle_c + \frac{1}{2}\sum_i H_i \langle S_i^- S_i^+ \rangle_c, \tag{24}$$

where we have set $H_\ell = 0$ and $\sum_i H_i = 0$. For structure averaging one can use the equation of motion (5b). In LOA and for $\langle S_i^z \rangle \simeq \sigma$ it results in the expressions

$$\text{I:} \quad E = E_0 + \frac{1}{2}\sum_{\vec{k}} \int d\omega\, \omega (1+2\sigma) \frac{\tilde{\rho}_{\vec{k}}(\omega)}{1 - 2\tilde{B}_{\vec{k}}} / (e^{\beta\omega} + 1),$$

$$\text{II:} \quad E = E_0 + \sigma \sum_{\vec{k}} \int d\omega\, (2\omega + \langle B_{ii} \rangle_c J_0 (1 - f_{\vec{k}})) \frac{\tilde{\rho}_{\vec{k}}(\omega)}{(1 - 2\tilde{B}_{\vec{k}})(e^{\beta\omega}+1)}, \tag{25}$$

where $E_0 = -(1/8)NJ_0$. The analysis of Eqs. (23) and (25) is not directly possible as some integrals diverge, i.e., $E(T) \to \infty$. Neglecting , however, the unphysical long tails of the QDS ($|\omega| < \omega_t$) the convergence of $E(T)$ is ensured. Then a linear T-dependence $C(T) \sim \gamma_M T$ follows, mainly, as a consequence of the finite $\tilde{\rho}_{\vec{k}}(\omega = 0)$. The contribution of $C(T)$ proportional to $\tilde{\rho}_{\vec{k}}(\omega = 0)$ is given by

$$\text{I:} \quad \frac{C_1(T)}{k_B} = D(1+2\sigma_0) \sum_{\vec{k}} \frac{\tilde{\rho}_{\vec{k}}(0)}{(1-2\tilde{B}^{\circ}_{\vec{k}})} \cdot k_B T \,,$$

$$\text{II:} \quad \frac{C_1(T)}{k_B} = 4\sigma_0 D \sum_{\vec{k}} \frac{\tilde{\rho}_{\vec{k}}(0)}{(1-2\tilde{B}^{\circ}_{\vec{k}})} \,. \qquad (26)$$

Moreover further linear T-contributions exist which are all proportional to $\partial\tilde{\rho}_{\vec{k}}(\omega)/\partial\omega|_{\omega=0}$, e.g., due to the T^2-dependence of $\sigma(T)$ and $\tilde{B}_{\vec{k}}(T)$. As the effect of these contributions is hard to estimate ($C(T) - C_1(T)$ increases with ω_t), here we do not want to analyse further the $C(T)$ -formula.

We note that the obtained $C(T)$-behaviour of our model system is in common with the linear $C(T)$ -law of a wide class of disordered and amorphous systems [39,40], for which several explanations are proposed [40,41]. In microscopic approaches the common point of the explanation is the finite DS at $\omega = 0$. In our case the high concentration of localized states at $\omega = 0$ plays an important role in producing a finite DS at $\omega = 0$. To distinguish the magnetic contribution γ_M from that one of the structural or electronic origin one can apply an external magnetic field which should switch off the magnetic contribution. In our model $\gamma_M \to 0$ if $H_\ell \to \infty$.

4.3. Finite Temperatures

For some typical amorphous FM the magnetization shows a flattening in the temperature variation predicted

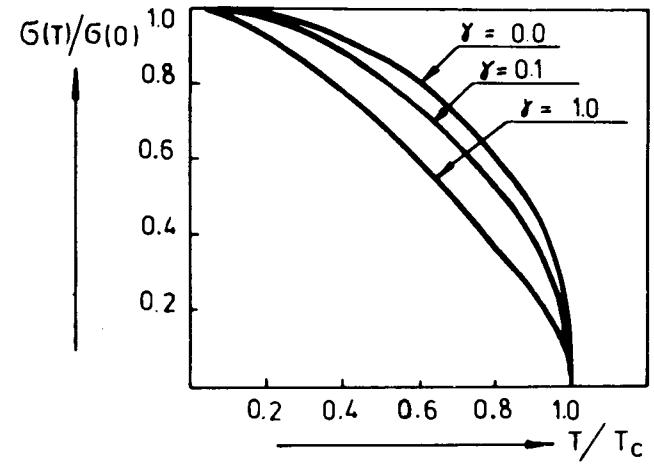

Fig. 3. Reduced magnetization $\sigma(T)/\sigma(0)$ in dependence of disorder strength in the case I for an elliptic band $f_{\vec{k}}$.

as well by the MFA [2]. In *Fig. 3* we present the results on $\sigma_r(t) = \sigma(T/T_c)/\sigma_0$ for the case I. This picture is similar to the MFA results [2]. On the other hand, the flattening seems to be typical only for the case of fluctuating J_{ij} , as we have not found this behaviour of $\sigma_r(t)$ for the case II ($\Gamma \leq \Gamma_c$ - all curves nearly coincide within the accuracy of numerical calculations).

In the ordered Heisenberg model the susceptibility shows a singular behaviour at $0 \leq T \leq T_c$ if $H_\ell \to 0$. It is therefore of interest to investigate the influence of disorder on this singular behaviour. From formula (15) the zero field susceptibility follows as

$$\chi(T) = \frac{4\sigma(0)/N \sum_{\vec{k}} B^{(1)}_{\vec{k}} / (1-2\tilde{B}_{\vec{k}}(0))^2}{1 - 4\sigma(0)/N \sum_{\vec{k}} B^{(2)}_{\vec{k}} / (1-2\tilde{B}_{\vec{k}}(0))^2} \,, \qquad (27)$$

where

$$B^{(1)}_{\vec{k}} = \beta \int d\omega \, \tilde{\rho}_{\vec{k}}(\omega)|_{H_\ell = 0} \, e^{\beta\omega} / (e^{\beta\omega} + 1)^2 \,,$$

$$B^{(2)}_{\vec{k}} = \int d\omega \, \frac{\partial\tilde{\rho}_{\vec{k}}(\omega)}{\partial\sigma}\Big|_{H_\ell = 0} / (e^{\beta\omega} + 1), \qquad (28)$$

If $\sigma(0)=0$ ($T > T_c$ or $\Gamma > \Gamma_c$) with (19) one gets

$$\chi(T) = \frac{1}{4} B(T) / (1/N \sum_{\vec{k}} (1 + \chi(T)(1 - f_{\vec{k}}))^{-1}). \qquad (29)$$

It is easy to check that the denominator in (27) vanishes only as $T \to T_c (\geq T)$ and $\chi(T)$ from Eq. (29) diverges as $\Gamma \to \Gamma_c$ or $T \to T_c (\leq T)$. Expanding $B_{\vec{k}}^{(1)}$ and $\tilde{B}_{\vec{k}}(0)$ up to an order of k^2, it can be shown that in the case II the nominator of (27) diverges for $\Gamma < \Gamma_c$. In the case I such a divergence was not found, i.e., in this case randomness of J_{ij} prevents the singular behaviour at $0 \leq T < T_c$. On the other hand, also in the case II $\chi(T)$ becomes finite at $0 \leq T < T_c$ if one uses the approximation leading to Eq. (18). That means the existence of localized states at $\omega \simeq 0$ seems to be the main reason why the singularity of $\chi(T)$ at $0 \leq T < T_c$ disappears.

5. *DISCUSSION*

5.1. *On the Inhomogeneous Unsaturated Ferromagnetism*

The results of the simple model case (Sec. 4) indicate that localized states in the disordered Heisenberg model ($\langle J_{ij} \rangle_c > 0$) can provide a finite DS at $\omega \leq 0$ while the ferromagnetic like long range order ($\sigma > 0$) will not be destroyed. On the other hand, this circumstance leads to the conjecture that a finite DS at $\omega \leq 0$ obtained within a RPA-LOA theory, i.e.,

$$\tilde{\rho}(\omega \leq 0) = \frac{1}{\pi} \operatorname{Im} \frac{1}{N} \sum_i \langle \tilde{G}_{ii}(\omega - i0^+) \rangle_c > 0 .$$

can reflect the existence of localized states at $\omega \leq 0$ with a macroscopic weight. Then the use of the AGF rather than the CGF would be required.

Just such a behaviour of the DS was obtained within some RPA-LOA based on the coherent potential approximations (CPA)[25-28,42-44]. Frequently, $\tilde{\rho}(\omega \leq 0) > 0$ was connected with an instability of the ordered phase as only the CGF was used. Therefore it would be desirable

to reexamine these theories on the basis of the AGF. Let us discuss some examples.

Applying a single bond CPA, some different kinds of J_{ij} -distribution (only nearest-neighbour exchange) in the random bond Heisenberg model have been investigated [25-28,43]. For discrete J_{ij} -distribution[25-28,43] $\tilde{\rho}(\omega)$ spreads to $\omega < 0$ if the concentration of negative J_{ij} ($\langle J_{ij} \rangle_c > 0$) exceeds a critical value. The same effect is obtained for a rectangular J_{ij} -distribution function[26,28] if the mean square fluctuations $\Delta = \langle (J_{ij} - \langle J_{ij} \rangle_c)^2 \rangle_c / \langle J_{ij} \rangle_c^2$ becomes larger than $\Delta_c \simeq 1.0$[28]. In two dimensions applying an external field H_ℓ the Gaussian distribution of J_{ij} yileds gap states, with energies smaller than the Zeeman gap energy H_ℓ as $\Delta \geq 0.5$[27]. However, in three dimensions ($H_\ell = 0$) this distribution reveals a particular behaviour. Here $\tilde{\rho}(\omega \leq 0) \leq 0$ (*Fig. 4*) for $\Delta > \Delta_c \simeq 1.0$[28]. This $\tilde{\rho}(\omega)$ enables furthermore a stable long range order $\sigma > 0$ obtained by the CGF calculations[28] (see Sec. 5.2). Probably, because of i) the smeared distribution function without critical long tails (as for the Lorentzian) and ii) the strong coupled DR and ODR, no localized states at

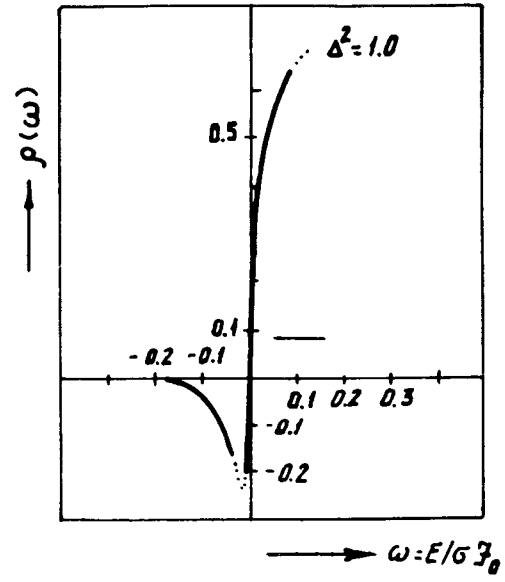

Fig. 4. Density of states as a function of energy according to[28].

$\omega \leq 0$ with a macroscopic weight will appear in this three dimensional case. Contrary to that, the enhanced localization tendency in lower dimensions [8,9;22] could explain the appearance of localized states at $\omega \leq H_f$ in two dimensions.

Besides in the case where negative J_{ij} are present in a substantial amount, also for some disordered alloy cases and for dilute ferromagnets $\tilde{\rho}(\omega \leq 0) > 0$ was detected by a single site [42] and a cluster CPA [44]. As for such systems any kind of localized states at $\omega \leq 0$ (connected with isolated spins as well with large but finite magnetic clusters) is really expected (see [27,44,45]), the fact $\tilde{\rho}(\omega \leq 0) > 0$ can indicate dense localized states at $\omega \leq 0$. If one would use the AGF these CPA theories would not break down if the response spreads to negative ω. For instance, within the theory of Harris et al. [44] one could hope to avoid the unphysical singularity in $T_c(x)$ at the concentration of non-magnetic atoms $x = x^*$ and for $x > x^*$ to describe the infleunce of existing localized states on the thermodynamics quite correctly.

Obviously, the used CPA versions can modify the results of the complicated interplay of coupled DR and ODR. As a single site and cluster CPA produce always only an effective band edge, i.e., cut up tails [8,9,18], the appearance of $\tilde{\rho}(\omega \leq 0) > 0$ can essentially depend on the assumptions about the coherent potential or effective medium. For instance, in some theories for the alloy and dilute FM case [46] $\tilde{\rho}(\omega \leq 0) > 0$ it has not been found what may be caused by the special ansatz for the coherent potential which automatically satisfied that the complex coherent excitation energy $\omega_{\vec{k}}$ goes to zero if $\vec{k} \to 0$. This assumption was based on the Goldstone theorem [47]. However, in our opinion such an approach projects out the influence of localized states at $\omega \leq 0$. Furthermore, the application of the Goldstone theorem in such a way calls forth an objection. Even if the total spin $\vec{S} = \sum \vec{S}_i$ commutes with $\mathcal{H}$ and the eigenstates of $\vec{S}$ can be characterized by $\vec{k} = 0$, it does not mean that $\mathcal{H}$ has a $\vec{k} = 0$ spin wave mode as an eigenstate. Because of the lack of translational symmetry $\vec{k}$ -vectors are no longer the actual quantum numbers. In dependence of the dis-

order strength rather different ground states may exist and, accordingly, the low lying excitations are either complicated extended or localized states. The Goldstone theorem, for our knowledge so far proved only for translational invariant systems, requires a gapless excitation spectrum if in the ground state the continuous symmetry of $\mathcal{H}$ was broken spontaneouslt. This statement, however, does not provide a direct assertion about the coherent potential at $\vec{k} \to 0$.

5.2. *On the Homogenized Unsaturated Ferromagnetism*

Let us return to the interesting case where J_{ij} are randomly distributed according to a Gaussian distribution function. The results of the single bond CPA [28] for $\tilde{G}_{ij}(\omega)$ would lead to unphysical magnetization ($\sigma > 1/2$) using the AGF. On the other hand, the obtained states at $\omega < 0$ (*Fig.4*) yield a zero point reduction of σ, i.e., $\sigma_0 < 1/2$, when applying the CGF. $\tilde{\rho}(\omega = 0) = 0$ means certainly that the states at $\omega \geq 0$ are extended ones which feel the three dimensional extension (analogously to spin waves in ordered systems: in one dimension $-\rho(\omega \to 0) \to \infty$; in two dimensions - $\tilde{\rho}(\omega = 0) \neq 0$, in three dimensions $\tilde{\rho}(\omega = 0) = 0$). The states at $\omega < 0$ with a spectral intensity $I^-(\omega) \sim \tilde{\rho}(\omega)/(e^{\beta\omega} - 1) \geq 0$ can be interpreted as in the case of ordered antiferromagnetism where the pure Neel-state is unstable against some spin wave excitations [17,18]. Here for $\Delta > \Delta_c$, i.e., if the amount of negative J_{ij} exceeds a critical value, the pure FM state is not further the actual ground state. One particle spin excitations (magnons) stabilize now the ground state. Assuming that these magnons are extended excitations caused by a sufficiently large delocalization effect of the ODR in this case, it is possible to explain the different results obtained by the CGF and the AGF in the sense of Sec. 3.2. We want therefore to call such a kind of ferromagnetic like order homogenized unsaturated FM (HUFM).

From the considered example in [28] it follows that HUFM should result from a smooth J_{ij}-distribution single peaked about $J_{ij} \sim \langle J_{ij} \rangle_c$. If we yet take into account correlations between neighbouring J_{ij} and $\langle S_i^z \rangle$ relaxations (Sec. 3.2) which both can enhance the delocalization tendency, the HUFM can certainly be observed in some amorphous transition metal-metalloids [1,3,4].

Assuming the existence of the HUFM one can resolve a discrepancy concerning spin wave and magnetization measurements for amorphous ferromagnetic alloys [3,4,48]. Inelastic neutron scattering studies [48] have permitted the spin wave dispersion relation $\omega_{\vec{k}} = Dk^2$ to be defined at small wave vectors. The discrepancy remains in that the $\vec{k} \to 0$ spin waves (within the usual spin wave theory) provide only about 70% of the density of magnon states necessary to explain $\sigma(T) = \sigma_0(1 - BT^{3/2} + \ldots)$. Within the RPA-LOA theory for the HUFM $\sigma(T)$ is given by

$$\sigma(T) = 1/2(1+2P)^{-1}, \qquad (30)$$

$$P = \int d\omega \tilde{\rho}(\omega)/(e^{\beta\omega} - 1),$$

where $\tilde{\rho}(\omega)$ has a form shown in *Fig. 4*. If we separate the zero temperature contribution, P can be rewritten as

$$P = \int_0^\infty d\omega(-\rho(-\omega)) + \int_0^\infty d\omega(\tilde{\rho}(\omega) - \tilde{\rho}(-\omega))/(e^{\beta\omega} - 1). \qquad (31)$$

As we have proposed that the states at $\omega \approx 0$ are extended, these low lying excitations can be quite well characterized as $\vec{k} \to 0$ spin waves (in an averaged sense). Their dispersion at $\omega > 0$ and $\omega < 0$ may be supposed as $\omega_{\vec{k}}^{+} = D^{+}k^2 + \ldots$ and $\omega_{\vec{k}}^{-} = D^{-}k^2 + \ldots$ respectively, while the damping of these modes can be neglected for $\vec{k} \to 0$. Then it is expected that for $\omega \to 0^{+}$ $\tilde{\rho}(\omega) \sim A^{+}\omega^{1/2}$ and $-\tilde{\rho}(-\omega) \sim A^{-}\omega^{1/2}$, where $A^{\pm}$ is proportional to $(D^{\pm})^{-3/2}$ and to a specific weighing factor depending on concrete calculations. As the neutron scattering has indicated only one response for small $\vec{k}$ it is to be expected that either $D^{+} \sim D^{-}$ or the

intensity of the $\omega_{\vec{k}}$-model is relative small (or damping large). On the other hand, according to Eqs. (30) and (31) both modes reduce the magnetization increasing temperature. Consequently, the ordinary spin wave theory ($\tilde{\rho}(\omega) = 0$ for $\omega \leq 0$), in which $B = 2.612\,(k_B/(4\pi D^{+}))^{3/2}\sigma_0$, could not explain the real situation. The remaining 30% of the observed B would then be caused by the already in the ground state existing extended magnons and not by localized spin excitations as it was proposed in [48].

To our knowledge, only within the single bond CPA $\tilde{\rho}(\omega \leq 0) \leq 0$ could be detected. Because of the crude approximation it will be necessary to check these results using improved CPA versions. In this connection, it will be of interest to include the field dependence of $\tilde{\rho}(\omega)$ which is believed to be important to understood the difference in neutron and NMR measurements [48,49].

5.3. A Phase Diagram

To complete our discussion, we want to summarize our results in one kind of phase diagram for amorphous ferromagnets (see the *Table*). The application of the CGF and the AGF in various cases of disordered Heisenberg systems suggests that just three qualitatively different forms of amorphous FM exist: i) the pure FM, ii) the unsaturated FM with a homogenized spin structure due to extended zero-point fluctuations (HUFM), and iii) the unsaturated FM with local or inhomogeneous spin structure and localized low lying excitations (IUFM). If the ferromagnetic order becomes unstable, only a complete random ordered phase (ROP) is possible. We note that it is of course difficult to set up sharp boundaries for any real systems, and a more complicated behaviour can be observed. Furthermore, with increasing disorder in a certain way transitions occur not only in the placed sequence: FM-HUFM-IUFM-ROP. One can as well imagine the following transitions: FM-IUFM-ROP and FM-HUFM-ROP. In the phase diagram we have also included some conjectures about that in which real systems each

Table. A phase diagram for amorphous magnets ($<J_{ij}>_c > 0$).

⟶ increasing randomness, e.g., $\Delta = <(J_{ij} - <J>_c)^2>_c / <J>_c^2$, or changing disorder			
Pure FM	Homogenized unsaturated FM	Inhomogeneous unsaturated FM	Random ordered phase (Micromagnetism, Spinglass)
$\sigma - \sigma_0 \sim T^{3/2}$	$\sigma - \sigma_0 \sim T^{3/2}$	$\sigma - \sigma_0 \sim T^2$	$\sigma_0 \neq 0 \quad (H_\ell \neq 0)$
$\sigma_0 = \sigma_S$	$\sigma_0 < \sigma_S$	$\sigma_0 < \sigma_S$	$\chi - \chi_0 \overset{?}{\sim} T,\ T^2,\ T^{3/2}$
$\chi \sim T^{3/2}, \chi_0 = 0$	$\chi - \chi_0 \sim T^{3/2}, \chi_0 > 0$	$\chi - \chi_0 \sim T^2, \chi_0 > 0$	$\chi_0 \geq 0$
$C \sim T^{3/2}$	$C \sim T^{3/2}$	$C \sim T$	$C \overset{?}{\sim} T,\ T^{3/2}$
$\omega_{\vec{k}\to 0} \sim D k^2$	$\omega^+_{\vec{k}\to 0} \sim D^+ k^2,\ \omega^-_{\vec{k}\to 0} \sim D^- k^2$	$\omega_{\vec{k}\to 0} \sim D k^2 + i\Gamma_{\vec{k}}$	
		$\Gamma_{\vec{k}\to 0} \sim \begin{cases} k^2 \\ \text{const} \end{cases}$	$\omega_{\vec{k}\to 0} \overset{?}{\sim} \begin{cases} D k^2 + i\Gamma_{\vec{k}} \\ D'\lvert\vec{k}\rvert + i\Gamma_{\vec{k}} \end{cases}$
$Fe_xGe_{1-x}, x > 0.6$ [37]	$Fe_xGe_{1-x}, 0.6 > x \gtrsim 0.5$ [37]	$Fe_xGe_{1-x}, 0.5 \gtrsim x \gtrsim 0.4$ [37]	$Fe_xGe_{1-x}, x \lesssim 0.4$ [37]
$Ni_{1-x}P_x, x < 0.18$ [38]	$(Fe_{93}Mo_7)B_{10}P_{10}$ [48]	$Ni_{1-x}P_x, x > 0.18$ [38]	$Ni_{1-x}P_x, x \gtrsim 0.25$ [38]
$Co_{1-x}P_x, x \to 0$ [48,49]	$Fe_{75}P_{15}C_{10}$ [cf. 51]	M_xAu_{1-x}, M=Ni,Co,Fe [52]	$Fe-Y$ [4]
$Gd_{80}Au_{20}$ [50]	Gd–Al [51]	$Fe_{1-x}Tb_x$ [cf. 4]	$Dy-Co_3$ [4]
		$Y_{1-x}Ni_x$ [cf. 4]	Gd–Al [51]
		Dy–Cu [4]	

phase is eventually realized. Obviously, the proof of such a phase diagram requires further theoretical investigations as well as detailed experimental analysis.

ACKNOWLEDGEMENT

The author thanks Dr. V.L.Aksenov, Dr. N.M.Plakida, Dr. S.Kobe and Dr. H.Rochter for helpful discussions and Dr. W.John for reading the manuscript.

REFERENCES

1. *Cargill G.S., III. AIP Conference Proceedings, 1975, 24, p.188.*
2. *Handrich K. In: Magnetische Eigenschaften von Festkorpern, VEB Deutscher Verlag Grundstoffindustrie, Leipzig, 1976, p.169.*
3. *Mizoguchu T. AIP Conf. Proc., 1976, 34, p.286; Hasegawa R., O"Handley R.C., Mendelsolm L.I. Ibid., p.298.*
4. *Coey L.M.D. J.Appl.Phys., 1978, 49, p.1646.*
5. *Krey U. J.Magn.Mat., 1978, 18, p.67.*
6. *Fischer K.H. Physica, 1977, 86-88B, p.813.*
7. *Anderson P.W. Phys.Rev., 1958, 109, p.1492.*
8. *Mott N.F., Davis E.A. "Electronic Processes in Non-Crystalline Materials", Clarendon Press, Oxford, 1971.*
9. *Thouless D.J. Phys.Rep., 1974, 13C, p.93.*
10. *Izyumov Y.A. Proc.Phys.Soc., 1966, 87, p.505; Izyumov Y.A., Medvedev M.V. JETF, 1968, 55, p.1123.*
11. *Economou E.N., Cohen M.H. Phys.Rev., 1972, B5, p.2931.*
12. *Adkins C.J., Pollitt S., Pepper M. J. de Phys., 1976, C4, p.343.*
13. *Buyers W.J.L., et al. Phys.Rev.Lett., 1971, 27, p.1442.*
14. *Economou E.N. Phys.Rev.Lett., 1972, 28, p.1206.*
15. *Kirkpatrick S.K., Harris A.B. Phys.Rev., 1975, B12, p.4980.*
16. *Zubarev D.N. Usp.Fiz.Nauk, 1960, 71, p.71.*
17. *Tjablikov S.V. "Methods in the Quantum Theory of Magnetism", Plenum Press, New York, 1967.*

18. *Elliott R.J., Krumhansl J.A., Leath P.L. Rev.Mod. Phys., 1974, 46, p.465.*
19. *Ramos J.G., Gomes A.A. Nuovo Cim., 1971, 3A, p.441.*
20. *Hermann H. phys. stat. sol. (b), 1974, 63, p.K65.*
21. *Schreiber J., John W. phys. stat. sol. (b), 1976, 78, p.199.*
22. *Economou E.N., Autoniou P.D. Phys.Rev., 1977, B16, p.3768.*
23. *Weaire D., Srivastava V. J.Phys., 1977, C10, p.4309. and to be published .*
24. *Montgomery C.G., Krugler J.I., Stubbs R.M. Phys. Rev.Lett., 1970, 25, p.669.*
25. *Ni Foo E., Wu D. Phys.Rev., 1972, B5, p.98.*
26. *Tahir-Kheli R.A. Phys.Rev., 1972, B6, p.2808; In: Amorphous Magnetism, Plenum Press, New York, 1973; Solid State Comm., 1976, 19, p.1213.*
27. *Krey U. Int. J.Magn., 1973, 5, p.137; Phys. Cond. Matter, 1974, 18, p.17; Krey U., Schlichting H.J. Z.Physik, 1975, B21, p.157.*
28. *Schreiber J. phys. stat. sol. (b), 1973, 59, p.K119; 1974, 61, p.K17; Wiss. Zeit. TU Dresden, 1974, 23, p.1014.*
29. *Butler W.H., Kohn W. J.Res.Nat.Bur. Std. (US), 1970, 74A, p.443.*
30. *Loyd P. J.Phys., 1969, C2, p.1717.*
31. *John W., Schreiber J. phys. stat. sol. (b), 1973, 66, p.193.*
32. *Pytte E., Schneider T. Phys.Rev., 1977, B15, p.1519.*
33. *Rhyne J.J., Price D.L., Mook H.A. AIP Conf. Proc., 1975, 24, p.121.*
34. *Riess I. Physica, 1974, 74, p.496.*
35. *Callen E., Lin Y.J., Cullen J.R. Phys.Rev., 1977, B16, p.263.*
36. *Richter J., Schreiber J., Handrich K. phys. stat. sol. (b), 1976, 74, p.K125.*
37. *Suran G., Daver H., Sztern J. AIP Conf. Proc., 1976, 34, p.310; Physica , 1977, 86-88B, p.810.*
38. *Berrada A. et al. Physica, 1977, 86-88B, p.790.*
39. *Stephens R.B. Phys.Rev., 1973, B8, p.2896.*
40. *Giaquinta P.V. et al. Phys.Rev.Lett., 1977, 39, p.41.*
41. *Anderson P.W., Halperin B.I., Varma C.M. Phil. Mag., 1972, 25, p.1.*
42. *Harris R., Plischke M., Zuckermann M.J. phys. stat. sol. (b), 1972, 53, p.269.*
43. *Oguchi T., Ono I., Ishikawa. J.Phys.Soc.Jap., 1973, 35, p.1586.*
44. *Harris A.B. et al. J.Phys., 1974, C7, p.1693.*

45. *Kirkpatrick S., Eggarter T.P. Phys.Rev., 1972, B6, p.3598.*
46. *Theumann A. J.Phys., 1973, C6, p.2822; 1974, C7, p.2328; Theumann A., Tahir-Kheli R.A. Phys.Rev., 1975, B12, p.1796.*
47. *Anderson P.W. Concepts in Solids, Benjamin, New York, 1963.*
48. *Axe J.D. et al. Phys.Rev., 1977, B15, p.2763.*
49. *McColl J.R. et al. AIP Conf. Proc., 1976, 29, p.172.*
50. *Poon S.J., Durand J. Phys.Rev., 1977, B16, p.316.*
51. *Coey J.M.D., von Molnar S., Gambino R.J. Sol. Stat. Comm., 1977, 24, p.167.*
52. *Bergmann G., Marquardt P. Phys.Rev., 1978, B17, p.1355.*

Received by Pyblishing Department on July 26 1978.