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# ON THE SPIN WAVE THEORY OF DISORDERED ITINERANT-ELECTRON FERROMAGNETS

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О спин-волновой теории неупорядоченных ферромагнетиков с делокализованными электронами

Ферромагнитные спиновые волны в неупорядоченных сплавах получены при нулевой температуре на основе микроскопического ферми-жидкостного описания делокализованных электронов. Коэффициент жесткости спиновых волн перенормирован в рамках модели Хаббарда со случайными параметрами с использованием когерентного локального лестничного приближения в канале частица-частица. Рассмотрены тождества Уорда, условие устойчивости и затухание магнонов. Настоящий подход справедлив для систем с сильной корреляцией электронов и малой концентрацией носителей.

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On the Spin Wave Theory of Disordered Itinerant-Electron Ferromagnets

Ferromagnetic spin waves in disordered alloys are derived at zero temperature from a microscopic Fermi liquid description of itinerant electrons. The spin wave stiffness constant is renormalized within the random Hubbard model using the coherent local ladder approximation in the particle-particle channel. Ward identities, the stability condition, and magnon damping are investigated. The present scheme is valid for systems with strong electron correlations and small carrier concentrations.

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## 1. INTRODUCTION

The spin wave excitations in ferromagnetic transition metals and their alloys are affected by the degree of itinerancy of the d-electron system. Such a problem can be described by the Hubbard Hamiltonian /1/ being rotationally invariant in the spin space. Hence in the long-wavelength region one can extract, in principle from the "broken symmetry" /2/, a gapless spectrum $\omega_q = Dq^2$ ($\vec{q}$: wave vector, D: spin wave stiffness constant) while single-particle (Stoner) excitations remain finite in energy. In particular, D is connected with the stability of the ferromagnetic ground state against the low-lying collective modes.

Correlation effects enter into the stiffness constant D. An explicit expression for D in terms of the transverse spin-current autocorrelation function was given by Edwards and Fisher /3/.

Approximations for deriving the magnon spectrum of itinerant-electron ferromagnets have been performed in the following directions:

(i) For pure systems the basis work /4/ was done in the random-phase approximation (RPA). The spin wave theories beyond the RPA involve, e.g., perturbative corrections to the RPA spectrum [5], the T-matrix approximation [6,7], and diagram analysis guided by the Ward relations [8,9]; compare also [10] and the sum-rule approach [11].

(ii) For disordered systems the configurational average can be carried out within the coherent potential approximation (CPA) [12]. The CPA-RPA treatments [13 to 17] (without CPA see [18]) of substitutionally disordered alloys are based on the Hartree-Fock (HF) approximation which completely neglects spin fluctuations. A RPA decoupling scheme was given in [19].

In the present paper we choose a microscopic Fermi liquid approach at zero temperature (cf. [20,21]) to derive the spin wave energy of disordered alloys in the random Hubbard model. The stiffness constant D is renormalized by the coherent ladder approximation [22], i.e., the self-consistent combination of the CPA and the local ladder approximation (LLA) [23].

## 2. SPIN WAVE STIFFNESS CONSTANT AND BETHE-SALPETER EQUATION

The itinerant-electron system in disordered $A_cB_{1-c}$ alloys can be described by the random Hubbard Hamiltonian (cf. [1])

$$H^{\{\nu\}} = \sum_{\substack{ij\sigma \\ (i \neq j)}} t_{ij} c^{+}_{i\sigma} c_{j\sigma} + \sum_{i\sigma} \varepsilon^{\nu}_{i} n_{i\sigma} + \sum_{i} U^{\nu}_{i} n_{i\uparrow} n_{i\downarrow} = H^{\{\nu\}}_{\Delta} + H^{\{\nu\}}_{U} , \qquad (2.1)$$

where $c^{+}_{i\sigma}$ ($c_{i\sigma}$) is the creation (annihilation) operator for a spin $\sigma$ electron in the Wannier state at lattice site i, and $n_{i\sigma} = c^{+}_{i\sigma} c_{i\sigma}$. Within the whole configuration $\{\nu\}$ the atomic energy $\varepsilon^{\nu}_{i}$ and the intra-atomic Coulomb repulsion $U^{\nu}_{i}$ take the random values $\varepsilon^{\nu}$ and $U^{\nu}$ ($\nu = A,B$), respectively, according to whether an A- or B-atom occupies the site i. The hopping integrals $t_{ij}$ are assumed to be independent of the atomic arrangement. The model (2.1) belongs to the class of exchange Hamiltonians, because $H^{\{\nu\}}$ commutes with the total spin. For the interaction part, this rotational invariance in the spin space can be expressed by $H_U^{\{\nu\}} = \sum_i U_i^{\nu} n_{i\uparrow} n_{i\downarrow} = \frac{1}{2} \sum_{i\sigma} U_i^{\nu} n_{i\sigma} - \frac{2}{3} \sum_i U_i^{\nu} \vec{S}_i \cdot \vec{S}_i$ ($\vec{S}_i$: local spin-density operator). Such a form refers to the possibility of collective modes.

Let us introduce the transverse susceptibility (causal spin-density response function) at zero temperature as

$$\chi^{+-\{\nu\}}(\vec{q},\omega) = -\langle\langle S_{\vec{q}}^{+}, S_{-\vec{q}}^{-}\rangle\rangle_{\omega}^{\{\nu\}} = i\int dt\, e^{i\omega t}\langle T S_{\vec{q}}^{+}(t) S_{-\vec{q}}^{-}(0)\rangle^{\{\nu\}} , \tag{2.2}$$

where $S_{\vec{q}}^{+} = \frac{1}{\sqrt{N}} \sum_i c_{i\uparrow}^{+} c_{i\downarrow} e^{-i\vec{q}\vec{R}_i}$ and $S_{-\vec{q}}^{-} = (S_{\vec{q}}^{+})^{+}$, $\vec{R}_i$ is the position vector of site i, and $\langle \ldots \rangle^{\{\nu\}}$ means the ground- state expectation value within $\{\nu\}$. Here $\chi^{+-\{\nu\}}(\vec{q},\omega)$ reflects the linear response to an external rotating magnetic field (rf) $H_i^{+} = H_i^{x}(t) + iH_i^{y}(t) = H^{+}(\vec{q},\omega)\, e^{i(\vec{q}\vec{R}_i - \omega t)}$ applied perpendicular to the direction of spontaneous magnetization (z-axis); i.e., the net transverse magnetization is given by $\langle M^{+}(\vec{q},\omega)\rangle^{\{\nu\}} = \chi^{+-\{\nu\}}(\vec{q},\omega) H^{+}(\vec{q},\omega)$ , where $M_{\vec{q}}^{+} = 2\mu_B S_{\vec{q}}^{+}$. Note that the factor $2\mu_B^2$ ( $\mu_B$: Bohr magneton) is omitted in (2.2). By exploiting the time-reversal invariance, one finds from the equation of motion that

$$\omega^2 \chi^{+-\{\nu\}}(\vec{q},\omega) = -\left[\omega \frac{1}{N}\sum_i 2\langle S_i^{z}\rangle^{\{\nu\}} + \langle [S_{\vec{q}}^{+}, qJ_{-\vec{q}}^{-}]\rangle^{\{\nu\}} + \langle\langle qJ_{\vec{q}}^{+}, qJ_{-\vec{q}}^{-}\rangle\rangle_{\omega}^{\{\nu\}}\right] , \tag{2.3}$$

where $S_i^{z} = \frac{1}{2}(n_{i\uparrow} - n_{i\downarrow})$, and the transverse spin current operator $J_{\vec{q}}^{+}$ (or $J_{-\vec{q}}^{-} = (J_{\vec{q}}^{+})^{+}$) takes the nonrandom form

$$q J^{+}_{\vec{q}} = [S^{+}_{\vec{q}}, H^{\{\nu\}}] = \frac{1}{\sqrt{N}} \sum_{ij} t_{ij} (e^{-i\vec{q}\vec{R}_i} - e^{-i\vec{q}\vec{R}_j}) c^{+}_{i\uparrow} c_{j\downarrow}$$
$$= \frac{1}{\sqrt{N}} \sum_{\vec{k}} (\varepsilon_{\vec{k}+\vec{q}} - \varepsilon_{\vec{k}}) c^{+}_{\vec{k}\uparrow} c_{\vec{k}+\vec{q}\downarrow} . \quad (2.4)$$

In the limit $q \to 0$ the "nonquasiparticle" contribution [24] to $\chi^{+-\{\nu\}}(\vec{q},\omega)$ is identically zero, because $S^{+}_{\vec{q}}$ is a quasiintegral of motion.

The definition of the spin wave stiffness constant D requires an explicit pole ansatz $\chi^{+-}_{\mathrm{pole}}(\vec{q},\omega) = \frac{-2\langle\langle S^z_i\rangle^{\{\nu\}}\rangle_c}{\omega - Dq^2}$ , being valid for small $\omega$ and q (here the imaginary part of the causal response is suppressed). $\chi^{+-}_{\mathrm{pole}}(\vec{q},\omega)$ is a pole part of $\chi^{+-}(\vec{q},\omega) \equiv \langle \chi^{+-\{\nu\}}(\vec{q},\omega) \rangle_c$, where $\langle \ldots \rangle_c$ denotes the configuration average. Note that $\langle\langle S^z_i \rangle^{\{\nu\}}\rangle_c$ is independent of site i. Thus, one can extract D via the prescription

$$D = -\frac{1}{2\langle\langle S^z_i\rangle^{\{\nu\}}\rangle_c} \lim_{\omega\to 0} \lim_{q\to 0} \left[ \frac{\omega^2}{q^2} \left( \chi^{+-}(\vec{q},\omega) + \frac{2\langle\langle S^z_i\rangle^{\{\nu\}}\rangle_c}{\omega} \right) \right]$$
$$= \frac{1}{2\langle\langle S^z_i\rangle^{\{\nu\}}\rangle_c} \left[ \lim_{q\to 0} \frac{1}{q^2} \langle\langle [S^{+}_{\vec{q}}, qJ^{-}_{-\vec{q}}] \rangle^{\{\nu\}}\rangle_c - \lim_{\omega\to 0} \lim_{q\to 0} \chi^{+-}_{J}(\vec{q},\omega) \right] , \quad (2.5)$$

where $2\langle\langle S^z_i\rangle^{\{\nu\}}\rangle_c = (n_\uparrow - n_\downarrow)$ is the magnetization per site ($n_\sigma$: average number of $\sigma$ electrons per site) and $\chi^{+-}_{J}(\vec{q},\omega) = -\langle \langle\langle J^{+}_{\vec{q}}, J^{-}_{-\vec{q}} \rangle\rangle^{\{\nu\}}_{\omega} \rangle_c$ is the transverse spin-current autocorrelation function. Such a relation between D and $\chi^{+-}_{J}$ was derived in [3] for pure systems, and applied to alloys by Hill and Edwards [14]. Whereas D follows exactly from $\chi^{+-}_{\mathrm{pole}}(\vec{q},\omega)$, the pole ansatz involves indeed an approximation; for instance a generalized RPA leads to $\chi^{+-}(\vec{q},\omega) = Z(\vec{q},\omega)/\Delta(\vec{q},\omega)$, where $\mathrm{Re}\,\Delta(\vec{q},\omega_q) = 0$ gives the spin wave spectrum $\omega_q = Dq^2$.

Explicitly, the commutator in (2.5) becomes

$$C_{\vec{q}} = \langle\langle [S^+_{\vec{q}}, qJ^-_{-\vec{q}}] \rangle^{\{\nu\}} \rangle_c = \frac{1}{N}\sum_{ij} t_{ij} \{ (e^{-i\vec{q}(\vec{R}_i-\vec{R}_j)} - 1) \langle\langle c^+_{i\uparrow} c_{j\uparrow} \rangle^{\{\nu\}} \rangle_c + (e^{i\vec{q}(\vec{R}_i-\vec{R}_j)} - 1) \langle\langle c^+_{i\downarrow} c_{j\downarrow} \rangle^{\{\nu\}} \rangle_c \}$$
$$= \frac{1}{N}\sum_{\vec{k}} \{ (\varepsilon_{\vec{k}+\vec{q}} - \varepsilon_{\vec{k}}) \langle\langle n_{\vec{k}\uparrow} \rangle^{\{\nu\}} \rangle_c + (\varepsilon_{\vec{k}-\vec{q}} - \varepsilon_{\vec{k}}) \langle\langle n_{\vec{k}\downarrow} \rangle^{\{\nu\}} \rangle_c \}. \tag{2.6}$$

By employing the cubic symmetry hereafter one obtains

$$\lim_{q\to 0} \frac{C_{\vec{q}}}{q^2} = -\frac{1}{6N}\sum_{ij\sigma} t_{ij} (\vec{R}_i - \vec{R}_j)^2 \langle\langle c^+_{i\sigma} c_{j\sigma} \rangle^{\{\nu\}} \rangle_C = \frac{1}{6N}\sum_{\vec{k}\sigma} (\nabla^2_{\vec{k}} \varepsilon_{\vec{k}}) \langle\langle n_{\vec{k}\sigma} \rangle^{\{\nu\}} \rangle_C , \tag{2.7}$$

since no term to order q contributes to $C_{\vec{q}}$ in (2.6) due to time-reversal invariance. The limiting procedure in

$$\chi^{+-}_J(\vec{q},\omega) = -\frac{1}{q^2}\frac{1}{N}\sum_{ijmn} t_{ij} (e^{-i\vec{q}\vec{R}_i} - e^{-i\vec{q}\vec{R}_j}) t_{mn} (e^{i\vec{q}\vec{R}_n} - e^{i\vec{q}\vec{R}_m}) \langle\langle\langle c^+_{i\uparrow} c_{j\downarrow}, c^+_{m\downarrow} c_{n\uparrow} \rangle\rangle^{\{\nu\}}_\omega \rangle_C$$
$$= -\frac{1}{q^2}\frac{1}{N}\sum_{\vec{k}\vec{k}'} (\varepsilon_{\vec{k}+\vec{q}} - \varepsilon_{\vec{k}})(\varepsilon_{\vec{k}'+\vec{q}} - \varepsilon_{\vec{k}'}) \langle\langle\langle c^+_{\vec{k}\uparrow} c_{\vec{k}+\vec{q}\downarrow}, c^+_{\vec{k}'+\vec{q}\downarrow} c_{\vec{k}'\uparrow} \rangle\rangle^{\{\nu\}}_\omega \rangle_C \tag{2.8}$$

leads to

$$\lim_{\omega\to 0}\lim_{q\to 0} \chi^{+-}_J(\vec{q},\omega) = -\frac{1}{3N}\sum_{ijmn} \vec{j}_{ij} \cdot \vec{j}_{mn} \langle\langle\langle c^+_{i\uparrow} c_{j\downarrow}, c^+_{m\downarrow} c_{n\uparrow} \rangle\rangle^{\{\nu\}}_{\omega=0} \rangle_C$$
$$= -\frac{1}{3N}\sum_{\vec{k}\vec{k}'} \nabla_{\vec{k}}\varepsilon_{\vec{k}} \cdot \nabla_{\vec{k}'}\varepsilon_{\vec{k}'} \langle\langle\langle c^+_{\vec{k}\uparrow} c_{\vec{k}\downarrow}, c^+_{\vec{k}'\downarrow} c_{\vec{k}'\uparrow} \rangle\rangle^{\{\nu\}}_{\omega=0} \rangle_C , \quad \vec{j}_{ij} = -i t_{ij} (\vec{R}_i - \vec{R}_j). \tag{2.9}$$

Now we give a way of attacking the correlation problem by means of the Bethe-Salpeter (BS) equation. Let us express the spin current-spin current response (2.8) in terms of the causal two-particle correlation function $L^{\{\nu\}}$ through

$$\langle\langle c^+_{i\uparrow} c_{j\downarrow}, c^+_{m\downarrow} c_{n\uparrow} \rangle\rangle^{\{\nu\}}_\omega = i\int \frac{dE\, dE'}{(2\pi)^2} L^{\{\nu\}}_{\substack{jnim \\ \downarrow\uparrow\uparrow\downarrow}} (E, E'; E-\omega, E'+\omega). \tag{2.10}$$

According to [20], $L^{\{\nu\}}$ satisfies a BS-type equation

$$L^{\{\nu\}}_{\substack{jnim\\ \downarrow\uparrow\uparrow\downarrow}}(E,E';-\omega) = -2\pi\delta(E-E'-\omega)G^{\{\nu\}}_{jm\downarrow}(E)G^{\{\nu\}}_{ni\uparrow}(E')$$

$$-\sum_{\ell} G^{\{\nu\}}_{j\ell\downarrow}(E)G^{\{\nu\}}_{\ell i\uparrow}(E-\omega)\int\frac{d\bar{E}}{2\pi}\, i\, I^{\{\nu\}}_{\substack{\ell\\ \downarrow\uparrow\uparrow\downarrow}}(E,\bar{E}-\omega;-\omega)L^{\{\nu\}}_{\substack{\ell n\ell m\\ \downarrow\uparrow\uparrow\downarrow}}(\bar{E},E';-\omega)\,, \qquad (2.11)$$

where the energy transfer $\omega$ is abbreviated by, e.g., $L^{\{\nu\}}(E,E';E-\omega,E'+\omega) = L^{\{\nu\}}(E,E';-\omega)$. Note that only the spin-diagonal one-particle (causal) Green functions $G^{\{\nu\}}$ are taken into account in (2.10) and (2.11); correspondingly, the mean value of the transverse spin current vanishes. The essential assumption in (2.11) consists in retaining only site-diagonal elements of the irreducible particle-hole vertex $I^{\{\nu\}}_{\substack{1\\ \downarrow\uparrow\uparrow\downarrow}}$ ; for instance a local ladder approximation fits to this scheme. The choice of the kernel for the spin-flip response in (2.11) involves only spin-transverse components of $I^{\{\nu\}}_1$ on account of the Pauli principle.

Analogously, we have

$$\chi^{+-\{\nu\}}(\vec{q},\omega) = -\frac{i}{N}\sum_{ij}\int\frac{dE\,dE'}{(2\pi)^2}L^{\{\nu\}}_{\substack{ijij\\ \downarrow\uparrow\uparrow\downarrow}}(E,E';E-\omega,E'+\omega)\,e^{-i\vec{q}(\vec{R}_i-\vec{R}_j)}. \qquad (2.12)$$

Since the ferromagnetic state is specified by $\frac{1}{N}\sum_i \langle S^z_i\rangle^{\{\nu\}} \neq 0$, (2.12) implies that $\langle S^{+(-)}_i\rangle^{\{\nu\}} = \langle c^+_{i\uparrow(\downarrow)}\, c_{i\downarrow(\uparrow)}\rangle^{\{\nu\}} = 0$.

## 3. EVALUATION OF THE TRANSVERSE SPIN CURRENT-SPIN CURRENT RESPONSE

Next we discuss within a local approximation the renormalization of the stiffness constant $D$ in the presence of disorder. Let us introduce the transverse spin current vertex $\vec{\Lambda}^{\{\nu\}}_{\downarrow\uparrow}$ by (compare

(2.8) and (2.9) )

$$\chi_J^{+-}(\vec{q}=0,\omega)=\frac{i}{3N}\int\frac{dE}{2\pi}\left\langle tr\left\{\vec{j}\,G_{\downarrow}^{(\nu)}(E+\omega)\vec{\Lambda}_{\downarrow\uparrow}^{(\nu)}(E+\omega,E)G_{\uparrow}^{(\nu)}(E)\right\}\right\rangle_C , \tag{3.1}$$

where the trace means the summation (without spin) over one-particle states; $\vec{j}$ and $\vec{\Lambda}_{\downarrow\uparrow}^{(\nu)}$ are understood to form a scalar product. On combining (2.9), (2.10), (2.11) and (3.1) one derives the integral equation for the effective spin-flip current as

$$\vec{\Lambda}_{\substack{ij\\ \downarrow\uparrow}}^{(\nu)}(E+\omega,E)=\vec{j}_{ij}-\delta_{ij}\,i\int\frac{d\bar{E}}{2\pi}I_{\substack{i\\ \downarrow\uparrow\uparrow\downarrow}}^{(\nu)}(E+\omega,\bar{E};-\omega)\sum_{mn}G_{im\downarrow}^{(\nu)}(\bar{E}+\omega)\vec{\Lambda}_{\substack{mn\\ \downarrow\uparrow}}^{(\nu)}(\bar{E}+\omega,\bar{E})\,G_{ni\uparrow}^{(\nu)}(\bar{E}). \tag{3.2}$$

Separating diagonal and off-diagonal elements of $\vec{\Lambda}_{\downarrow\uparrow}^{(\nu)}$ in (3.1) we get

$$\chi_J^{+-}(\vec{q}=0,\omega)=\frac{i}{3N}\int\frac{dE}{2\pi}\left\langle tr\left\{\vec{j}\,G_{\downarrow}^{(\nu)}(E+\omega)\vec{j}\,G_{\uparrow}^{(\nu)}(E)\right\}\right\rangle_C+\tilde{\chi}_J^{+-}(\vec{q}=0,\omega), \tag{3.3}$$

where

$$\tilde{\chi}_J^{+-}(\vec{q}=0,\omega)=\frac{i}{3N}\int\frac{dE}{2\pi}\left\langle\sum_i\vec{K}_{\substack{ii\\ \uparrow\downarrow}}^{(\nu)}(E,E+\omega)\cdot\vec{\Lambda}_{\substack{ii\\ \downarrow\uparrow}}^{(\nu)}(E+\omega,E)\right\rangle_C , \tag{3.4}$$

$$\vec{K}_{\substack{ii\\ \uparrow\downarrow}}^{(\nu)}(E,E+\omega)=\sum_{mn}G_{im\uparrow}^{(\nu)}(E)\,\vec{j}_{mn}\,G_{ni\downarrow}^{(\nu)}(E+\omega). \tag{3.5}$$

As it was argued in /20/, the problem of averaging configurationally in (3.4) is beyond the CPA; therefore, we proceed with the factorization $\langle\vec{K}^{(\nu)}\vec{\Lambda}^{(\nu)}\rangle_c=\langle\vec{K}^{(\nu)}\rangle_c\langle\vec{\Lambda}^{(\nu)}\rangle_c$. From (3.5) we obtain the CPA result

$$\begin{gathered}\vec{K}_{\uparrow\downarrow}(z_1,z_2)=\langle G_{\uparrow}^{(\nu)}(z_1)\vec{j}\,G_{\downarrow}^{(\nu)}(z_2)\rangle_{cii}=\frac{1}{N}\sum_{\vec{k}}\mathcal{G}_{\vec{k}\uparrow}(z_1)\mathcal{G}_{\vec{k}\downarrow}(z_2)\nabla_{\vec{k}}\varepsilon_{\vec{k}}=0,\\ \tilde{\chi}_J^{+-}(\vec{q}=0,\omega)=0,\end{gathered} \tag{3.6}$$

that tends to zero due to time-reversal symmetry [12]. The subscript "ii" means taking the site-diagonal element after averaging. $\mathcal{G}_{\vec{k}\sigma}(z)$ denotes the CPA averaged Green function renormalized by correlations (see below). For the ordered case a similar proof yields immediately $\tilde{\chi}_J^{+-}(\vec{q}=0,\omega)$. Hence, in the local approximation, $\chi_J^{+-}(\vec{q}=0,\omega)$ is equal to its irreducible part (cf., for a gas with short-range interactions [5]).

Going over from the causal Green functions in (3.3) to the advanced „a" and retarded „r" ones and substituting the CPA result for diagonal disorder $\langle \mathrm{tr}\,\{\vec{j}G_\downarrow^{(\nu)}(z_1)\,\vec{j}G_\uparrow^{(\nu)}(z_2)\}\rangle_c = \sum_{\vec{k}} \mathcal{G}_{\vec{k}\downarrow}(z_1)\,\mathcal{G}_{\vec{k}\uparrow}(z_2)\,(\nabla_{\vec{k}}\,\varepsilon_{\vec{k}})^2$ we can write

$$\chi_J^{+-}(\vec{q}=0,\omega)=\frac{i}{3N}\sum_{\vec{k}}(\nabla_{\vec{k}}\varepsilon_{\vec{k}})^2\int_{-\infty}^{\infty}\frac{dE}{2\pi}\Big[f(E+\omega)\mathcal{G}^{a}_{\vec{k}\downarrow}(E+\omega)\mathcal{G}^{a}_{\vec{k}\uparrow}(E)-f(E)\mathcal{G}^{r}_{\vec{k}\downarrow}(E+\omega)\mathcal{G}^{r}_{\vec{k}\uparrow}(E) \\ +\big(f(E)-f(E+\omega)\big)\mathcal{G}^{r}_{\vec{k}\downarrow}(E+\omega)\mathcal{G}^{a}_{\vec{k}\uparrow}(E)\Big]\quad,\quad \omega\geq 0, \tag{3.7}$$

where $f(E)=\Theta(\mu-E)$ is the Fermi function with $\mu$ being the chemical potential. The limit $\omega\to 0$ in (3.7) yields

$$\lim_{\omega\to 0}\lim_{q\to 0}\chi_J^{+-}(\vec{q},\omega)=\frac{2}{3N}\sum_{\vec{k}}(\nabla_{\vec{k}}\varepsilon_{\vec{k}})^2\int_{-\infty}^{\mu}\frac{dE}{2\pi}\,\mathrm{Im}\{\mathcal{G}^{r}_{\vec{k}\downarrow}(E)\,\mathcal{G}^{r}_{\vec{k}\uparrow}(E)\}. \tag{3.8}$$

From (2.7), it follows the averaged expression

$$\lim_{q\to 0}\frac{C_{\vec{q}}}{q^2}=-\frac{1}{6\pi N}\sum_{\vec{k}\sigma}(\nabla^2_{\vec{k}}\varepsilon_{\vec{k}})\int_{-\infty}^{\mu}dE\,\mathrm{Im}\,\mathcal{G}^{r}_{\vec{k}\sigma}(E), \tag{3.9}$$

which can be rewritten by $\sum_{\vec{k}}\mathcal{G}_{\vec{k}\sigma}(z)\,(\nabla^2_{\vec{k}}\,\varepsilon_{\vec{k}}) = -\sum_{\vec{k}}\mathcal{G}^2_{\vec{k}\sigma}(z)(\nabla_{\vec{k}}\,\varepsilon_{\vec{k}})^2$ provided that $\mathcal{G}_{\vec{k}\sigma}(z)$ depends on $\vec{k}$ only via $\varepsilon(\vec{k})$ (see section 4).

Finally, by inserting (3.9) and (3.8) into (2.5) we arrive at

$$D = \frac{1}{6\pi(n_\uparrow - n_\downarrow)} \operatorname{Im} \int_{-\infty}^{\mu} dE \, \frac{1}{N} \sum_{\vec{k}} \left( \mathcal{G}_{\vec{k}\uparrow}(E+i0) - \mathcal{G}_{\vec{k}\downarrow}(E+i0) \right)^2 (\nabla_{\vec{k}} \varepsilon_{\vec{k}})^2. \tag{3.10}$$

This result agrees formally with D obtained in the CPA-RPA scheme /13,14,17/ based on the HF approximation. However, in contrast to /13,14,17/, $\mathcal{G}_{\vec{k}\sigma}(z)$ is dressed self-consistently in the framework of the coherent LLA, as will be outlined in the following.

## 4. COHERENT LOCAL LADDER APPROXIMATION

Assume a local approximation for the multiple scatterings in the particle-particle channel given in terms of conditionally averaged causal functions by /21,22/

$$\Sigma^{\nu}_{Uii\sigma}(E) = \int \frac{dE'}{2\pi i} G^{\nu}_{ii-\sigma}(E')\, T^{\nu}_{i}(E+E'), \qquad (\nu = A, B) \tag{4.1}$$

$$T^{\nu}_{i}(E) = \frac{U^{\nu}_{i}}{1 + U^{\nu}_{i} \int \frac{dE'}{2\pi i} G^{\nu}_{ii\sigma}(E') G^{\nu}_{ii-\sigma}(E-E')}, \tag{4.2}$$

where $T^{\nu}_{i} \equiv T^{\nu}_{\substack{i \\ \sigma-\sigma\sigma-\sigma}}$ is the effective two-particle vertex. The local Green function $G^{\nu}_{ii\sigma}(z)$ written as resolvent is renormalized by

$$G^{\nu}_{ii\sigma}(z) = \frac{F_{\sigma}(z)}{1 - \left(\tilde{\varepsilon}^{\nu}_{i\sigma}(z) - \Sigma_{\sigma}(z)\right) F_{\sigma}(z)}, \tag{4.3}$$

$$\tilde{\varepsilon}^{\nu}_{i\sigma}(z) = \varepsilon^{\nu}_{i} + \Sigma^{\nu}_{Uii\sigma}(z), \tag{4.4}$$

$$F_\sigma(z) = \frac{1}{N}\sum_{\vec{k}} \mathcal{G}_{\vec{k}\sigma}(z) \tag{4.5}$$

$$\mathcal{G}_{\vec{k}\sigma}(z) = \left(z - \varepsilon_{\vec{k}} - \Sigma_\sigma(z)\right)^{-1}, \tag{4.6}$$

$$\Sigma_\sigma(z) = c\,\tilde{\varepsilon}^A_\sigma(z) + (1-c)\,\tilde{\varepsilon}^B_\sigma(z) - \left[\tilde{\varepsilon}^A_\sigma(z) - \Sigma_\sigma(z)\right] F_\sigma(z) \left[\tilde{\varepsilon}^B_\sigma(z) - \Sigma_\sigma(z)\right]. \tag{4.7}$$

Here $\Sigma_\sigma$ is the coherent potential satisfying the single-site CPA condition (4.7), $\mathcal{G}_{\vec{k}\sigma}$ is the totally averaged Green function entering into the stiffness formula (3.10). Contrary to the usual CPA [12], the atomic potential $\tilde{\varepsilon}^{\nu}_{i\sigma}(z)$ (i is dropped in (4.7) ) becomes energy-dependent through the self-energy $\Sigma^{\nu}_{Uii\sigma}(z)$ caused by correlations. The set of self-consistent equations is closed by

$$n = \sum_\sigma n_\sigma = -\frac{1}{\pi}\sum_\sigma \int_{-\infty}^{\mu} dE \, \mathrm{Im}\, F_\sigma(E + i0), \tag{4.8}$$

where n is the average number of electrons per site. Note that $n_\sigma$ in (3.10) is calculated from (4.8).

For a pure system (i.e., $\Sigma^{\nu}_{Uii\sigma} \xrightarrow[c\to 0]{} \Sigma_{U\sigma}$ ) we get

$$G_{ii\sigma}(z) \equiv F_\sigma(z) = \frac{1}{N}\sum_{\vec{k}} \left[z - \varepsilon_{\vec{k}} - \Sigma_{U\sigma}(z)\right]^{-1}, \tag{4.9}$$

and the correlation problem must be now solved from (4.1), (4.2), (4.8), and (4.9).

In the Hartree-Fock approximation only the CPA problem from (4.3) to (4.8) is retained which is completed by the constant self-energy $\Sigma^{\nu HF}_{Uii\sigma} = U^{\nu}_i n^{\nu}_{i-\sigma}$, where $n^{\nu}_{i\sigma}$ is the average electron number with spin $\sigma$ at $\nu$ sites given by

$$n_{i\sigma}^{\nu} = -\frac{1}{\pi}\int_{-\infty}^{\mu} dE\ \mathrm{Im}\, G_{ii\sigma}^{\nu}(E+i0). \tag{4.10}$$

## 5. EFFECTIVE VERTICES AND WARD IDENTITIES

In proof of the gauge invariance of transverse susceptibilities in the ferromagnetic phase we are looking for the Ward identities compatible with the continuity equation. Working within an arbitrary configuration $\{\nu\}$ we derive relations between effective spin-flip vertices. The special case of the ordered system is involved, too.

From (2.2) and (2.12) one can define (cf. (3.1)) the effective vertices $\Lambda_o^{\{\nu\}}$ of the spin-flip density by

$$\chi^{+-\{\nu\}}(\vec{q},\omega) = -\langle\langle S_{\vec{q}}^{+}, S_{-\vec{q}}^{-}\rangle\rangle_{\omega}^{\{\nu\}} = \frac{i}{N}\int\frac{dE}{2\pi}\mathrm{tr}\left\{\lambda_o(\vec{q})\,G_{\downarrow}^{\{\nu\}}(E+\omega)\Lambda_{o\downarrow\uparrow}^{\{\nu\}}(E+\omega,E;-\vec{q})\,G_{\uparrow}^{\{\nu\}}(E)\right\}$$

$$= \frac{i}{N}\int\frac{dE}{2\pi}\mathrm{tr}\left\{\Lambda_{o\uparrow\downarrow}^{\{\nu\}}(E,E+\omega;\vec{q})\,G_{\downarrow}^{\{\nu\}}(E+\omega)\lambda_o(-\vec{q})\,G_{\uparrow}^{\{\nu\}}(E)\right\}, \tag{5.1}$$

$$\lambda_{oij}(\vec{q}) = \lambda_{oi}(\vec{q})\delta_{ij} = e^{-i\vec{q}\vec{R}_i}\,\delta_{ij},$$

where in getting the second line we have used the symmetry relation

$$L_{\substack{jnim\\ \downarrow\uparrow\uparrow\downarrow}}^{\{\nu\}}(E,E';-\omega) = L_{\substack{njmi\\ \uparrow\downarrow\downarrow\uparrow}}^{\{\nu\}}(E',E;\omega). \tag{5.2}$$

By comparing the BS equation (2.11) with the analogue for $L_{\uparrow\downarrow\downarrow\uparrow}^{\{\nu\}}$ one concludes that (5.2) implies $I_{\downarrow\uparrow\uparrow\downarrow}^{\{\nu\}}(E,E';-\omega) = I_{\uparrow\downarrow\downarrow\uparrow}^{\{\nu\}}(E',E;\omega)$. Note that (2.12) with (5.2) leads, in terms of causal functions, to $\chi^{+-\{\nu\}}(\vec{q},\omega) = \chi^{-+\{\nu\}}(-\vec{q},-\omega)$.

Now we introduce on the basis of (2.4) and (5.2) the effective spin-flip current $\Lambda_1^{\{\nu\}}$ through

$$\chi_1^{+-\{\nu\}}(\vec{q},\omega)=-\langle\langle qJ^+_{\vec{q}},S^-_{-\vec{q}}\rangle\rangle^{\{\nu\}}_\omega=\frac{i}{N}\int\frac{dE}{2\pi}\mathrm{tr}\left\{\lambda_1(\vec{q})G_\downarrow^{\{\nu\}}(E+\omega)\Lambda_{0\downarrow\uparrow}^{\{\nu\}}(E+\omega,E;-\vec{q})G_\uparrow^{\{\nu\}}(E)\right\}$$

$$=\frac{i}{N}\int\frac{dE}{2\pi}\mathrm{tr}\left\{\Lambda_{1\uparrow\downarrow}^{\{\nu\}}(E,E+\omega;\vec{q})G_\downarrow^{\{\nu\}}(E+\omega)\lambda_0(-\vec{q})G_\uparrow^{\{\nu\}}(E)\right\}, \qquad (5.3)$$

$$\lambda_{1ij}(\vec{q})=t_{ij}(e^{-i\vec{q}\vec{R}_i}-e^{-i\vec{q}\vec{R}_j}).$$

Another version of (5.3) mediated by the time-reversal symmetry is

$$\chi_1^{+-\{\nu\}}(\vec{q},\omega)=-\langle\langle S^+_{\vec{q}},qJ^-_{-\vec{q}}\rangle\rangle^{\{\nu\}}_\omega=-\frac{i}{N}\int\frac{dE}{2\pi}\mathrm{tr}\left\{\lambda_0(\vec{q})G_\downarrow^{\{\nu\}}(E+\omega)\Lambda_{1\downarrow\uparrow}^{\{\nu\}}(E+\omega,E;-\vec{q})G_\uparrow^{\{\nu\}}(E)\right\}$$

$$=-\frac{i}{N}\int\frac{dE}{2\pi}\mathrm{tr}\left\{\Lambda_{0\uparrow\downarrow}^{\{\nu\}}(E,E+\omega;\vec{q})G_\downarrow^{\{\nu\}}(E+\omega)\lambda_1(-\vec{q})G_\uparrow^{\{\nu\}}(E)\right\}. \qquad (5.4)$$

In place of (2.8) one finds with (2.10) and (5.2) the expressions

$$q^2\chi_J^{+-\{\nu\}}(\vec{q},\omega)=-\langle\langle qJ^+_{\vec{q}},qJ^-_{-\vec{q}}\rangle\rangle^{\{\nu\}}_\omega=-\frac{i}{N}\int\frac{dE}{2\pi}\mathrm{tr}\left\{\lambda_1(\vec{q})G_\downarrow^{\{\nu\}}(E+\omega)\Lambda_{1\downarrow\uparrow}^{\{\nu\}}(E+\omega,E;-\vec{q})G_\uparrow^{\{\nu\}}(E)\right\}$$

$$=-\frac{i}{N}\int\frac{dE}{2\pi}\mathrm{tr}\left\{\Lambda_{1\uparrow\downarrow}^{\{\nu\}}(E,E+\omega;\vec{q})G_\downarrow^{\{\nu\}}(E+\omega)\lambda_1(-\vec{q})G_\uparrow^{\{\nu\}}(E)\right\}. \qquad (5.5)$$

The definitions of the effective vertices $\Lambda_{\alpha\downarrow\uparrow}^{\{\nu\}}$ and $\Lambda_{\alpha\uparrow\downarrow}^{\{\nu\}}$ ($\alpha = 0,1$), respectively, involved into (5.1), (5.3), (5.4) and (5.5) are quoted as

$$\sum_{mn}G_{jm\downarrow}^{\{\nu\}}(E)\Lambda_{\alpha mn\atop\downarrow\uparrow}^{\{\nu\}}(E,E-\omega;-\vec{q})G_{ni\uparrow}^{\{\nu\}}(E-\omega)=-\sum_{mn}\int\frac{dE'}{2\pi}L_{jnim\atop\downarrow\uparrow\uparrow\downarrow}^{\{\nu\}}(E,E';-\omega)\lambda_{\alpha mn}(-\vec{q}), \quad (\alpha=0,1) \qquad (5.6)$$

and

$$\sum_{ij}G_{ni\uparrow}^{\{\nu\}}(E')\Lambda_{\alpha ij\atop\uparrow\downarrow}^{\{\nu\}}(E',E'+\omega;\vec{q})G_{jm\downarrow}^{\{\nu\}}(E'+\omega)=-\sum_{ij}\int\frac{dE}{2\pi}L_{njmi\atop\uparrow\downarrow\downarrow\uparrow}^{\{\nu\}}(E',E;\omega)\lambda_{\alpha ij}(\vec{q}). \qquad (5.7)$$

By inserting (5.6) into (2.11) one verifies that

$$\Lambda^{\{\nu\}}_{\alpha ij \atop \downarrow\uparrow}(E+\omega,E;-\vec{q}) = \lambda_{\alpha ij}(-\vec{q}) - \delta_{ij}\int\frac{d\bar{E}}{2\pi}\, i\, I^{\{\nu\}}_{i \atop \downarrow\uparrow\uparrow\downarrow}(E+\omega,\bar{E};-\omega)\sum_{mn} G^{\{\nu\}}_{im\downarrow}(\bar{E}+\omega)\Lambda^{\{\nu\}}_{\alpha mn \atop \downarrow\uparrow}(\bar{E}+\omega,\bar{E};-\vec{q})\, G^{\{\nu\}}_{ni\uparrow}(\bar{E}). \tag{5.8}$$

At $\vec{q} = 0$ eq. (5.8) for $\alpha = 1$ and the first line of (5.5) after averaging are in agreement with (3.2) and (3.1), respectively. On the other hand, the BS equation for $L^{\{\nu\}}_{\uparrow\downarrow\downarrow\uparrow}$ (cf. (2.11)) and (5.7) give rise to

$$\Lambda^{\{\nu\}}_{\alpha ij \atop \uparrow\downarrow}(E,E+\omega;\vec{q}) = \lambda_{\alpha ij}(\vec{q}) - \delta_{ij}\int\frac{d\bar{E}}{2\pi}\, i\, I^{\{\nu\}}_{i \atop \uparrow\downarrow\downarrow\uparrow}(E,\bar{E}+\omega;\omega)\sum_{mn} G^{\{\nu\}}_{im\uparrow}(\bar{E})\Lambda^{\{\nu\}}_{\alpha mn \atop \uparrow\downarrow}(\bar{E},\bar{E}+\omega;\vec{q})\, G^{\{\nu\}}_{ni\downarrow}(\bar{E}+\omega). \tag{5.9}$$

Formally, (5.9) goes over into (5.8) by replacing $\omega \to -\omega$, $\vec{q} \longrightarrow -\vec{q}$, and $\uparrow\downarrow \longrightarrow \downarrow\uparrow$.

The lattice-space description in (5.8) and (5.9) was chosen since the translational invariance is broken within $\{\nu\}$, and only the energy, but not the momentum is conserved in the scattering process (e.g., reflected by $I_i^{\{\nu\}}(E+\omega,\bar{E};\, E,\, \bar{E}+\omega)$). Until this point, no specific assumptions have been made about the BS interaction kernel $I^{\{\nu\}}$ except for its zero range (locality). To get consistency with the LLA in the completely random version (cf. the partially averaged form (4.1), (4.2), and the ordered case[23])

$$\Sigma^{\{\nu\}}_{Uii\sigma}(E) = \int\frac{dE'}{2\pi i} G^{\{\nu\}}_{ii-\sigma}(E')\, T^{\{\nu\}}_i(E+E'), \tag{5.10}$$

$$T^{\{\nu\}}_i(E) = \frac{U^{\nu}_i}{1 + U^{\nu}_i\int\frac{dE'}{2\pi i} G^{\{\nu\}}_{ii\sigma}(E')\, G^{\{\nu\}}_{ii-\sigma}(E-E')}, \tag{5.11}$$

we have to put for the irreducible particle-hole vertex

$$I_{\substack{i\\ \downarrow\uparrow\uparrow\downarrow}}^{\{\nu\}}(E+\omega,\bar{E}_j-\omega)=-T_{\substack{i\\ \uparrow\downarrow\uparrow\downarrow}}^{\{\nu\}}(E+\bar{E}+\omega)\equiv -T_i^{\{\nu\}}(E+\bar{E}+\omega). \tag{5.12}$$

In (5.12) the contribution of $O(T_i^{\{\nu\}2})$ is neglected (cf. the scheme given in /21/). By substituting the approximated $I^{\{\nu\}}$ from (5.12) into (5.8), and using (5.10) and the Dyson equation (see (5.15)), we find

$$\begin{aligned}\sum_{mn} G_{im\downarrow}^{\{\nu\}}(E+\omega)\Big[\omega\Lambda_{\substack{om\\ \downarrow\uparrow}}^{\{\nu\}}(E+\omega,E;-\vec{q})\delta_{mn}+\Lambda_{\substack{1mn\\ \downarrow\uparrow}}^{\{\nu\}}(E+\omega,E;-\vec{q})\Big]G_{nj\uparrow}^{\{\nu\}}(E)&\\ =e^{i\vec{q}\vec{R}_i}G_{ij\uparrow}^{\{\nu\}}(E)-G_{ij\downarrow}^{\{\nu\}}(E+\omega)e^{i\vec{q}\vec{R}_j}&,\end{aligned} \tag{5.13a}$$

or

$$\omega\Lambda_{\substack{oi\\ \downarrow\uparrow}}^{\{\nu\}}(E+\omega,E;-\vec{q})\delta_{ij}+\Lambda_{\substack{1ij\\ \downarrow\uparrow}}^{\{\nu\}}(E+\omega,E;-\vec{q})=G_{ij\downarrow}^{\{\nu\}-1}(E+\omega)e^{i\vec{q}\vec{R}_j}-e^{i\vec{q}\vec{R}_i}G_{ij\uparrow}^{\{\nu\}-1}(E), \tag{5.13b}$$

where the site-diagonality of $\Lambda_{\substack{omn\\ \downarrow\uparrow}}^{\{\nu\}}(E+\omega,E;-\vec{q})\equiv\Lambda_{\substack{om\\ \downarrow\uparrow}}^{\{\nu\}}(E+\omega,E;-\vec{q})\,\delta_{mn}$ has been taken into account.

An analogous procedure can be performed on the basis of (5.9) yielding

$$\begin{aligned}\sum_{mn} G_{im\uparrow}^{\{\nu\}}(E)\Big[\omega\Lambda_{\substack{om\\ \uparrow\downarrow}}^{\{\nu\}}(E,E+\omega;\vec{q})\delta_{mn}-\Lambda_{\substack{1mn\\ \uparrow\downarrow}}^{\{\nu\}}(E,E+\omega;\vec{q})\Big]G_{nj\downarrow}^{\{\nu\}}(E+\omega)&\\ =G_{ij\uparrow}^{\{\nu\}}(E)e^{-i\vec{q}\vec{R}_j}-e^{-i\vec{q}\vec{R}_i}G_{ij\downarrow}^{\{\nu\}}(E+\omega)&,\end{aligned} \tag{5.14a}$$

or

$$\omega\Lambda_{\substack{oi\\ \uparrow\downarrow}}^{\{\nu\}}(E,E+\omega;\vec{q})\delta_{ij}-\Lambda_{\substack{1ij\\ \uparrow\downarrow}}^{\{\nu\}}(E,E+\omega;\vec{q})=e^{-i\vec{q}\vec{R}_i}G_{ij\downarrow}^{\{\nu\}-1}(E+\omega)-G_{ij\uparrow}^{\{\nu\}-1}(E)e^{-i\vec{q}\vec{R}_j}. \tag{5.14b}$$

The Dyson equation used in (5.13) and (5.14) reads

$$(G^{\{\nu\}-1}(E))_{ij\sigma} = (E-\varepsilon_i^{\nu})\delta_{ij} - t_{ij} - \Sigma^{\{\nu\}}_{Uii\sigma}(E)\delta_{ij} \,, \tag{5.15}$$

where the site-diagonal self-energy $\Sigma^{\{\nu\}}_{Uii\sigma}(E)$ obeys (5.10).

From (5.13b) and (5.15), we have

$$\begin{gathered}\omega\Lambda^{\{\nu\}}_{\substack{oi\\ \downarrow\uparrow}}(E+\omega,E;-\vec{q}) + \Lambda^{\{\nu\}}_{\substack{1ii\\ \downarrow\uparrow}}(E+\omega,E;-\vec{q}) = \left[\omega + \Sigma^{\{\nu\}}_{Uii\uparrow}(E) - \Sigma^{\{\nu\}}_{Uii\downarrow}(E+\omega)\right] e^{i\vec{q}\vec{R}_i} \,, \\ \Lambda^{\{\nu\}}_{\substack{1ij\\ \downarrow\uparrow}}(E+\omega,E;-\vec{q}) = \lambda_{1ij}(-\vec{q}) \,, \quad (i \neq j) \,,\end{gathered} \tag{5.16}$$

where $\lambda_1$ is defined in (5.3).

Likewise, (5.14b) with (5.15) can be rewritten as

$$\begin{gathered}\omega\Lambda^{\{\nu\}}_{\substack{oi\\ \uparrow\downarrow}}(E,E+\omega;\vec{q}) - \Lambda^{\{\nu\}}_{\substack{1ii\\ \uparrow\downarrow}}(E,E+\omega;\vec{q}) = \left[\omega + \Sigma^{\{\nu\}}_{Uii\uparrow}(E) - \Sigma^{\{\nu\}}_{Uii\downarrow}(E+\omega)\right] e^{-i\vec{q}\vec{R}_i} \,, \\ \Lambda^{\{\nu\}}_{\substack{1ij\\ \uparrow\downarrow}}(E+\omega,E;\vec{q}) = \lambda_{1ij}(\vec{q}) \,, \qquad (i \neq j) .\end{gathered} \tag{5.17}$$

The equations (5.13), (5.14), (5.16), and (5.17) are random modifications of the generalized Ward (or Ward-Takahashi) identities (cf., e.g., /25/). Note that the scalar product $\vec{q}\cdot\vec{\Lambda}_1(\vec{q})$ could also be used instead of the present $\Lambda_1(\vec{q})$. Although these Ward relations have been derived within a local scheme, (5.13) and (5.14) remain valid even generally (compare /8/), whereas (5.16) and (5.17) are explicitly restricted to the local approximation. Here it has been proved that the random LLA satisfies the Ward relations; especially, (5.16) and (5.17) impose constraints on the partially averaged vertices, too.

*As a consequence of (5.13a), (5.1), and (5.4) (or (5.14a), (5.1),* and (5.3)) the first moment equation becomes

$$\omega\chi^{+-\{\nu\}}(\vec{q},\omega)-\chi_1^{+-\{\nu\}}(\vec{q},\omega)=\frac{1}{N}\sum_i(n_{i\downarrow}^{\{\nu\}}-n_{i\uparrow}^{\{\nu\}}), \tag{5.18}$$

where the average number of spin $\sigma$ electrons at site i is given by $n_{i\sigma}^{\{\nu\}}=\int\frac{dE}{2\pi i}G_{ii\sigma}^{\{\nu\}}(E)$ within $\{\nu\}$. When the r.h.s. of (5.18) does not vanish, $\chi^{+-\{\nu\}}$ and $\chi_1^{+-\{\nu\}}$ (notice the usual form $\vec{q}\cdot\vec{\chi}_1$) must have singular parts in the limit $\omega\to 0$, $\vec{q}\to 0$, refering to Goldstone-type modes.

The Ward relation (5.13a) with (5.3) and (5.5) (or (5.14a) with (5.4) and (5.5)) leads to the second moment equation

$$\begin{aligned}&\omega\chi_1^{+-\{\nu\}}(\vec{q},\omega)-q^2\chi_J^{+-\{\nu\}}(\vec{q},\omega)\\&=\frac{1}{N}\int\frac{dE}{2\pi}\sum_{ij}t_{ij}\left\{(e^{-i\vec{q}(\vec{R}_i-\vec{R}_j)}-1)G_{ji\uparrow}^{\{\nu\}}(E)+(e^{i\vec{q}(\vec{R}_i-\vec{R}_j)}-1)G_{ji\downarrow}^{\{\nu\}}(E+\omega)\right\}=-C_{\vec{q}}^{\{\nu\}},\end{aligned} \tag{5.19}$$

where the abbreviation $C_{\vec{q}}^{\{\nu\}}=\langle[S_{\vec{q}}^{+},qJ_{-\vec{q}}^{-}]\rangle^{\{\nu\}}$ follows from (2.6). Then the combination of (5.18) and (5.19) gives (2.3).

## 6. STABILITY CONDITION. DAMPING OF THE SPIN WAVES.

The ferromagnetic ground state can be unstable against both the collective excitations (spin waves) and the individual (Stoner) excitations. In the long-wavelength region, especially, we must bring out the connection between the stability of the ground state and spin waves.

At zero temperature, the spectral representation of the transverse susceptibility $\chi^{+-\{\nu\}}(\vec{q},\omega)$ is given by

$$\chi^{+-\{\nu\}}(\vec{q},\omega) = -\langle\langle S^+_{\vec{q}}, S^-_{-\vec{q}}\rangle\rangle^{\{\nu\}}_{\omega} = -\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega' \frac{\mathrm{sign}\,\omega'}{\omega-\omega'+i\varepsilon\,\mathrm{sign}\,\omega'}\, I^{\{\nu\}}_{S^+_{\vec{q}}S^-_{-\vec{q}}}(\omega'), \tag{6.1}$$

where the spectral density $I^{\{\nu\}}_{S^+_{\vec{q}}S^-_{-\vec{q}}}(\omega)$ within $\{\nu\}$ takes the form

$$I^{\{\nu\}}_{S^+_{\vec{q}}S^-_{-\vec{q}}}(\omega) = \begin{cases} 2\pi\sum_m |\langle m|S^+_{\vec{q}}|0\rangle^{\{\nu\}}|^2\,\delta(\omega^{\{\nu\}}_{mo}+\omega), & \omega<0 \\ \\ 2\pi\sum_m |\langle 0|S^+_{\vec{q}}|m\rangle^{\{\nu\}}|^2\,\delta(\omega^{\{\nu\}}_{mo}-\omega), & \omega>0. \end{cases} \tag{6.2}$$

Here $\omega^{\{\nu\}}_{mo}$ is the excitation energy of the m-th eigenstate of $H^{\{\nu\}}$, and $\langle 0|S^+_{\vec{q}}|m\rangle^{\{\nu\}}$ is the transition element of $S^+_{\vec{q}}$ between the ground (0) and m-th state. Insertion of (6.2) into (6.1) yields

$$\chi^{+-\{\nu\}}(\vec{q},\omega) = -\sum_m\left\{\frac{|\langle 0|S^+_{\vec{q}}|m\rangle^{\{\nu\}}|^2}{\omega-\omega^{\{\nu\}}_{mo}+i\varepsilon} - \frac{|\langle m|S^+_{\vec{q}}|0\rangle^{\{\nu\}}|^2}{\omega+\omega^{\{\nu\}}_{mo}-i\varepsilon}\right\}. \tag{6.3}$$

$\chi^{+-\{\nu\}}(\vec{q},\omega)$ in (6.3) involves both „quasiparticle" (quasiboson) and „nonquasiparticle" contributions, i.e., pole singularities from the states with $\lim_{q\to 0}\omega^{\{\nu\}}_{mo} = 0$ and cut singularities from the states with $\omega^{\{\nu\}}_{mo} > 0$ in the limit $q \to 0$, respectively. In other words, the magnon pole which we are interested in must be now separated from the Stoner continuum. Since $S^+_{\vec{q}=0}$ is a conserved quantity (cf. (2.4)), the Stoner excitations have vanishing spectral weight for $\vec{q}\to 0$ /8/. Thus, we can pick up from (6.3) the spin wave for small q and $\omega$ as

$$\chi^{+-\{\nu\}}_{\text{pole}}(\vec{q},\omega) = -\frac{\frac{1}{N}\sum_i 2\langle S_i^z\rangle^{\{\nu\}}}{\omega - D^{\{\nu\}}q^2 + i\varepsilon\,\text{sign}\,D^{\{\nu\}}}\quad, \tag{6.4}$$

where the damping was neglected. The residue in (6.4) is written down only in the lowest order of q.

The stability condition of the ground state can be expressed by $\omega^{\{\nu\}}_{\text{mo}} > 0$. Consequently, the spectral representation (6.1) with (6.2) yields $\text{Im}\,\chi^{+-\{\nu\}}(\vec{q},\omega) = \frac{1}{2} I^{\{\nu\}}_{S^+_{\vec{q}} S^-_{-\vec{q}}}(\omega) > 0$. To achieve the stability criterion we notice from (6.4) that $\text{Im}\,\chi^{+-\{\nu\}}_{\text{pole}} = \frac{2\pi}{N}\sum_i \langle S_i^z\rangle^{\{\nu\}}\,\text{sign}\,D^{\{\nu\}}\,\delta(\omega - D^{\{\nu\}}q^2)$. Then by comparing $\chi^{+-\{\nu\}}$ and $\chi^{+-\{\nu\}}_{\text{pole}}$, one concludes that the condition

$$\hat{D}^{\{\nu\}} = \lim_{q\to 0}\frac{1}{q^2}\left\langle [S^+_{\vec{q}}, qJ^-_{-\vec{q}}]\right\rangle^{\{\nu\}} - \lim_{\omega\to 0}\lim_{q\to 0}\chi_J^{+-\{\nu\}}(\vec{q},\omega) > 0\,,$$

with (6.5)

$$D^{\{\nu\}} = \frac{\hat{D}^{\{\nu\}}}{\frac{1}{N}\sum_i 2\langle S_i^z\rangle^{\{\nu\}}}\quad,$$

ensures the stability of the ferromagnetic ground state, while $\hat{D}^{\{\nu\}} < 0$ signifies instabilities induced by the spin waves. Note that the explicit form of $\hat{D}^{\{\nu\}}$ in (6.5) was derived in (2.5). By inserting into (6.1) the spectral weight function $2\,\text{Im}\,\chi^{+-\{\nu\}}_{\text{pole}}(\vec{q},\omega) = \frac{4\pi}{N}\left|\sum_i \langle S_i^z\rangle^{\{\nu\}}\right|\delta(\omega - D^{\{\nu\}}q^2)$ based on the condition (6.5) one recovers immediately (6.4). Another formulation of (6.4) with sign $D^{\{\nu\}}$ replaced by sign $\omega$ can also be chosen to get (6.5).

The configuration average of the retarded susceptibility can be written in spectral representation as

$$\chi^{+-r}(\vec{q},\omega) = -\left\langle\langle\langle S^+_{\vec{q}}, S^-_{-\vec{q}}\rangle\rangle^{r\{\nu\}}_{\omega}\right\rangle_c = -\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega'\,\frac{\text{sign}\,\omega'}{\omega - \omega' + i\varepsilon}\, I_{S^+_{\vec{q}} S^-_{-\vec{q}}}(\omega')\,. \tag{6.6}$$

Especially, the spin-wave pole

$$\chi^{+-r}_{pole}(\vec{q},\omega) = -\frac{2\langle\langle S_i^z\rangle^{\{\nu\}}\rangle_c}{\omega - Dq^2 + i\varepsilon} \qquad (6.7)$$

involves the average magnetisation per site $2\langle\langle S_i^z\rangle^{\{\nu\}}\rangle_c$ and the stiffness constant D introduced into (2.5). Notice that (6.7) satisfies the sum rule $\int \frac{d\omega}{\pi} \operatorname{Im} \chi^{+-r}_{pole}(\vec{q},\omega) = 2\langle\langle S_i^z\rangle^{\{\nu\}}\rangle_c$. We are then led to a comparison between $\operatorname{Im} \chi^{+-r}(\vec{q},\omega) = \frac{1}{2} \operatorname{sign}\omega\, I_{S^+_{\vec{q}} S^-_{-\vec{q}}}(\omega)$ and $\operatorname{Im} \chi^{+-r}_{pole}(\vec{q},\omega) = 2\pi \langle\langle S_i^z\rangle^{\{\nu\}}\rangle_c\, \delta(\omega - Dq^2)$. It holds $I_{S^+_{\vec{q}} S^-_{-\vec{q}}}(\omega) \geq 0$ as a postulate for an averaged effective medium, too. Assuming $\langle\langle S_i^z\rangle^{\{\nu\}}\rangle_c > 0$ we obtain for $\omega > 0$ the inequality $D > 0$ (for $\omega < 0$ we are left with the trivial result $I_{S^+_{\vec{q}} S^-_{-\vec{q}}}(\omega) = 0$); putting $\langle\langle S_i^z\rangle^{\{\nu\}}\rangle_c < 0$ we find $D < 0$ for $\omega < 0$ (for $\omega > 0$ it results $I_{S^+_{\vec{q}} S^-_{-\vec{q}}}(\omega) = 0$). Thus the stability condition is expressed by

$$\hat{D} = \lim_{q\to 0} \frac{C\vec{q}}{q^2} - \lim_{\omega\to 0}\lim_{q\to 0} \chi_J^{+-}(\vec{q},\omega) > 0, \qquad D = \frac{\hat{D}}{2\langle\langle S_i^z\rangle^{\{\nu\}}\rangle_c}, \qquad (6.8a)$$

or, in the approximated form (3.10), as

$$\hat{D} = \frac{1}{6\pi} \operatorname{Im} \int_{-\infty}^{\mu} dE\, \frac{1}{N} \sum_{\vec{k}} \left(\mathcal{G}_{\vec{k}\uparrow}(E+i0) - \mathcal{G}_{\vec{k}\downarrow}(E+i0)\right)^2 (\nabla_{\vec{k}}\, \varepsilon_{\vec{k}})^2 > 0. \qquad (6.8b)$$

Eq. (6.8a) can also be obtained directly from (6.5) by averaging configurationally.

In generalising the undamped case (6.7) we introduce the damping $\gamma_q$ of the collective mode by

$$\chi^{+-r}_{pole}(\vec{q},\omega) = -\frac{2\langle\langle S_i^z\rangle^{\{\nu\}}\rangle_c}{\omega - \omega_q + i\gamma_q}, \qquad (6.9)$$

where $\omega_q = Dq^2$ denotes the spin wave energy. To determine $\gamma_q$ we separate the real and imaginary parts in (2.3), and hence

$$\gamma_q = -\frac{1}{2\langle\langle S_i^z\rangle^{(\nu)}\rangle_c} \mathrm{Im}\left\langle\langle\langle qJ^+_{\vec q}, qJ^-_{-\vec q}\rangle\rangle_\omega^{r\{\nu\}}\right\rangle_c\Big|_{\omega=\omega_q} = \frac{q^2}{2\langle\langle S_i^z\rangle^{(\nu)}\rangle_c}\, \mathrm{sign}\,\omega\, \mathrm{Im}\,\chi_J^{+-}(\vec q,\omega)\Big|_{\omega=\omega_q)} \tag{6.10}$$

where it can be proved that $C_{\vec q}$ defined in (2.6) is a real quantity. Here small damping ($\gamma_q \ll \omega_q$) is considered; $\gamma_q^{-1}$ describes the lifetime of the spin waves. In the second part of (6.10) we have used the relation $\mathrm{Im}\left\langle\langle\langle qJ^+_{\vec q}, qJ^-_{-\vec q}\rangle\rangle_\omega^{r\{\nu\}}\right\rangle_c = \mathrm{sign}\,\omega\ \mathrm{Im}\left\langle\langle\langle qJ^+_{\vec q}, qJ^-_{-\vec q}\rangle\rangle_\omega^{\{\nu\}}\right\rangle_c$ ($\omega$ being real) and the definition (2.8).

A similar analysis as proposed in handling (3.1) can be carried out for the imaginary part of $\chi_J^{+-}(\vec q,\omega)$. This means that vertex corrections due to electron correlations will not appear (compare the arguments leading from (3.3) to (3.6)), so that

$$q^2\chi_J^{+-}(\vec q,\omega) = -\frac{i}{N}\int\frac{dE}{2\pi}\left\langle \mathrm{tr}\left\{\lambda_1(\vec q)G_\downarrow^{\{\nu\}}(E+\omega)\lambda_1(-\vec q)G_\uparrow^{\{\nu\}}(E)\right\}\right\rangle_c , \tag{6.11}$$

which retains only the „irreducible" part of $\chi_J^{+-}(\vec q,\omega)$ from (5.5) with (5.9). In the lowest order of q, eq. (6.11) can be reduced via (3.3) without CPA vertex corrections to

$$\chi_J^{+-}(\vec q=0,\omega) = \frac{i}{3N}\sum_{\vec k}(\nabla_{\vec k}\varepsilon_{\vec k})^2\int\frac{dE}{2\pi}\mathcal{G}_{\vec k\downarrow}(E+\omega)\mathcal{G}_{\vec k\uparrow}(E) , \tag{6.12}$$

where $\mathcal{G}_{\vec k\sigma}$ represents the causal coherent Green function. From (3.7) (and its analogue for $\omega < 0$) it results

$$\mathrm{Im}\,\chi_J^{+-}(\vec q=0,\omega) = \frac{1}{3\pi N}\sum_{\vec k}(\nabla_{\vec k}\varepsilon_{\vec k})^2\,\mathrm{sign}\,\omega\int_{\mu-\omega}^{\mu}dE\ \mathrm{Im}\,\mathcal{G}^r_{\vec k\downarrow}(E+\omega)\,\mathrm{Im}\,\mathcal{G}^r_{\vec k\uparrow}(E). \tag{6.13}$$

By expanding (6.13) to first order of $\omega$ we find

$$\mathrm{Im}\,\chi_{J}^{+-}(\vec{q}=0,\omega)=\frac{1}{3\pi N}\sum_{\vec{k}}(\nabla_{\vec{k}}\varepsilon_{\vec{k}})^{2}\omega\,\mathrm{sign}\,\omega\,\mathrm{Im}\,\mathcal{G}^{r}_{\vec{k}\downarrow}(\mu)\,\mathrm{Im}\,\mathcal{G}^{r}_{\vec{k}\uparrow}(\mu). \qquad (6.14)$$

The combination of (6.10) and (6.13) provides the lowest order description of the damping

$$\gamma_{q}=\frac{1}{3\pi(n_{\uparrow}-n_{\downarrow})}Dq^{4}\frac{1}{N}\sum_{\vec{k}}\mathrm{Im}\,\mathcal{G}^{r}_{\vec{k}\downarrow}(\mu)\,\mathrm{Im}\,\mathcal{G}^{r}_{\vec{k}\uparrow}(\mu)(\nabla_{\vec{k}}\varepsilon_{\vec{k}})^{2}. \qquad (6.15)$$

The same expression was found in the CPA-RPA by Fukuyama /15/. The damping is caused by impurity scattering ($\mathrm{Im}\,\mathcal{G}^{r}_{\vec{k}\sigma}(\mu)\neq 0$) including here electron-electron scatterings as a generalization of the treatment /15/. It holds $\gamma_q \geq 0$ in the stable case (6.8); especially $\gamma_q$ vanishes for saturated ferromagnets in $O(q^4)$.

## 7. CONCLUSION

The present derivation of a renormalized spin wave spectrum of disordered alloys assumes the locality of the effective four-leg vertex originated by the random intra-atomic interaction. The physical content is confirmed by a stability criterion, small damping, and the fulfillment of Ward identities. According to the horizontal ladder approximation the result for the stiffness constant can be justified for strong correlations and small electron (or hole) densities; it may therefore be applied, e.g., to Ni and some Ni-based alloys. The present scheme is appropriate to numerical calculations, as will be shown in a subsequent paper.

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