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THE SUPEROPERATOR METHOD IN THE THEORY OF OSCILLATOR WEAKLY INTERACTING WITH MEDIUM

II. Definite Interaction Mechanisms

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Метод супероператоров в теории осциллятора, слабо взаимодействующего со средой. II. Конкретные механизмы взаимодействия

На основе общего подхода, развитого автором ранее, исследуются корреляционные функции, описывающие динамику осциллятора, слабо связанного со средой-термостатом. В качестве среды-термостата рассматривается система большого числа как угодно взаимодействующих между собой осцилляторов. Получены уравнения для корреляционных функций типа "плотность" и "плотность-плотность". Все расчеты проведены в квадратичном по взаимодействию осциллятора с флуктуациями среды приближении.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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The Superoperator Method in the Theory of Oscillator Weakly Interacting with Medium. II. Definite Interaction Mechanisms

On the basis of a general approach developed by the author in an early work the correlation functions, describing the dynamics of an oscillator weakly coupled to a thermostat are investigated. As a thermostat is considered a system of a large number of arbitrarily interacting oscillators. Equations are obtained for "density" and "density-density" type correlation functions. A square on the interaction of oscillator with fluctuations approximation was used in all calculations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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I. Introduction.

In the preceding work /1/ (references to which will be marked as /1/) the general formulas for correlation functions were obtained by the superoperator method, describing dynamics of the weakly bound with arbitrary medium oscillator. These results are true for any power of the displacement operator $\vec{\mathcal{U}}_0$ of the singled out particle in the Hamiltonian of the system. Oscillator non-linearity is supposed to be little in comparison with its frequency.

In the present work the cases of linear (on $\vec{\mathcal{U}}_0$) and quadratic interaction of oscillator with medium fluctuations are considered as an application of the general theory. Equation systems for functions were obtained, determining the spectral distribution of singled out oscillators. These equations permit one to explore oscillator $\varkappa$ spectral distribution described by the correlation function $\langle b_{\varkappa}(t) b_{\varkappa}^{+}(o)\rangle_{\omega}$ in a general case of arbitrary relation between oscillator non-linearity (which determines the distance betweem fine structure lines of non-linear oscillator) and broadening of lines. Depending on relation between these magnitudes, the spectral distribution either splits up into a set of fine structure lines or merges into single broadening distribution. Equation systems have also been obtained for the correlation function of occupation numbers $\langle \hat{n}_\varkappa(t)\,\tilde{\hat{n}}_\varkappa(0)\rangle_\omega$ which is smooth function without fine structure.

Oscillator correlation functions in time representation for the considered types of interaction were investigated by means of kinetic equations, obtained from general kinetic equations of paper /I/.

The correlation functions for the operator of medium potential energy are parameters, describing in equations for oscillator correlation functions interaction with medium fluctuations.

All used notations are given in work /I/ . References to formulas of work /I/ will be marked by dual number of (I.15) type.

2. Linear interaction of oscillator with medium.

To pass in general formulas of work /I/ to the case of linear interaction, it is necessary to expand $e^{\vec{u}_o\vec{\nabla}_o}$ in the interaction Hamiltonian H_{int} (I.2) or (I.3) into a series up to a linear term, i.e.,to perform the substitution

$$e^{\vec{u}_o\vec{\nabla}_o}-1 \longrightarrow \vec{u}_o\vec{\nabla}_o=\sum_{\varkappa}(\vec{\alpha}_\varkappa b_\varkappa+\vec{\alpha}^*_\varkappa b^+_\varkappa)\vec{\nabla}_o . \tag{1}$$

Then taking into consideration (I.40), (1),the matrix element of "mass" superoperator $\hat{M}(\omega)$ which determines averaged

Green superoperator $\bar{\hat{G}}(\omega)$ in the second order on H_{int} of perturbation theory can be written as:

$$\left[\hat{M}(\omega+i\varepsilon)\right]^{n'm'}_{nm}=\int\limits_{-\infty}^{\infty}d\omega'\sum_{\varkappa_1}\sum_{j=\pm1}\Big\{\Big[\frac{n_{\varkappa_1}+\frac{1}{2}(1+j)}{\omega-\omega_{n+j\delta_{\varkappa_1\varkappa'}}+\omega_m-\omega'+i\varepsilon}+\frac{m_{\varkappa_1}+\frac{1}{2}(1+j)}{\omega-\omega_n+\omega_{m+j\delta_{\varkappa_1\varkappa'}}+\omega'+i\varepsilon}\Big]\delta_{nn'}\delta_{mm'}-$$
$$-\sqrt{\left[n_{\varkappa_1}+\tfrac{1}{2}(1+j)\right]\left[m_{\varkappa_1}+\tfrac{1}{2}(1+j)\right]}\cdot \tag{2}$$
$$\cdot\Big(\frac{1}{\omega-\omega_{n+j\delta_{\varkappa_1\varkappa'}}+\omega_m-\omega'+i\varepsilon}+\frac{1}{\omega-\omega_n+\omega_{m+j\delta_{\varkappa_1\varkappa'}}+\omega'+i\varepsilon}\Big)\cdot$$
$$\cdot\,\delta_{n',n+j\delta_{\varkappa_1\varkappa'}}\,\delta_{m',m+j\delta_{\varkappa_1\varkappa'}}\Big\}\,|\vec{\alpha}_{\varkappa_1}\vec{\nabla}_o|^2\,\varphi^o_\omega(\omega')$$

To obtain formula (2),the conditions of applicability of equation (I.40), in particular, the condition of noncoincidence of different singled out oscillation frequences $(\omega_\varkappa\neq\omega_{\varkappa'})$ and condition of non-coincidence of different frequency combinations (for example, $\omega_\varkappa\neq\omega_{\varkappa_1}\pm\omega_{\varkappa_2}$ and so on) were taken into consideration. These restrictions lead to reducing the dual sums $\sum\limits_{\varkappa_1\varkappa_2}$, appearing with substituting (1) into (I.40), to ordinary sum over $\varkappa_1$ in (2).

Let us write down equation (I.35) for the correlation function $\langle b_\varkappa(t)b^+_\varkappa(0)\rangle_\omega$ in the actual frequency range $\omega\approx\omega_\varkappa$ as

$$\langle b_\varkappa(t)b^+_\varkappa(0)\rangle_\omega=-\frac{1}{\pi}\,Im\sum_m g_\varkappa(m,\omega+i\varepsilon);$$
$$g_\varkappa(m,\omega+i\varepsilon)=\sqrt{1+m_\varkappa}\,(\rho_o)^m_m\sum_{m'}\sqrt{1+m'_\varkappa}\;\cdot \tag{3}$$
$$\cdot\left[\bar{\hat{G}}(\omega+i\varepsilon)\right]^{m'+\delta_{\varkappa\varkappa'},m'}_{m+\delta_{\varkappa\varkappa'},m}$$

From equation (2) it follows, that the matrix $\hat{M}_{nm}^{n'm'}$ for the linear interaction has the following non-zero matrix elements: diagonal with $n=n'$, $m=m'$ and nondiagonal of only two types with $n'=n+\delta_{\varkappa_1\varkappa'}$, $m'=m+\delta_{\varkappa_1\varkappa'}$ and with $n'=n-\delta_{\varkappa_1\varkappa'}$, $m'=m-\delta_{\varkappa_1\varkappa'}$ for every oscillation $\varkappa_1$. This circumstance (and diagonality of $\hat{L}_0$) permits one, as is easy to see, to write the system of equations (I.42) as a system of equations for the function $g_\varkappa(m,\omega+i\varepsilon)$

$$[\hat{G}^{-1}(\omega+i\varepsilon)]^{m+\delta_{\varkappa\varkappa'},m}_{m+\delta_{\varkappa\varkappa'},m}\, g_\varkappa(m,\omega+i\varepsilon)+$$
$$+\sum_{j=\pm 1}\sum_{\varkappa_1}[\hat{G}^{-1}(\omega+i\varepsilon)]^{m+\delta_{\varkappa\varkappa'}+j\delta_{\varkappa_1\varkappa'},\,m+j\delta_{\varkappa_1\varkappa'}}_{m+\delta_{\varkappa\varkappa'},m}\sqrt{\frac{1+m_\varkappa}{1+m_\varkappa+j\delta_{\varkappa\varkappa_1}}}\,\cdot$$
$$\cdot e^{\lambda j\omega_{\varkappa_1}}\, g_\varkappa(m+j\delta_{\varkappa_1\varkappa'},\omega+i\varepsilon)=(1+m_\varkappa)(\rho_0)^m_m. \qquad (4)$$

Here, the function $g_\varkappa(m+j\delta_{\varkappa_1\varkappa'},\omega+i\varepsilon)$ is determined by equation (3) with the substitution $m_\varkappa\to m_\varkappa+j\delta_{\varkappa_1\varkappa}$, $m\to m+j\delta_{\varkappa_1\varkappa'}$.

The real and imaginary parts of the superoperator $\hat{G}^{-1}(\omega+i\varepsilon)=\omega-P\hat{L}P-\hat{M}(\omega+i\varepsilon)$ matrix elements, which are coefficients of equation (4), are determined according to (I.14), (I.36), (2) as

$$\hat{G}^{-1}(\omega+i\varepsilon)=\mathrm{Re}\,\hat{G}^{-1}(\omega+i\varepsilon)+i\,\mathrm{Im}\,\hat{G}^{-1}(\omega+i\varepsilon);$$
$$\mathrm{Re}\,[\hat{G}^{-1}(\omega+i\varepsilon)]^{m+\delta_{\varkappa\varkappa'},m}_{m+\delta_{\varkappa\varkappa'},m}=\omega-\omega_\varkappa(m)-P_\varkappa;$$
$$P_\varkappa=2\int_{-\infty}^{\infty}d\omega'\,\frac{\omega'}{\omega_\varkappa^2-\omega'^2}\,|\vec{\mathcal{I}}_\varkappa\vec{\nabla}_0|^2\,\varphi^0_\omega(\omega'); \qquad (5)$$

$$\mathrm{Im}\,[\hat{G}^{-1}(\omega+i\varepsilon)]^{m+\delta_{\varkappa\varkappa'},m}_{m+\delta_{\varkappa\varkappa'},m}=\Gamma^{(1)}_\varkappa(m);$$
$$\Gamma^{(1)}_\varkappa(m)=\pi\sum_{\varkappa'}|\vec{\mathcal{I}}_{\varkappa'}\vec{\nabla}_0|^2\sum_{j=\pm1}(2m_{\varkappa'}+\delta_{\varkappa\varkappa'}+1+j)\,\varphi^0_\omega(-j\omega_{\varkappa'});$$
$$\mathrm{Re}\,[\hat{G}^{-1}(\omega+i\varepsilon)]^{m+\delta_{\varkappa\varkappa'}+j\delta_{\varkappa_1\varkappa'},\,m+j\delta_{\varkappa_1\varkappa'}}_{m+\delta_{\varkappa\varkappa'},m}=0;$$
$$\mathrm{Im}\,[\hat{G}^{-1}(\omega+i\varepsilon)]^{m+\delta_{\varkappa\varkappa'}+j\delta_{\varkappa_1\varkappa'},\,m+j\delta_{\varkappa_1\varkappa'}}_{m+\delta_{\varkappa\varkappa'},m}=\gamma^j_{\varkappa\varkappa_1}(m);$$
$$\gamma^j_{\varkappa\varkappa_1}(m)=-2\pi\sqrt{[m_{\varkappa_1}+\delta_{\varkappa\varkappa_1}+\tfrac{1}{2}(1+j)][m_{\varkappa_1}+\tfrac{1}{2}(1+j)]}\,\cdot$$
$$\cdot\,|\vec{\mathcal{I}}_{\varkappa_1}\vec{\nabla}_0|^2\,\varphi^0_\omega(-j\omega_{\varkappa_1}).$$

Equations (5) for real and imaginary parts of "mass" superoperator $\hat{M}(\omega+i\varepsilon)$ are written in the frequency range $\omega\approx\omega_\varkappa$ actual for the function $\langle b_\varkappa(t)b^+_\varkappa(0)\rangle_\omega$, i.e., $\omega=\omega_\varkappa$ is put in them. Besides, non-linear corrections to the frequencies which are unessential here are thrown away in formulas (5) for these magnitudes.

The system (4) of equations may easily be solved in some particular cases.

Let's consider, for example, the harmonic (linear)oscillator, when all non-linear constants $V_{\varkappa\varkappa'}$, $V_{\varkappa\varkappa'\varkappa''}$, ... in the Hamiltonian (I.33) are equal to zero. In this case, the system of equations (4) has the exact solution of type

$$g_\varkappa(m,\omega+i\varepsilon)=\frac{1}{Z_0}\,\frac{(m_\varkappa+1)\exp\{-\lambda\sum_{\varkappa'}\omega_{\varkappa'}m_{\varkappa'}\}}{\Omega_\varkappa+i\Gamma_\varkappa};$$

$$Z_0=\mathrm{Sp}\exp\{-\lambda\sum_{\varkappa'}\omega_{\varkappa'}\hat{n}_{\varkappa'}\}=\prod_\varkappa(1+\bar{n}_\varkappa);\ \Omega_\varkappa=\omega-\omega_\varkappa-P_\varkappa,$$

where $\bar{n}_{\varkappa} = Z_0^{-1} \sum\limits_{m} m_{\varkappa} \exp\{-\lambda \sum\limits_{\varkappa'} \omega_{\varkappa'} m_{\varkappa'}\} = (e^{\lambda\omega_{\varkappa}} - 1)^{-1}$ is the mean occupation numb..r for oscillator $\varkappa$. Here the definition

$$\Gamma_{\varkappa} = \pi (1 - e^{-\lambda\omega_{\varkappa}}) \, |\vec{I}_{\varkappa} \vec{V}_0|^2 \, \varphi_{\omega}^{0}(\omega_{\varkappa}) \tag{6}$$

is introduced. Consequently, the correlation function (3) for the harmonic oscillator is determined by the expression

$$\langle b_{\varkappa}(t) b_{\varkappa}^{+}(0) \rangle_{\omega} = \frac{1}{\pi} (\bar{n}_{\varkappa} + 1) \frac{\Gamma_{\varkappa}}{\Omega_{\varkappa}^2 + \Gamma_{\varkappa}^2} . \tag{7}$$

Expression (7) describes Lorentz spectral distribution of harmonic oscillator $\varkappa$, whose frequency shift $P_{\varkappa}$ and broadening $\Gamma_{\varkappa}$ are conditioned by the linear interaction with the medium and determined by the medium correlation function $\varphi_{\omega}^{0}(\omega)$ (see (I.20), (I.21)).

Lets's consider now the system of equations (4) for non-linear oscillator in case, when $\Gamma_{\varkappa}^{(1)}(m) \ll \Delta_{\varkappa}(m)$ ($\Delta_{\varkappa}(m)$ is determined by expression (I.41) and represents non-equidistancy measure of non-linear oscillator levels). As is seen from equation (5), the coefficients at non-diagonal terms of equation (4) are proportional $\gamma_{\varkappa\varkappa_1}^{j}(m) \sim \Gamma_{\varkappa}^{(1)}(m)$. On the other hand, functions $g_{\varkappa}(m + j\delta_{\varkappa_1\varkappa'}, \omega + i\varepsilon)$, as it follows from the discussion given in work /I/ (after formula (I,42)), are the magnitudes of an order of $\frac{1}{\Delta_{\varkappa}(m)}$ at frequences $\omega \approx \omega_{\varkappa}(m)$. The correctness of this statement will be seen further in calculations. Taking the function

$$g_{\varkappa}^{0}(m, \omega + i\varepsilon) = \frac{(m_{\varkappa} + 1)(\rho_0)_m^m}{\omega - \omega_{\varkappa}(m) - P_{\varkappa} + i\Gamma_{\varkappa}^{(1)}(m)}$$

as zero approximation and solving the system of equations (4) by iteration method we obtain with an accuracy up to terms of the first order in small paramter $\Gamma_{\varkappa}^{(1)}(m)/\Delta_{\varkappa}(m)$ the solution for correlation function (3)

$$\langle b_{\varkappa}(t) b_{\varkappa}^{+}(0) \rangle_{\omega} = \frac{1}{\pi Z_0} \sum_{m} (m_{\varkappa} + 1) e^{-\lambda \sum_{\varkappa'} \omega_{\varkappa'} m_{\varkappa'}} \frac{\Gamma_{\varkappa}^{(1)}(m)}{\Omega_{\varkappa}^2(m) + \Gamma_{\varkappa}^{(1)2}(m)} \cdot$$
$$\cdot \Big[1 + \sum_{j=\pm 1} \sum_{\varkappa_1} \gamma_{\varkappa\varkappa_1}^{j}(m) \sqrt{\frac{m_{\varkappa} + 1 + j\delta_{\varkappa\varkappa_1}}{m_{\varkappa} + 1}} \cdot$$
$$\cdot \frac{\Gamma_{\varkappa}^{(1)}(m) \Gamma_{\varkappa}^{(1)}(m + j\delta_{\varkappa_1\varkappa'}) - \Omega_{\varkappa}(m) \Omega_{\varkappa}(m + j\delta_{\varkappa_1\varkappa'})}{\Gamma_{\varkappa}^{(1)}(m) \Gamma_{\varkappa}^{(1)}(m + j\delta_{\varkappa_1\varkappa'})} \cdot \tag{8}$$
$$\cdot \frac{\Gamma_{\varkappa}^{(1)}(m + j\delta_{\varkappa_1\varkappa'})}{\Omega_{\varkappa}^2(m + j\delta_{\varkappa_1\varkappa'}) + \Gamma_{\varkappa}^{(1)2}(m + j\delta_{\varkappa_1\varkappa'})} \Big] .$$

Here, the following definition is introduced:

$$\Omega_{\varkappa}(m + j\delta_{\varkappa_1\varkappa'}) = \omega - \omega_{\varkappa}(m + j\delta_{\varkappa_1\varkappa'}) - P_{\varkappa} \; ; \; j = 0, \pm 1 .$$

The magnitude $\omega_{\varkappa}(m + j\delta_{\varkappa_1\varkappa'})$ is determined by the expression (I.37) for $\omega_{\varkappa}(m)$, in which $m_{\varkappa'}$ is substituted by $m_{\varkappa'} + j\delta_{\varkappa_1\varkappa'}$. By analogy, the magnitude $\Gamma_{\varkappa}^{(1)}(m + j\delta_{\varkappa_1\varkappa'})$ is determined by formula (5) with substitution $m_{\varkappa'} + j\delta_{\varkappa_1\varkappa'}$ for $m_{\varkappa'}$.

The first addend of equation (8) (zero approximation over $\Gamma_{\varkappa}^{(1)}(m)/\Delta_{\varkappa}(m)$) according to the general results, obtained in work /I/ (see formula (I.44), is the function describing the set of fine structure Lorentz lines of non-linear oscillator with the distance $\Delta_{\varkappa}(m)$ between them. The shift $P_{\varkappa}$ of lines with respect to the points $\omega = \omega_{\varkappa}(m)$ and their widths $\Gamma^{(1)}_{\varkappa}(m)$ are determined according to formulas (5) (corresponding to general formulas (I.45)) through Fourier component of the medium correlation function $\varphi^{\circ}_{\omega}(\omega)$. As it was noted in work /I/, in case of linear interaction line broadening is conditioned only by transitions between oscillator levels appearing due to interaction with the medium excitations, and described by the medium correlation function on the oscillator frequencies $\pm\,\omega_{\varkappa_1}$. Modulational broadening, which the correlation function $\varphi^{\circ}_{\omega}(\omega)$ on zero frequency corresponds to, is absent in this case. For linear interaction, as it is seen from (5), line widths $\Gamma^{(1)}_{\varkappa}(m)$ depend linearly on oscillation occupation numbers $m_{\varkappa'}$.

Correction terms of the first order in $\Gamma^{(1)}_{\varkappa}(m)/\Delta_{\varkappa}(m)$ in formula (8) bring to slight asymmetry of separate lines, change of their intensities, and in some shift of maxima with respect to points $\Omega_{\varkappa}(m)=0$.

It is easy to obtain also an analytic expression for the correlation function $\langle b_{\varkappa}(t)b^{+}_{\varkappa}(0)\rangle_{\omega}$ on distribution wings, when $|\omega-\omega_{\varkappa}-P_{\varkappa}-\frac{1}{2}V_{\varkappa\varkappa}|=|\Omega'_{\varkappa}|\gg\sum_{\varkappa'}V_{\varkappa\varkappa'}m_{\varkappa'}+\ldots$ (i.e., it is possible to neglect in equation (I.37) for $\omega_{\varkappa}(m)$ the terms, depending on numbers $m_{\varkappa'}$) and also $|\Omega'_{\varkappa}|\gg\Gamma^{(1)}_{\varkappa}(m)$. Then iterating the system (4) equations and taking into consideration that terms with $j=\pm 1$ are magnitudes of an order of $\Gamma^{(1)}_{\varkappa}(m)|g_{\varkappa}|\ll|\Omega'_{\varkappa}g_{\varkappa}|$, for function (3) in the first approximation over $\Gamma^{(1)}_{\varkappa}(m)/|\Omega'_{\varkappa}|$ we get

$$\langle b_{\varkappa}(t)b^{+}_{\varkappa}(0)\rangle_{\omega}=\frac{1}{\pi}\left(\bar{n}_{\varkappa}+1\right)\frac{\Gamma_{\varkappa}}{\Omega'^{2}_{\varkappa}} \qquad (9)$$

In the general case the system (4) of equations may be solved by breaking the chain of equations. It appears to be possible as far as according to definition (3) the function $g_{\varkappa}(m,\omega+i\varepsilon)$ is proportional to $\exp\{-\lambda\sum_{\varkappa'}\omega_{\varkappa'}m_{\varkappa'}\}$. Thus, only one line will appear in spectrum at low temperatures, corresponding to the set $m=0$ of occupation numbers (intensities of the rest lines are exponentially small). With increasing temperature other spectrum lines with $m_{\varkappa'}\neq 0$ begin to develop and spectrum fine structure may appear, described by equation (8). With further increase of temperature lines with large $m_{\varkappa'}$ begin to develop, but line widths also grow, that brings to gradual disappearance of fine structure and appearance of single broadening spectral distribution.

Consequently, depending on temperature it is possible to cut the system of equations (4) at any equation due to the factor $\exp\{-\lambda\sum_{\varkappa'}\omega_{\varkappa'}m_{\varkappa'}\}$ and to solve the remaining system of equations either analytically (if there are few equations) or numerically at computer.

Let us illustrate the aforesaid by a simple example, when there is only one singled out oscillation $\varkappa$ (or interaction of the given oscillator with other oscillations may be neglected) so in sum over $\varkappa_1$ of expression (4) there is only one term with $\varkappa_1=\varkappa$. Let's take such temperature $(k_B T\ll\omega_{\varkappa})$, that it would be possible to take into consideration only functions $g_{\varkappa}(0,\omega+i\varepsilon)$, $g_{\varkappa}(1,\omega+i\varepsilon)$ and throw off the rest.

Then the system of equations (4) is reduced to two equations for the pointed out functions, and its solution is

$$g_{\varkappa}(0,\omega+i\varepsilon)+g_{\varkappa}(1,\omega+i\varepsilon)=$$

$$=\frac{\Omega_{\varkappa}(1)+i\Gamma_{\varkappa}^{(1)}(1)+2[\Omega_{\varkappa}(0)+i\Gamma_{\varkappa}^{(1)}(0)]\bar{n}_{\varkappa}+8i\Gamma_{\varkappa}\bar{n}_{\varkappa}}{[\Omega_{\varkappa}(0)+i\Gamma_{\varkappa}^{(1)}(0)][\Omega_{\varkappa}(1)+i\Gamma_{\varkappa}^{(1)}(1)]+8\Gamma_{\varkappa}^{2}\bar{n}_{\varkappa}}\,. \qquad (10)$$

Formulas (3), (10) define the correlation function of $\varkappa$ oscillation at low temperature with any relation between line form parameters of oscillator $\varkappa$ spectral distribution. Line form, defined by equation (10), is a composite curve in the general case. If the distance between the frequencies, defined by the conditions $\Omega_{\varkappa}(0)=0$, $\Omega_{\varkappa}(1)=0$, is significantly larger than the magnitudes $\Gamma_{\varkappa}^{(1)}(0)$, $\Gamma_{\varkappa}^{(1)}(1)$, then spectral distribution decomposes into two fine structure lines with maxima on the pointed out frequencies and widths $\Gamma_{\varkappa}^{(1)}(0)$, $\Gamma_{\varkappa}^{(1)}(1)$. Line intensities are essentially different $(K_B T \ll \omega_{\varkappa})$. If inverse inequality between $|\Omega_{\varkappa}(0)-\Omega_{\varkappa}(1)|$ and $\Gamma_{\varkappa}^{(1)}(0), \Gamma_{\varkappa}^{(1)}(1)$ is satisfied, the expression (10) describes one asymmetric line.

Classical approximation for describing oscillator is applicable at high temperatures $K_B T \gg \omega_{\varkappa'}$ and in case of absence of fine structure. As in the classical limit the main contribution into correlation function is given by $g_{\varkappa}(m,\omega+i\varepsilon)$ with large quantum numbers $m_{\varkappa'}$, let us introduce the continuous variable $X_i = m_{\varkappa_i}$ ($i=0,1,\ldots$ defines number of normal oscillation and let's put $\varkappa_0=\varkappa$). Then going in functions $g_{\varkappa}(m+j\delta_{\varkappa_1\varkappa'},\omega+i\varepsilon)$ $(j=0,\pm 1)$ to continuous variables X_i and expanding them into power series in the vicinity of $j=0$, we get instead of the system of equations (4) the differential equation

$$[\Omega_{\varkappa}(X)-i\sum_i(2-\delta_{\varkappa\varkappa_i})\Gamma_{\varkappa_i}]\,g_{\varkappa}(X,\omega+i\varepsilon)-$$

$$-2i\sum_i X_i\Gamma_{\varkappa_i}\frac{dg_{\varkappa}(X,\omega+i\varepsilon)}{dX_i}-2i\sum_{i\neq 0}\frac{\Gamma_{\varkappa_i}}{\lambda\omega_{\varkappa_i}}\frac{dg_{\varkappa}(X,\omega+i\varepsilon)}{dX_i}-$$

$$-2i\sum_i X_i\frac{\Gamma_{\varkappa_i}}{\lambda\omega_{\varkappa_i}}\frac{d^2 g_{\varkappa}(X,\omega+i\varepsilon)}{dX_i^{2}}=\frac{1}{Z_0}X_0\,e^{-\lambda\sum_i\omega_{\varkappa_i}X_i}; \qquad (11)$$

$$g_{\varkappa}(-X,\omega+i\varepsilon)=g_{\varkappa}(\infty,\omega+i\varepsilon)=0\,;\quad Z_0=\prod_i\frac{1}{\lambda\omega_{\varkappa_i}},$$

where X denotes the set of numbers X_i, $\Gamma_{\varkappa_i}$ is defined by the expression (6).

The correlation function $\langle b_{\varkappa}(t)b_{\varkappa}^{+}(0)\rangle_{\omega}$ in the classical limit is defined as

$$\langle b_{\varkappa}(t)b_{\varkappa}^{+}(0)\rangle_{\omega}=-\frac{1}{\pi}\,Im\int_0^{\infty}g_{\varkappa}(X,\omega+i\varepsilon)\,dX \qquad (12)$$

and may be found by numerical calculation of equations (11),(12) at the computer.

If there is only one oscillation $\varkappa$, then the function $g_{\varkappa}(X,\omega+i\varepsilon)$ becomes the function of one variable X_0 and the expression (11) is essentially simplified. One more simplification is possible, if we restrict ourselves to oscillator quadratic non-linearity in occupation number operator $\hat{n}_{\varkappa}=b_{\varkappa}^{+}b_{\varkappa}$. Then function $\Omega_{\varkappa}(X)=\Omega'_{\varkappa}-\sum_i V_{\varkappa\varkappa_i}X_i$.

As it was shown in paper /I/, the oscillator correlation function in time representation is defined by kinetic equation (I.30). Let's consider the function

$$\langle b_{\varkappa}(t)b_{\varkappa}^{+}(0)\rangle=\sum_m(\rho_0)_m^m(1+m_{\varkappa})\,G_{\varkappa}(m,t)\,;$$

$$G_{\varkappa}(m+j\delta_{\varkappa_1\varkappa'},t)=\sum_{m'}\sqrt{\frac{1+m'_{\varkappa}}{1+m_{\varkappa}+j\delta_{\varkappa_1\varkappa}}}\;\cdot \qquad (13)$$

$$\cdot\left(\rho\, e^{-iLt}\rho\right)^{m'+\delta_{\varkappa\varkappa'},\,m'}_{m+\delta_{\varkappa\varkappa'}+j\delta_{\varkappa_1\varkappa'},\,m+j\delta_{\varkappa_1\varkappa'}}\,; \qquad (j=0,\pm 1)$$

According to equations (I.30) (I.31) (5) kinetic equation for the function $G_{\varkappa}(m,t)$ in the second order of perturbation theory in H_{int} is

$$\frac{\partial}{\partial t} G_{\varkappa}(m,t) = -i[\omega_{\varkappa}(m) + P_{\varkappa} - i\Gamma^{(1)}_{\varkappa}(m)]G_{\varkappa}(m,t) +$$
$$+2\sum_{\varkappa_1}\sum_{j=\pm1}[(1+\delta_{\varkappa\varkappa_1})\tfrac{1}{2}(1+j)+m_{\varkappa_1}][\bar n_{\varkappa_1}+\tfrac{1}{2}(1-j)]\Gamma_{\varkappa_1}\cdot$$
$$\cdot G_{\varkappa}(m+j\delta_{\varkappa_1\varkappa'},t);\ (t\gg\omega^{-1}_{med}),\quad G_{\varkappa}(m,0)=1. \tag{14}$$

Following paper /2/ let us consider the function of continuously varying parameters $y_{\varkappa'}$

$$\Phi(y,t) = \frac{1}{z_0}\sum_m (1+m_{\varkappa})G_{\varkappa}(m,t)e^{-\sum_{\varkappa'} m_{\varkappa'}y_{\varkappa'}} \tag{15}$$

and let's restrict ourselves to the case when $\omega_{\varkappa}(m)$ is equal to $\omega_{\varkappa}+\frac{1}{2}V_{\varkappa\varkappa}+\sum_{\varkappa_1}V_{\varkappa\varkappa_1}m_{\varkappa_1}$ (in the expression (I.37) all non-linear constants besides $V_{\varkappa\varkappa_1}$ are put equal to zero). In (15) y denotes a definite set of parameters $y_{\varkappa'}$.

Then using equation (14) we may derive an equation for the function $\Phi(y,t)$

$$\frac{\partial\Phi(y,t)}{\partial t} + \sum_{\varkappa_1}\beta_{\varkappa_1}\frac{\partial\Phi(y,t)}{\partial y_{\varkappa_1}} = \gamma_{\varkappa}\Phi(y,t);$$
$$\Phi(y,0) = \frac{1}{z_0}\sum_m(1+m_{\varkappa})\exp(-\sum_{\varkappa'}m_{\varkappa'}y_{\varkappa'}). \tag{16}$$

The coefficients of this equation are defined by the formulas

$$\beta_{\varkappa_1} = -iV_{\varkappa\varkappa_1} - 2\Gamma_{\varkappa_1}(1+2\bar n_{\varkappa_1}) + 2e^{y_{\varkappa_1}}\Gamma_{\varkappa_1}\bar n_{\varkappa_1} +$$
$$+2e^{-y_{\varkappa_1}}\Gamma_{\varkappa_1}(1+\bar n_{\varkappa_1});$$
$$\gamma_{\varkappa} = -i(\omega_{\varkappa}+P_{\varkappa}+\tfrac{1}{2}V_{\varkappa\varkappa}) - \Gamma_{\varkappa}(\bar n_{\varkappa}+1) - \sum_{\varkappa'}(2+\delta_{\varkappa\varkappa'})\Gamma_{\varkappa'}\bar n_{\varkappa'} +$$
$$+2\sum_{\varkappa'}(1+\delta_{\varkappa\varkappa'})\Gamma_{\varkappa'}(1+\bar n_{\varkappa'})e^{-y_{\varkappa'}},$$

and the correlation function $\langle b_{\varkappa}(t)b^{+}_{\varkappa}(0)\rangle$ by the value of $\Phi(y,t)$ at $y_{\varkappa'} = \lambda\omega_{\varkappa'}$.

The equation of type (16) was solved in paper /2/ by the method of characteristics. Using this solution, we get the expression for the correlation function (13)

$$\langle b_{\varkappa}(t)b^{+}_{\varkappa}(0)\rangle = (1+\bar n_{\varkappa})\Psi^{-1}_{\varkappa\varkappa}(t)\prod_{\varkappa'}\Psi^{-1}_{\varkappa\varkappa'}(t)\cdot$$
$$\cdot\exp[-i(\omega_{\varkappa}+P_{\varkappa})\delta_{\varkappa\varkappa'}t + (\tfrac{i}{2}V_{\varkappa\varkappa'}+\Gamma_{\varkappa'})t]; \tag{17}$$
$$(t\gg\omega^{-1}_{med}).$$

Here the notation is the same as in work/2/

$$\Psi_{\varkappa\varkappa'}(t) = ch\,a_{\varkappa\varkappa'}t + [1+i\frac{V_{\varkappa\varkappa'}}{2\Gamma_{\varkappa'}}(1+2\bar n_{\varkappa'})]\frac{\Gamma_{\varkappa'}}{a_{\varkappa\varkappa'}}\,sh\,a_{\varkappa\varkappa'}t;$$
$$a^2_{\varkappa\varkappa'} = \Gamma^2_{\varkappa'} + i\Gamma_{\varkappa'}V_{\varkappa\varkappa'}(1+2\bar n_{\varkappa'}) - \frac{1}{4}V^2_{\varkappa\varkappa'}. \tag{18}$$

Solution (17) formally coincides with the solution obtained in work/2/ but is more general. The parameters $P_{\varkappa}$, $\Gamma_{\varkappa'}$, entering into equations (14),(16) and the solution (17), are defined by expressions (5),(6) through the correlation function for the arbitrary medium (in/2/ the magnitudes $P_{\varkappa}$, $\Gamma_{\varkappa'}$ are defined for the medium as a set of harmonic oscillators with continuous spectrum).

Thus, in the considered case of oscillator linear interaction with the fluctuations of the arbitrary medium there is a solution for the correlation function $\langle b_{\varkappa}(t)b^{+}_{\varkappa}(0)\rangle$, which is true for any relation between magnitudes $V_{\varkappa\varkappa'}$ (defining non-equidistancy of oscillator levels) and $\Gamma_{\varkappa'}$ (defining level broadening, conditioned by the interaction with the medium).
In particular, at $|V_{\varkappa\varkappa'}| \gg \Gamma^{(1)}_{\varkappa}(m)$ from formula (17) after

the Fourier transformation the oscillator $\varkappa$ spectral distribution as a set of fine structure lines is obtained, which is described by equation (8).

If the inverse limiting relation $|V_{\varkappa\varkappa'}| \ll \Gamma_{\varkappa'}$ is carried out, then expanding equations (17), (18) to this little parameter, we obtain /2/

$$\langle b_\varkappa(t) b_\varkappa^+(0)\rangle_\omega = \frac{1}{\pi}(1+\bar n_\varkappa)\left\{\frac{\Gamma_\varkappa}{\tilde\Omega'^2_\varkappa+\Gamma^2_\varkappa} + \sum_{\varkappa'}\frac{V^2_{\varkappa\varkappa'}}{4\Gamma^2_{\varkappa'}}\bar n_{\varkappa'}(1+\bar n_{\varkappa'})\cdot\right.$$

$$\left.\cdot(1+\delta_{\varkappa\varkappa'})\left[\frac{\Gamma_\varkappa}{\tilde\Omega'^2_\varkappa+\Gamma^2_\varkappa} - \frac{\Gamma_\varkappa+2\Gamma_{\varkappa'}}{\tilde\Omega'^2_\varkappa+(\Gamma_\varkappa+2\Gamma_{\varkappa'})^2} - 2\frac{\Gamma_{\varkappa'}(\Gamma^2_\varkappa-\tilde\Omega'^2_\varkappa)}{(\tilde\Omega'^2_\varkappa+\Gamma^2_\varkappa)^2}\right]\right\};$$

$$\tilde\Omega'_\varkappa = \Omega'_\varkappa - \sum_{\varkappa'} V_{\varkappa\varkappa'}\bar n_{\varkappa'}(1+\delta_{\varkappa\varkappa'}).$$

In the case of intermediate relation between $V_{\varkappa\varkappa'}$ and $\Gamma_{\varkappa'}$ the oscillator $\varkappa$ spectral distribution may be obtained by numerical integration of expression (17) over time t. Such calculation at computer was carried out for quantum and classical cases in works /2,3/ respectively. As it was said above, in the general case for finding spectral distribution by means of numerical calculation at computer the system of equations (4) or differential equation (11)(in classical limit) may be used instead of integration of equation (17).

Let's consider now the time correlation function for the operator of the oscillation $\varkappa$ occupation number

$$\langle \hat n_\varkappa(t)\,\tilde{\hat n}_\varkappa(0)\rangle = \sum_m (\rho_0)^m_m\, m_\varkappa\, g'_\varkappa(m,t);$$

$$g'_\varkappa(m,t) = \sum_{m'} \tilde m'_\varkappa\,(\rho e^{-i\hat L t}\rho)^{m'm'}_{mm}.$$

Kinetic equation for the function $g'_\varkappa(m,t)$ is determined by the expression (I.30), (I.31), (2) and is as follows (in the second approximation of perturbation theory):

$$\frac{\partial}{\partial t} g'_\varkappa(m,t) = -2\sum_{\varkappa_1}\Gamma_{\varkappa_1}\Big\{[m_{\varkappa_1}(1+2\bar n_{\varkappa_1})+\bar n_{\varkappa_1}]\,g'_\varkappa(m,t) -$$

$$-\sum_{j=\pm1}[\bar n_{\varkappa_1}+\tfrac12(1-j)][m_{\varkappa_1}+\tfrac12(1+j)]\,g'_\varkappa(m+j\delta_{\varkappa_1\varkappa'},t)\Big\};$$

$$(t\gg\omega^{-1}_{med}) \qquad (19)$$

$$g'_\varkappa(m,0) = m_\varkappa - \bar n_\varkappa.$$

If the magnitudes

$$N^{\kappa\kappa_1}_{\varkappa\varkappa_1}(t) = \sum_m m^\kappa_\varkappa\, m^{\kappa_1}_{\varkappa_1}\,(\rho_0)^m_m\, g'_\varkappa(m,t)$$

are introduced, then from equation (19) we have equations for these magnitudes

$$\frac{\partial N^{00}_{\varkappa\varkappa}(t)}{\partial t} = 0\,;\quad N^{00}_{\varkappa\varkappa}(0) = N^{00}_{\varkappa\varkappa}(t) = 0\,;$$

$$\frac{\partial N^{10}_{\varkappa\varkappa}(t)}{\partial t} = -2\Gamma_\varkappa N^{10}_{\varkappa\varkappa}(t)\,;\quad N^{10}_{\varkappa\varkappa}(0) = \bar n_\varkappa(1+\bar n_\varkappa). \qquad (20)$$

Thus, for the case of linear interaction with the medium fluctuations the correlation function $\langle\hat n_\varkappa(t)\tilde{\hat n}_\varkappa(0)\rangle \equiv N^{10}_{\varkappa\varkappa}(t)$ depends on time exponentially with characteristic time $(2\Gamma_\varkappa)^{-1}$ and its spectral representation has the Lorentz shape

$$\langle\hat n_\varkappa(t)\tilde{\hat n}_\varkappa(0)\rangle_\omega = \frac{1}{\pi}\,\bar n_\varkappa(1+\bar n_\varkappa)\,\frac{2\Gamma_\varkappa}{\omega^2+(2\Gamma_\varkappa)^2}. \qquad (21)$$

3. Quadratic interaction of the oscillator with medium.

To consider non-linear in displacement operator $\vec u_0$ oscillator interaction with the medium fluctuations, it is necessary to take into consideration in the operator $e^{\vec u_0\vec\nabla_0}$ expansion (in the Hamiltonian of interaction) nonlinear terms.

Let us restrict ourselves to quadratic in $\vec{u}_0$ terms. Then, in general formulas of paper /I/, it is necessary to perform the substitution

$$e^{\vec{u}_0\vec{\nabla}_0} - 1 \longrightarrow \vec{u}_0\vec{\nabla}_0 + \frac{1}{2}(\vec{u}_0\vec{\nabla}_0)^2 = \sum_{\varkappa}(\vec{\alpha}_\varkappa b_\varkappa + \vec{\alpha}^*_\varkappa b^+_\varkappa)\vec{\nabla}_0 + \tag{22}$$

$$+\frac{1}{2}\sum_{\varkappa\varkappa' ij}(\alpha^i_\varkappa b_\varkappa + \alpha^{i*}_\varkappa b^+_\varkappa)(\alpha^j_{\varkappa'} b_{\varkappa'} + \alpha^{j*}_{\varkappa'} b^+_{\varkappa'})\nabla^i_0\nabla^j_0,$$

where i, j denote Cartesian coordinates.

"Mass" superoperator $\hat{M}$ in the second order of perturbation theory in H_{int} is defined by square of expression (22). Since (as it was pointed out in work /I/) it is necessary to preserve only terms containing equal number of operators $b_\varkappa$, $b^+_\varkappa$ for every oscillation $\varkappa$, terms of linear $(\vec{u}_0\vec{\nabla}_0)$ and quadratic interaction will not overlap. Taking into consideration the aforesaid and also expressions (I.40), (22) we have the contribution of quadratic interaction in the matrix element of the superoperator $\hat{M}(\omega)$

$$\left[\hat{M}^{(2)}(\omega)\right]^{n'm'}_{nm} = \frac{1}{4}\int_{-\infty}^{\infty}d\omega'\sum_{\varkappa_1\varkappa_2}\Big\{(2-\delta_{\varkappa_1\varkappa_2})\Big[\frac{(1+n_{\varkappa_1})(1+n_{\varkappa_2}+\delta_{\varkappa_1\varkappa_2})}{\omega-\omega_{n+\delta_{\varkappa_1\varkappa'}+\delta_{\varkappa_2\varkappa'}}+\omega_m-\omega'} +$$

$$+\frac{n_{\varkappa_1}(n_{\varkappa_2}-\delta_{\varkappa_1\varkappa_2})}{\omega-\omega_{n-\delta_{\varkappa_1\varkappa'}-\delta_{\varkappa_2\varkappa'}}+\omega_m-\omega'} + \frac{(1+m_{\varkappa_1})(1+m_{\varkappa_2}+\delta_{\varkappa_1\varkappa_2})}{\omega-\omega_n+\omega_{m+\delta_{\varkappa_1\varkappa'}+\delta_{\varkappa_2\varkappa'}}+\omega'} +$$

$$+\frac{m_{\varkappa_1}(m_{\varkappa_2}-\delta_{\varkappa_1\varkappa_2})}{\omega-\omega_n+\omega_{m-\delta_{\varkappa_1\varkappa'}-\delta_{\varkappa_2\varkappa'}}+\omega'}\Big]\delta_{nn'}\delta_{mm'} + \Big[\frac{4(1+n_{\varkappa_1})n_{\varkappa_2}+\delta_{\varkappa_1\varkappa_2}}{\omega-\omega_{n+\delta_{\varkappa_1\varkappa'}-\delta_{\varkappa_2\varkappa'}}+\omega_m-\omega'} +$$

$$+\frac{4(1+m_{\varkappa_1})m_{\varkappa_2}+\delta_{\varkappa_1\varkappa_2}}{\omega-\omega_n+\omega_{m+\delta_{\varkappa_1\varkappa'}-\delta_{\varkappa_2\varkappa'}}+\omega'}\Big]\delta_{nn'}\delta_{mm'} - (2-\delta_{\varkappa_1\varkappa_2})\cdot$$

$$\cdot\sqrt{(1+n_{\varkappa_1})(1+n_{\varkappa_2}+\delta_{\varkappa_1\varkappa_2})(1+m_{\varkappa_1})(1+m_{\varkappa_2}+\delta_{\varkappa_1\varkappa_2})}\cdot$$

$$\cdot\Big(\frac{1}{\omega-\omega_{n+\delta_{\varkappa_1\varkappa'}+\delta_{\varkappa_2\varkappa'}}+\omega_m-\omega'} + \frac{1}{\omega-\omega_n+\omega_{m+\delta_{\varkappa_1\varkappa'}+\delta_{\varkappa_2\varkappa'}}+\omega'}\Big)\cdot$$

$$\cdot\delta_{n',n+\delta_{\varkappa_1\varkappa'}+\delta_{\varkappa_2\varkappa'}}\delta_{m',m+\delta_{\varkappa_1\varkappa'}+\delta_{\varkappa_2\varkappa'}} - (2-\delta_{\varkappa_1\varkappa_2})\cdot$$

$$\cdot\sqrt{n_{\varkappa_1}(n_{\varkappa_2}-\delta_{\varkappa_1\varkappa_2})m_{\varkappa_1}(m_{\varkappa_2}-\delta_{\varkappa_1\varkappa_2})}\Big(\frac{1}{\omega-\omega_{n-\delta_{\varkappa_1\varkappa'}-\delta_{\varkappa_2\varkappa'}}+\omega_m-\omega'} +$$

$$+\frac{1}{\omega-\omega_n+\omega_{m-\delta_{\varkappa_1\varkappa'}-\delta_{\varkappa_2\varkappa'}}+\omega'}\Big)\delta_{n',n-\delta_{\varkappa_1\varkappa'}-\delta_{\varkappa_2\varkappa'}}\delta_{m',m-\delta_{\varkappa_1\varkappa'}-\delta_{\varkappa_2\varkappa'}} -$$

$$-\Big[\sqrt{(1+n_{\varkappa_1})(n_{\varkappa_2}+\delta_{\varkappa_1\varkappa_2})} + \sqrt{(1+n_{\varkappa_1}-\delta_{\varkappa_1\varkappa_2})n_{\varkappa_2}}\Big]\cdot$$

$$\cdot\Big[\sqrt{(1+m_{\varkappa_1})(m_{\varkappa_2}+\delta_{\varkappa_1\varkappa_2})} + \sqrt{(1+m_{\varkappa_1}-\delta_{\varkappa_1\varkappa_2})m_{\varkappa_2}}\Big]\cdot \tag{23}$$

$$\cdot\Big(\frac{1}{\omega-\omega_{n+\delta_{\varkappa_1\varkappa'}-\delta_{\varkappa_2\varkappa'}}+\omega_m-\omega'} + \frac{1}{\omega-\omega_n+\omega_{m+\delta_{\varkappa_1\varkappa'}-\delta_{\varkappa_2\varkappa'}}+\omega'}\Big)\cdot$$

$$\cdot\delta_{n',n+\delta_{\varkappa_1\varkappa'}-\delta_{\varkappa_2\varkappa'}}\delta_{m',m+\delta_{\varkappa_1\varkappa'}-\delta_{\varkappa_2\varkappa'}}\Big\}|\vec{\alpha}_{\varkappa_1}\vec{\nabla}_0|^2|\vec{\alpha}_{\varkappa_2}\vec{\nabla}_0|^2\varphi^0_\omega(\omega').$$

From equation (23) it is seen that the superoperator $\hat{M}^{(2)}(\omega)$ has diagonal matrix elements with $n = n'$, $m = m'$ different from zero and also with $n' = n + j_1\delta_{\varkappa_1\varkappa'} + j_2\delta_{\varkappa_2\varkappa'}$, $m' = m + j_1\delta_{\varkappa_1\varkappa'} + j_2\delta_{\varkappa_2\varkappa'}$ $(j_1, j_2 = \pm 1)$. Consequently, the system of equations (I.42) for the function $g_\varkappa(m, \omega + i\epsilon)$ taking into account linear and quadratic interactions may be written as

$$\left[\Omega_{\varkappa}(m) - P_{\varkappa}^{(2)}(m) + i\Gamma_{\varkappa}^{(1)}(m) + i\Gamma_{\varkappa}^{(2)}(m)\right] g_{\varkappa}(m, \omega + i\varepsilon) +$$

$$+ i \sum_{j=\pm 1} \sum_{\varkappa_1} \gamma^{j}_{\varkappa\varkappa_1}(m) \sqrt{\frac{1+m_{\varkappa}}{1+m_{\varkappa}+j\delta_{\varkappa\varkappa_1}}}\; e^{\lambda j \omega_{\varkappa_1}} g_{\varkappa}(m + j\delta_{\varkappa_1\varkappa'}, \omega + i\varepsilon) +$$

$$+ i \sum_{\substack{j_1, j_2 = \pm 1 \\ (j_1 \geqslant j_2)}} \sum_{\varkappa_1\varkappa_2} \gamma^{j_1 j_2}_{\varkappa\varkappa_1\varkappa_2}(m) \sqrt{\frac{1+m_{\varkappa}}{1+m_{\varkappa}+j_1\delta_{\varkappa\varkappa_1}+j_2\delta_{\varkappa\varkappa_2}}}\; \exp[\lambda(j_1\omega_{\varkappa_1} + j_2\omega_{\varkappa_2})] \cdot \tag{24}$$

$$\cdot g_{\varkappa}(m + j_1\delta_{\varkappa_1\varkappa'} + j_2\delta_{\varkappa_2\varkappa'}, \omega + i\varepsilon) = (1+m_{\varkappa})(\rho_0)^{m}_{m}.$$

Equation (24) is written for actual frequency range $\omega \approx \omega_{\varkappa}$ and using formulas (4),(5) for linear interaction. Magnitudes $P_{\varkappa}^{(2)}(m)$ and $\Gamma_{\varkappa}^{(2)}(m)$, $\gamma^{j_1 j_2}_{\varkappa\varkappa_1\varkappa_2}(m)$, determining the correction to the frequency and broadening due to quadratic interaction respectively, are given by the equations

$$P_{\varkappa}^{(2)}(m) = \frac{1}{2}\int_{-\infty}^{\infty} d\omega' \sum_{\varkappa_1\varkappa_2} \Big[(2 - \delta_{\varkappa_1\varkappa_2})\delta_{\varkappa\varkappa_1} \cdot$$

$$\cdot \frac{(1+\delta_{\varkappa\varkappa_2}+2m_{\varkappa_2})\omega' - (1+\delta_{\varkappa\varkappa_2})(\omega_{\varkappa_1}+\omega_{\varkappa_2})}{(\omega_{\varkappa_1}+\omega_{\varkappa_2})^2 - \omega'^2} +$$

$$+ 2\,\frac{(1+m_{\varkappa_1})\delta_{\varkappa\varkappa_2} + (m_{\varkappa_2}+\delta_{\varkappa\varkappa_2})\delta_{\varkappa\varkappa_1}}{\omega_{\varkappa_2} - \omega_{\varkappa_1} - \omega'}\Big] |\vec{\alpha}_{\varkappa_1}\vec{\nabla}_0|^2 |\vec{\alpha}_{\varkappa_2}\vec{\nabla}_0|^2 \varphi^{\circ}_{\omega}(\omega'); \tag{25}$$

$$\Gamma_{\varkappa}^{(2)}(m) = \frac{\pi}{2}\sum_{\varkappa_1\varkappa_2} |\vec{\alpha}_{\varkappa_1}\vec{\nabla}_0|^2 |\vec{\alpha}_{\varkappa_2}\vec{\nabla}_0|^2 \Big\{(2 - \delta_{\varkappa_1\varkappa_2})\big[m_{\varkappa_1}(m_{\varkappa_2} - \delta_{\varkappa_1\varkappa_2} +$$

$$+ \delta_{\varkappa\varkappa_2}) + e^{-\lambda(\omega_{\varkappa_1}+\omega_{\varkappa_2})}(1 + m_{\varkappa_1} + \delta_{\varkappa\varkappa_1})(1 + m_{\varkappa_2} + \delta_{\varkappa_1\varkappa_2})\big] \cdot$$

$$\cdot \varphi^{\circ}_{\omega}(\omega_{\varkappa_1}+\omega_{\varkappa_2}) + \big[2(1+m_{\varkappa_1}+\delta_{\varkappa\varkappa_1})(m_{\varkappa_2}+\delta_{\varkappa\varkappa_2}) + 2(1+m_{\varkappa_1})m_{\varkappa_2} +$$

$$+ \delta_{\varkappa_1\varkappa_2}\big]\varphi^{\circ}_{\omega}(\omega_{\varkappa_2} - \omega_{\varkappa_1}) - \delta_{\varkappa_1\varkappa_2}(1 + 2m_{\varkappa_1})(1 + 2m_{\varkappa_1} + 2\delta_{\varkappa\varkappa_1}) \cdot$$

$$\cdot \varphi^{\circ}_{\omega}(0)\Big\};$$

$$\gamma^{j_1 j_2}_{\varkappa\varkappa_1\varkappa_2}(m) = -\frac{\pi}{2}\left[1 + \frac{3}{2}(j_1 - j_2)\right]\left[2 - \frac{1}{2}(j_1 - j_2) - \delta_{\varkappa_1\varkappa_2}\right] \cdot$$

$$\cdot \left[m_{\varkappa_1} + \frac{1}{2}(1+j_1)\right]^{1/2} \left[m_{\varkappa_1} + \delta_{\varkappa\varkappa_1} + \frac{1}{2}(1+j_1)\right]^{1/2} \Big[m_{\varkappa_2} + \delta_{\varkappa\varkappa_2} + \frac{1}{2}(1+j_2) +$$

$$+ j_2\delta_{\varkappa_1\varkappa_2}\Big]^{1/2} \left[m_{\varkappa_2} + \frac{1}{2}(1+j_2) + j_2\delta_{\varkappa_1\varkappa_2}\right]^{1/2} |\vec{\alpha}_{\varkappa_1}\vec{\nabla}_0|^2 |\vec{\alpha}_{\varkappa_2}\vec{\nabla}_0|^2 \cdot$$

$$\cdot \varphi^{\circ}_{\omega}(-j_1\omega_{\varkappa_1} - j_2\omega_{\varkappa_2}); \quad (j_1 \geqslant j_2;\ j_1, j_2 = \pm 1)$$

From equation (25) for $\gamma^{j_1 j_2}_{\varkappa\varkappa_1\varkappa_2}(m)$ it is seen, that this magnitude equals zero at $\varkappa_1 = \varkappa_2$ and $j_1 = -j_2 = 1$, so that diagonal term in the last addend of the left part of equation (24) is already separated and put into the first addend of the left part of this equation.

Let us note, that quadratic interaction besides broadening conditioned by transitions between oscillator levels and described by the medium correlation function $\varphi^{\circ}_{\omega}(\omega)$ on the combinations of frequencies $\omega_{\varkappa_1}$ and $\omega_{\varkappa_2}$ leads to modulational broadening of levels, which is described by the function $\varphi^{\circ}_{\omega}(0)$.

Besides, broadening contains now together with the terms, linear in numbers $m_{\varkappa'}$, also terms, depending on $m_{\varkappa'}$ quadratically.

Let us consider the solution of equation (24) in the same cases, as for pure linear interaction.

In the case of harmonic oscillator ($V_{\varkappa\varkappa'}, V_{\varkappa\varkappa'\varkappa''}, \ldots$ are equal to zero) with nonlinear interaction the system (24) of equations has no exact solution in contrast with the case of harmonic oscillator with linear interaction, when there is exact solution (see (7)).

When the fine structure is revealed, the oscillator $\varkappa$ spectral distribution is defined by the solution of the system of equations (24) by the iteration method and is described by formula (8), in which, nevertheless, it is necessary to make the substitution

$$\Gamma_{\varkappa}^{(1)}(m) \to \tilde{\Gamma}_{\varkappa}(m) = \Gamma_{\varkappa}^{(1)}(m) + \Gamma_{\varkappa}^{(2)}(m);$$

$$\Omega_{\varkappa}(m) \to \tilde{\Omega}_{\varkappa}(m) = \Omega_{\varkappa}(m) - P_{\varkappa}^{(2)}(m)$$

and add the correction term of kind

$$\sum_{\substack{j_1, j_2 = \pm 1 \\ (j_1 \geq j_2)}} \sum_{\varkappa_1 \varkappa_2} \gamma_{\varkappa\varkappa_1\varkappa_2}^{j_1 j_2}(m) \sqrt{\frac{1 + m_{\varkappa} + j_1\delta_{\varkappa\varkappa_1} + j_2\delta_{\varkappa\varkappa_2}}{1 + m_{\varkappa}}} \cdot$$

$$\cdot \frac{\tilde{\Gamma}_{\varkappa}(m + j_1\delta_{\varkappa_1\varkappa'} + j_2\delta_{\varkappa_2\varkappa'})}{\tilde{\Omega}_{\varkappa}^2(m + j_1\delta_{\varkappa_1\varkappa'} + j_2\delta_{\varkappa_2\varkappa'}) + \tilde{\Gamma}_{\varkappa}^2(m + j_1\delta_{\varkappa_1\varkappa'} + j_2\delta_{\varkappa_2\varkappa'})} \cdot \tag{26}$$

$$\cdot \frac{\tilde{\Gamma}_{\varkappa}(m)\tilde{\Gamma}_{\varkappa}(m + j_1\delta_{\varkappa_1\varkappa'} + j_2\delta_{\varkappa_2\varkappa'}) - \tilde{\Omega}_{\varkappa}(m)\tilde{\Omega}_{\varkappa}(m + j_1\delta_{\varkappa_1\varkappa'} + j_2\delta_{\varkappa_2\varkappa'})}{\tilde{\Gamma}_{\varkappa}(m)\tilde{\Gamma}_{\varkappa}(m + j_1\delta_{\varkappa_1\varkappa'} + j_2\delta_{\varkappa_2\varkappa'})} .$$

Correction (26) due to quadratic interaction (the same as the correction from linear interaction) has the order of magnitude $\tilde{\Gamma}_{\varkappa}(m)/\Delta_{\varkappa}(m)$ (iteration is carried out now over this little parameter).

The correlation function $\langle b_{\varkappa}(t) b_{\varkappa}^{+}(0)\rangle_{\omega}$ on distribution wings is defined by the equation

$$\langle b_{\varkappa}(t) b_{\varkappa}^{+}(0)\rangle_{\omega} = \frac{1}{\pi}(1 + \bar{n}_{\varkappa}) \frac{\tilde{\Gamma}_{\varkappa}}{\tilde{\Omega}_{\varkappa}^2};\tag{27}$$

$$\tilde{\Gamma}_{\varkappa} = \Gamma_{\varkappa} + 4\sum_{\varkappa_1} [(\bar{n}_{\varkappa_1} - \bar{n}_{\varkappa\varkappa_1}^{+})\Gamma_{\varkappa\varkappa_1}^{+} + (\bar{n}_{\varkappa_1} + \bar{n}_{\varkappa\varkappa_1}^{-} + 1)\Gamma_{\varkappa\varkappa_1}^{-}] .$$

Here the following notation is introduced

$$\Gamma_{\varkappa\varkappa_1}^{\pm} = \frac{\pi}{4}[1 - e^{-\lambda(\omega_{\varkappa} \pm \omega_{\varkappa_1})}] |\vec{\alpha}_{\varkappa}\vec{\nabla}_0|^2 |\vec{\alpha}_{\varkappa_1}\vec{\nabla}_0|^2 \varphi_{\omega}^{0}(\omega_{\varkappa} \pm \omega_{\varkappa_1});$$

$$\bar{n}_{\varkappa\varkappa_1}^{\pm} = [e^{\lambda(\omega_{\varkappa} \pm \omega_{\varkappa_1})} - 1]^{-1} \tag{28}$$

Formula (27) is obtained by analogy with equation (9) by iteration (in the first approximation) with respect to parameter $\tilde{\Gamma}_{\varkappa}(m)/|\tilde{\Omega}_{\varkappa}|$, and $\tilde{\Omega}_{\varkappa}$ differs from $\Omega_{\varkappa}'$ by additions from independent of $m_{\varkappa'}$ terms of $P_{\varkappa}^{(2)}(m)$, which make renormalization of $P_{\varkappa}$ and $V_{\varkappa\varkappa}$ (terms of $P_{\varkappa}^{(2)}(m)$, depending on $m_{\varkappa'}$, perform renormalization of $V_{\varkappa\varkappa'}$ in the expression for $\omega_{\varkappa}(m)$).

In the case when it is possible to consider one oscillation $\varkappa$ and take into consideration only functions $g_{\varkappa}(0, \omega + i\varepsilon)$ and $g_{\varkappa}(1, \omega + i\varepsilon)$ (low temperatures), the system of equations (24) has the solution (with arbitrary relation between parameters of the line form) of kind (10) where it is necessary to make substitutions

$$\Omega_{\varkappa}(m_{\varkappa}) \to \tilde{\Omega}_{\varkappa}(m_{\varkappa}); \quad \Gamma_{\varkappa}^{(1)}(m_{\varkappa}) \to \tilde{\Gamma}_{\varkappa}(m_{\varkappa}).$$

In the general case the system of equations (24) may be solved at computer.

In classical limit the system of equations (24) is reduced to the differential equation (if we restriot ourselves, for simplicity, only to one oscillator $\varkappa$)

$$[\tilde{\Omega}_{\varkappa}(x_0) - i\Gamma_{\varkappa} + 2i\Gamma_{\varkappa\varkappa}^{+}(\frac{1}{\lambda\omega_{\varkappa}} - 3x_0) + 4i\Gamma_{\varkappa\varkappa}^{0}] g_{\varkappa}(x_0, \omega + i\varepsilon) -$$

$$- 2ix_0[\Gamma_{\varkappa} + 2\Gamma_{\varkappa\varkappa}^{+}(\frac{1}{\lambda\omega_{\varkappa}} + x_0)] \frac{dg_{\varkappa}(x_0, \omega + i\varepsilon)}{dx_0} - \tag{29}$$

$$- 2i\frac{x_0}{\lambda\omega_{\varkappa}}(\Gamma_{\varkappa} + 2\Gamma_{\varkappa\varkappa}^{+}x_0)\frac{d^2 g_{\varkappa}(x_0, \omega + i\varepsilon)}{dx_0^2} = \lambda\omega_{\varkappa} x_0 e^{-\lambda\omega_{\varkappa}x_0};$$

$$g_{\varkappa}(-X_0, \omega + i\varepsilon) = g_{\varkappa}(\infty, \omega + i\varepsilon) = 0 \, .$$

Here $\Gamma^{0}_{\varkappa\varkappa}$ is the magnitude, describing modulational broadening. It is defined by the equation

$$\Gamma^{0}_{\varkappa\varkappa} = \frac{\pi}{4} |\vec{\mathcal{I}}_{\varkappa} \vec{\nabla}_0|^2 |\vec{\mathcal{I}}_{\varkappa} \vec{\nabla}_0|^2 \varphi^{0}_{\omega}(0) = (\bar{n}^{-}_{\varkappa\varkappa} + 1)\Gamma^{-}_{\varkappa\varkappa} \, .$$

Differential equation (29) may be solved numerically at computer. Equation (29) formally coincides with that obtained in work[4] for particular case, when $\widetilde{\Omega}_{\varkappa}(X_0) = \widetilde{\Omega}_{\varkappa} - \widetilde{V}_{\varkappa\varkappa} X_0$, the medium is considered as a set of harmonic oscillators with continuous spectrum and $\Gamma^{0}_{\varkappa\varkappa} = 0$ (a particular kind of interaction Hamiltonian with respect to the displacement operators of medium particles). In this case in work [4] calculation of correlation function $\langle b_{\varkappa}(t) b^{+}_{\varkappa}(0) \rangle_{\omega}$ was carried out at computer by means of formulas (29), (12) for pure non-linear interaction $(\Gamma_{\varkappa} = 0)$. It is shown, that correlation function is described by symmetric curve which has essentially non-Lorentz shape, even for the case of harmonic oscillator $(\widetilde{V}_{\varkappa\varkappa} = 0)$ (compare with formula (7)).

In time representation, it is possible to obtain equation for function $\Phi(y, t)$ in the same way as in case of pure linear interaction. But consideration of quadratic interaction leads to equation for $\Phi(y, t)$, which unlike equation (16) will contain besides the first also the second derivatives with respect to $y_{\varkappa_1}$. This more complicated equation will not be considered here. It is convenient to present the spectral representation of correlation function of oscillator $\varkappa$

occupation numbers as $(\omega \approx 0)$

$$\langle \hat{n}_{\varkappa}(t) \tilde{\hat{n}}_{\varkappa}(0) \rangle_{\omega} = -\frac{1}{\pi} \operatorname{Im} \sum_{m} m_{\varkappa} \, g'_{\varkappa}(m, \omega + i\varepsilon) \, ;$$

$$g'_{\varkappa}(m, \omega + i\varepsilon) = (\rho_0)^{m}_{m} \sum_{m'} \widetilde{m}'_{\varkappa} \, [\bar{\widetilde{G}}(\omega + i\varepsilon)]^{m'm'}_{mm} \, . \qquad (30)$$

The system of equations for the function $g'_{\varkappa}(m, \omega + i\varepsilon)$ may be obtained by means of expressions (I.47),(2),(23). The result has the form in actual frequency range $\omega \approx 0$

$$[\omega + i\widetilde{\Gamma}'_{\varkappa}(m)] g'_{\varkappa}(m, \omega + i\varepsilon) + i \sum_{j=\pm 1} \sum_{\varkappa_1} \gamma'^{j}_{\varkappa\varkappa_1}(m) \cdot$$

$$\cdot e^{\lambda \omega_{\varkappa_1} j} g'_{\varkappa}(m + j\delta_{\varkappa_1\varkappa'}, \omega + i\varepsilon) + i \sum_{\substack{j_1, j_2 = \pm 1 \\ (j_1 \geqslant j_2)}} \sum_{\varkappa_1 \varkappa_2} \gamma'^{j_1 j_2}_{\varkappa\varkappa_1\varkappa_2}(m) \cdot \qquad (31)$$

$$\cdot e^{\lambda(j_1 \omega_{\varkappa_1} + j_2 \omega_{\varkappa_2})} g'_{\varkappa}(m + j_1 \delta_{\varkappa_1\varkappa'} + j_2 \delta_{\varkappa_2\varkappa'}, \omega + i\varepsilon) = (m_{\varkappa} - \bar{n}_{\varkappa})(\rho_0)^{m}_{m} \, .$$

Here magnitudes $\widetilde{\Gamma}'_{\varkappa}(m) = \Gamma'^{(1)}_{\varkappa}(m) + \Gamma'^{(2)}_{\varkappa}(m)$, $\gamma'^{j}_{\varkappa\varkappa_1}(m)$, $\gamma'^{j_1 j_2}_{\varkappa\varkappa_1\varkappa_2}(m)$ are defined by formulas (5),(25) for $\Gamma^{(1)}_{\varkappa}(m)$, $\Gamma^{(2)}_{\varkappa}(m)$, $\gamma^{j}_{\varkappa\varkappa_1}(m)$, $\gamma^{j_1 j_2}_{\varkappa\varkappa_1\varkappa_2}(m)$ respectively, in which it is necessary to put $\delta_{\varkappa\varkappa'} = 0$.

The system of equations (31) may be solved also as the system (24) by means of computer with breaking it at any equation.

In classical limit of high temperatures the system of equations (31) may be written as a differential equation (for one oscillation $\varkappa$)

$$(\omega - 2i\Gamma_{\varkappa} - 8iX_0 \Gamma^{+}_{\varkappa\varkappa}) g'_{\varkappa}(X_0, \omega + i\varepsilon) - 2i[(X_0 + \frac{1}{\lambda\omega_{\varkappa}})\Gamma_{\varkappa} +$$

$$+2X_0(X_0+\frac{2}{\lambda\omega_\varkappa})\Gamma^+_{\varkappa\varkappa}]\frac{dg'_\varkappa(X_0,\omega+i\varepsilon)}{dX_0}-2i\frac{X_0}{\lambda\omega_\varkappa}(\Gamma_\varkappa+$$

$$+2X_0\Gamma^+_{\varkappa\varkappa})\frac{d^2g'_\varkappa(X_0,\omega+i\varepsilon)}{dX_0^2}=(\lambda\omega_\varkappa X_0-1)e^{-\lambda\omega_\varkappa X_0}; \quad (32)$$

$$g'_\varkappa(-X_0,\omega+i\varepsilon)=g'_\varkappa(\infty,\omega+i\varepsilon)=0$$

The correlation function of occupation number, as it follows from (30), is defined in classical case by the equation

$$\langle\hat{n}_\varkappa(t)\tilde{\hat{n}}_\varkappa(0)\rangle_\omega=-\frac{1}{\pi}Im\int_0^\infty X g'_\varkappa(X,\omega+i\varepsilon)dX .$$

The equation of type (32) is solved in work /4/ numerically for different values of parameters. The obtained spectral distribution of function $\langle\hat{n}_\varkappa(t)\tilde{\hat{n}}_\varkappa(0)\rangle_\omega$ differs from the Lorentz distribution which takes place in case of pure linear interaction (see (21)), the stronger the larger is contribution of non-linear interaction $(\Gamma^+_{\varkappa\varkappa})$.

Taking into account non-linear interaction the non-Lorentz character of distribution of correlation function $\langle\hat{n}_\varkappa(t)\tilde{\hat{n}}_\varkappa(0)\rangle_\omega$ is seen also from kinetic equation for the function $g'_\varkappa(m,t)$. The quadratic interaction leads to addend in the right part of equation (19)

$$-\Gamma'^{(2)}_\varkappa(m)g'_\varkappa(m,t)-\sum_{\substack{j_1,j_2=\pm1\\(j_1\geqslant j_2)}}\sum_{\varkappa_1\varkappa_2}\gamma'^{j_1j_2}_{\varkappa\varkappa_1\varkappa_2}(m)\cdot \quad (33)$$

$$\cdot g'_\varkappa(m+j_1\delta_{\varkappa_1\varkappa'}+j_2\delta_{\varkappa_2,\varkappa'},t).$$

Now the correlation function $N^{10}_{\varkappa\varkappa}(t)$ does not satisfy the equation of type (20), and thus the distribution is not Lorentzian.

Let us note, that modulational broadening, defined by the function $\varphi^0_\omega(0)$, falls out of equations (31),(32),(33) for correlation function of occupation numbers.

4. Conclusion

The general theory, developed in work /I/, gives a possibility of calculating the correlation functions, which determine the dynamics of a particle weakly interacting with arbitrary medium. The obtained formulas permit one to get the results in any order of perturbation theory in Hamiltonian interaction H_{int} (see expansion (I.15), and also to consider any power of the displacement operator $\vec{u}_0$ of the singled out particle in terms, describing non-linearity of the considered oscillator, as well as in Hamiltonian H_{int}.

Transition to a definite (with respect to power $\vec{u}_0$) interaction of oscillator with the medium fluctuations is easily performed by means of expansion of the magnitude $e^{\vec{u}_0\vec{\nabla}_0}-1$ in general formulas to needed power of $\vec{u}_0$. As it is shown in the present work, the consideration of definite interaction mechanisms permits one to investigate the dynamics of non-linear oscillator in details. For every interaction mechanism there appears its own set of parameters expressed through the correlation functions for the medium potential energy. For example, we have the magnitude $\Gamma_\varkappa$

for linear interaction (see (6)), whereas $\Gamma^{\pm}_{\varkappa\varkappa_1}$ for quadratic interaction (see (28)). These magnitudes may be calculated at any definite medium that will make possible to obtain the dependence of the oscillator spectral distribution from the medium parameters (temperature and so on).

For example, to pass to the case of medium as a set of harmonic oscillators of continuous spectrum, which was considered in work /2-4/, it is necessary in formulas of the present work to carry out the substitutions for linear and quadratic interactions respectively

$$|\vec{\alpha}_{\varkappa}\vec{\nabla}_0|^2 \varphi^0_{\omega}(\omega_{\varkappa}) \longrightarrow \sum_{K} V^2_{\varkappa K}\left[(1+\bar{n}_{\varkappa})\delta(\omega_{\varkappa}-\omega_K) + \right.$$
$$\left. + \bar{n}_{\varkappa}\delta(\omega_{\varkappa}+\omega_K)\right];$$
$$\frac{1}{4}|\vec{\alpha}_{\varkappa_1}\vec{\nabla}_0|^2|\vec{\alpha}_{\varkappa_2}\vec{\nabla}_0|^2\varphi^0_{\omega}(\omega_{\varkappa_1}\pm\omega_{\varkappa_2}) \longrightarrow \sum_{K} V^2_{\varkappa_1\varkappa_2 K}\left[(1+ \right. \quad (34)$$
$$\left. +\bar{n}^{\pm}_{\varkappa_1\varkappa_2})\delta(\omega_{\varkappa_1}\pm\omega_{\varkappa_2}-\omega_K) + \bar{n}^{\pm}_{\varkappa_2\varkappa_1}\delta(\omega_{\varkappa_1}\pm\omega_{\varkappa_2}+\omega_K)\right],$$

where ω_K is the oscillator frequency of the continuous spectrum, $V_{\varkappa K}$, $V_{\varkappa_1\varkappa_2 K}$ are the constants of linear and quadratic interaction of the considered oscillator with the continuous spectrum. The result (34) is obtained by means of expansion in the medium correlation functions (I.20), (I.21) of the potential energy $\tilde{U}(\vec{\ell}_0, \vec{R}_1(t), \dots, \vec{R}_N(t))$ or $\tilde{\varphi}(\vec{\ell}_0, \vec{R}_\ell(t))$ in power series in displacements of medium particles $\vec{u}_n$ up to the linear term. The consideration of the next terms of expansion in $\vec{u}_n$ permits one to examine the interaction involving more then one (in contrast with (34)) quantum of medium oscillations.

Let us note, that the results of the present work formally coincide with these of work /2-4/ for the corresponding particular cases, considered in these works. The coincidence becomes actual, if we pass to the set of harmonic oscillators of continuous spectrum as a medium by means of the procedure described above.

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