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THE SUPEROPERATOR METHOD IN THE THEORY OF OSCILLATOR WEAKLY INTERACTING WITH MEDIUM.

I. General Results

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I. Introduction

The theory of the harmonic oscillator, whose weak interaction with medium is described by the expression linear in oscillator coordinates (or displacement operators), is developed in a number of works /1-5/. Classical and quantum theory of nonlinear oscillators interacting with medium by means of the linear and quadratic (on oscillator displacements) friction is constructed in works /6-8/. The system of harmonic oscillators belonging to continuous spectrum was considered as a medium model in works /6-8/.

A new method of constructing the weakly interacting with medium oscillator quantum theory based on the use of superoperators /9/ is presented in this work. By this method, it is possible (as it will be seen below) to construct the theory of the weakly bound oscillator with arbitrary power of the oscillator nonlinearity in its displacement operator (nonlinearity, however, is supposed to be small in comparison with the oscillator frequency) and taking into account any power of the oscillator displacement operator in the interaction Hamiltonian of the oscillator with medium. The medium and its spectral properties are supposed to be rather general.

It is convenient to carry out the investigation of dynamics of the singled out particle by means of the correlation functions for its displacement operators. Below we will obtain the general formulas for spectral representation $\langle U_0^{\alpha}(t) U_0^{\beta}(0)\rangle_{\omega}$ of the correlation function $\langle U_0^{\alpha}(t) U_0^{\beta}(0)\rangle$ ($\vec{U}_0(t)$ is the displacement operator of the singled out particle in the Heisenberg representation; α, β denote Cartesian coordinates x, y, z; $\langle \ldots \rangle$ denotes quantum-statistical averaging with full Hamiltonian). The consideration is carried out under the supposition of fast medium relaxation with respect to the relaxation frequency of the singled out particle due to its weak bond with the medium. This supposition permits one to consider the particle (in zero approximation) as being in some average field produced by the medium particles. The interaction of the singled out particle with the average field fluctuations is considered as a perturbation. The perturbation influence on the particle dynamics is described by spectral representations of the correlation functions for the medium potential energy.

The oscillation spectral distribution of the singled out particle is investigated by calculating spectral representation $\langle b_{\varkappa}(t) b_{\varkappa}^{+}(0)\rangle_{\omega}$ of the correlation function for normal oscillation of number $\varkappa$ ($b_{\varkappa}(t)$, $b_{\varkappa}^{+}(t)$ are annihilation and creation operators of oscillator $\varkappa$ in the Heisenberg representation) taking into consideration the presence of other modes of particle oscillations. The

general formulas are derived determining this correlation function. It is shown that the spectral distribution of the singled out oscillator may have the fine structure due to oscillator nonlinearity. The fine structure line widths are conditioned by the oscillator interaction with the medium fluctuations and expressed by the medium correlation functions.

The general formulas are also found, determining the spectral representation of the correlation function for oscillator $\varkappa$ occupation number $\langle \hat{n}_{\varkappa}(t) \tilde{\hat{n}}_{\varkappa}(0)\rangle_{\omega}$

$$(\hat{n}_{\varkappa} = b_{\varkappa}^{+} b_{\varkappa},\ \tilde{\hat{n}}_{\varkappa} = \hat{n}_{\varkappa} - \langle \hat{n}_{\varkappa}\rangle).$$

The quantum kinetic equation determining the time correlation functions $\langle U_0^{\alpha}(t) U_0^{\beta}(0)\rangle$, $\langle b_{\varkappa}(t) b_{\varkappa}^{+}(0)\rangle$, $\langle \hat{n}_{\varkappa}(t) \tilde{\hat{n}}_{\varkappa}(0)\rangle$ of the particle interacting with medium is obtained.

All obtained results are true for the examined general case when any power of the displacement operator of the singled out particle $\vec{U}_0$ is taken into consideration in the Hamiltonian.

The present theory may be applied to various physical situations. For example, if the singled out particle is an impurity atom in crystal, then obtained formulas describe local or quasilocal oscillations in crystals. Other applications are also possible, for example, in the theory of lasers and so on.

2. The general formulas determining the oscillator dynamics

Let's consider the Hamiltonian for the system, consisting of the singled out particle and medium, of the following type

$$H = \frac{P_o^2}{2M_o} + \sum_{\ell \neq 0} \frac{P_\ell^2}{2M_\ell} + U(\vec{R}_o, \vec{R}_1, \dots, \vec{R}_N) \,. \quad (1)$$

Here $\vec{P}_o$, M_o are, respectively, the momentum and mass of the singled out particle; $\vec{P}_\ell$, M_ℓ are the momentum and mass of an ℓ particle of medium consisting of N particles; U is the potential energy of all particles of the system; $\vec{R}_n$ are the radius-vectors of particles $(n=0,1,\dots,N)$.

Let's define the displacement vectors of particles from the equilibrium positions in the following way

$$\vec{R}_n = \langle \vec{R}_n \rangle + \vec{u}_n \equiv \vec{\ell}_n + \vec{u}_n \,,$$

where $\vec{\ell}_n \equiv \langle \vec{R}_n \rangle$ are equilibrium positions of particles.

Let's put down the system potential energy as an infinite series of displacements of singled out particle $\vec{u}_o$ as

$$U(\vec{R}_o, \vec{R}_1, \dots, \vec{R}_N) = e^{\vec{u}_o \vec{\nabla}_o}\, U(\vec{\ell}_o, \vec{R}_1, \dots, \vec{R}_N),$$

where $\vec{\nabla}_o$ denotes gradient operator with respect to particle coordinates. Now it is convenient to rewrite (1) in the form

$$\begin{aligned}
H &= H_o + H_{med} + H_{int}\,; \\
H_o &= \frac{P_o^2}{2M_o} + \left(e^{\vec{u}_o \vec{\nabla}_o} - 1\right) \langle U(\vec{\ell}_o, \vec{R}_1, \dots, \vec{R}_N) \rangle_{med}\,; \\
H_{med} &= \sum_{\ell \neq 0} \frac{P_\ell^2}{2M_\ell} + U(\vec{\ell}_o, \vec{R}_1, \dots, \vec{R}_N)\,; \qquad (2) \\
H_{int} &= \left(e^{\vec{u}_o \vec{\nabla}_o} - 1\right) \left[U(\vec{\ell}_o, \vec{R}_1, \dots, \vec{R}_N) - \langle U(\vec{\ell}_o, \vec{R}_1, \dots, \vec{R}_N) \rangle_{med} \right].
\end{aligned}$$

Here $\langle \dots \rangle_{med} \equiv Sp\, \rho_{med} (\rho_{med} \dots)$ denotes averaging over the Hamiltonian H_{med} states, $\lambda = \frac{1}{k_B T}$, $\rho_{med} = \left(Sp\, e^{-\lambda H_{med}}\right)^{-1} e^{-\lambda H_{med}}$. Hamiltonian H_o describes the singled out particle, which is in the mean field, which is determined by the averaged over the medium states potential energy of the particle-medium interaction. Hamiltonian H_{med}, in fact, describes the medium, which the considered particle interacts with. Hamiltonian H_{int} describes the interaction of the singled out particle with the field fluctuations operating on the particle.

In the case of pairing forces, when the system potential energy looks like

$$U(\vec{R}_o, \vec{R}_1, \dots, \vec{R}_N) = \sum_{\ell \neq 0} \varphi(\vec{R}_o, \vec{R}_\ell) + \frac{1}{2} \sum_{\ell \neq m \neq 0} \varphi(\vec{R}_\ell, \vec{R}_m),$$

putting down $\varphi(\vec{R}_o,\vec{R}_\ell)$ as

$$\varphi(\vec{R}_o,\vec{R}_\ell)=e^{\vec{u}_o\vec{\nabla}_o}\varphi(\vec{\ell}_o,\vec{R}_\ell),$$

we receive the following expressions for Hamiltonians

$$H_o=\frac{P_o^2}{2M_o}+(e^{\vec{u}_o\vec{\nabla}_o}-1)\sum_{\ell\neq 0}\langle\varphi(\vec{\ell}_o,\vec{R}_\ell)\rangle_{med};$$

$$H_{med}=\sum_{\ell\neq 0}\frac{P_\ell^2}{2M_\ell}+\frac{1}{2}\sum_{\ell\neq m\neq 0}\varphi(\vec{R}_\ell,\vec{R}_m)+\sum_{\ell\neq 0}\varphi(\vec{\ell}_o,\vec{R}_\ell); \tag{3}$$

$$H_{int}=(e^{\vec{u}_o\vec{\nabla}_o}-1)\sum_{\ell\neq 0}\left[\varphi(\vec{\ell}_o,\vec{R}_\ell)-\langle\varphi(\vec{\ell}_o,\vec{R}_\ell)\rangle_{med}\right].$$

Let's consider now the spectral representation of the correlation function for the displacement operator $\vec{u}_o(t)$, which describes the dynamics of the singled out particle. It is convenient to express the correlation function through retarded and advanced Green functions by the following known formula

$$\langle u_o^\alpha(t)u_o^\beta(0)\rangle_\omega=i(1-\eta e^{-\lambda\omega})^{-1}\left[G^r_{\alpha\beta}(\omega)-G^a_{\alpha\beta}(\omega)\right], \tag{4}$$

where

$$\langle u_o^\alpha(t)u_o^\beta(0)\rangle_\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}\langle u_o^\alpha(t)u_o^\beta(0)\rangle e^{i\omega t}dt,\ \eta=\pm 1.$$

Fourier-components of retarded $G^r_{\alpha\beta}(\omega)$ and advanced $G^a_{\alpha\beta}(\omega)$ Green functions may be represented in the following way

$$G^r_{\alpha\beta}(\omega)=-\frac{i}{2\pi}\int_0^{\infty}e^{i(\omega+i\varepsilon)t}\langle\{u_o^\alpha(t),u_o^\beta(0)\}_\eta\rangle dt;$$

$$G^a_{\alpha\beta}(\omega)=\frac{i}{2\pi}\int_{-\infty}^{0}e^{i(\omega-i\varepsilon)t}\langle\{u_o^\alpha(t),u_o^\beta(0)\}_\eta\rangle dt. \tag{5}$$

Here the $\{A,B\}_\eta=AB-\eta BA$ is introduced and also $\hbar=1$, $\varepsilon\to+0$ are put.

Let's introduce Liouville superoperator $\hat{L}$ by the definition

$$\hat{L}A=HA-AH, \tag{6}$$

where A is a usual quantum-mechanical operator, H is the Hamiltonian of the system. As far as the operator time dependence in the Heisenberg representation can be written via $\hat{L}$ as:

$$e^{iHt}Ae^{-iHt}=e^{i\hat{L}t}A, \tag{7}$$

then the correlation function (4) after the substitution into it the expressions (5) and time integration will take the form

$$\langle u_o^\alpha(t)u_o^\beta(0)\rangle_\omega=\frac{i}{2\pi}(1-\eta e^{-\lambda\omega})^{-1}\langle\{u_o^\alpha,[\hat{G}(\omega+i\varepsilon)-\hat{G}(\omega-i\varepsilon)]u_o^\beta\}_\eta\rangle. \tag{8}$$

Here Green superoperator $\hat{G}(\omega)$ is introduced according to

$$\hat{G}(\omega)=\frac{1}{\omega-\hat{L}}.$$

Let's suppose further, that the temperature satisfies inequality

$$K_B T\gg\omega_{int}, \tag{9}$$

where ω_{int} is of an order of the relaxation frequency of the singled out oscillator, conditioned by interaction H_{int}. The restriction (9) is not essential for the particle weakly bound with medium.

The supposition (9) permits one to write down the density matrix of system $\rho = (Sp\, e^{-\lambda H})^{-1} e^{-\lambda H}$ as

$$\rho \approx \rho_o \rho_{med}, \qquad (10)$$

where

$$\rho_o = (Sp\, e^{-\lambda H_o})^{-1} e^{-\lambda H_o}.$$

Let's note, that neglecting H_{int} for the density matrix while keeping H_{int} for time-dependent factors of (7) type, doesn't influence the dynamics of the considered oscillator.

Taking into consideration (10) the expression for the correlation function (8) may be rewritten in the following way

$$\langle u_o^\alpha(t) u_o^\beta(0)\rangle_\omega = \frac{i}{2\pi}(1-\eta e^{-\lambda\omega})^{-1}\langle\{u_o^\alpha, [\bar{\hat{G}}(\omega+i\varepsilon) - \bar{\hat{G}}(\omega-i\varepsilon)]\, u_o^\beta\}_\eta\rangle_o . \qquad (11)$$

Here $\langle\ldots\rangle_o = Sp\rho_o(\rho_o\ldots)$ is averaging over the states of Hamiltonian H_o. Averaged Green superoperator is also introduced into the expression (11)

$$\bar{\hat{G}}(\omega) = P\frac{1}{\omega - \hat{L}}P, \qquad (12)$$

where $P \equiv \langle\ldots\rangle_{med}$. Operator P possesses all the properties of the projecting operator, for example, $P^2 = P$. It is possible to show /10/ that in general case the following exact expressions for averaged Green superoperator take place

$$\bar{\hat{G}}(\omega) = \frac{P}{\omega - P\hat{L}P - \hat{M}(\omega)};$$

$$\hat{M}(\omega) = P\hat{L}_{int}Q\frac{1}{\omega - Q\hat{L}Q}Q\hat{L}_{int}P, \qquad (13)$$

where $Q = 1 - P$.

While putting down the formula for "mass" superoperator $\hat{M}(\omega)$ (which in general case keeps the superoperator $\hat{L}$ instead of $\hat{L}_{int}$) one takes into consideration, that $\hat{L}_{med}P = 0$, $P\hat{L}_oQ = 0$, $Q\hat{L}_oP = 0$ ($\hat{L} = \hat{L}_o + \hat{L}_{med} + \hat{L}_{int}$ respectively to Hamiltonian (2) terms and definition (6)), and also trace invariance under the cycle permutation of operators.

Taking into consideration, that for Hamiltonian (2) or (3) $P\hat{L}_{int}P = 0$, for the averaged Liouville superoperator $P\hat{L}P$, which according to the expression (13) defines the oscillation frequency spectrum of the considered particle, we have

$$P\hat{L}P = P\hat{L}_oP . \qquad (14)$$

"Mass" superoperator $\hat{M}(\omega \pm i\varepsilon)$ defines oscillator frequency shift (the real part $\hat{M}(\omega \pm i\varepsilon)$) and broadening of the spectrum line (imaginary part), conditioned by interaction H_{int} . It may be expanded in the infinite power series over $\hat{L}_{int}$ as

$$\hat{M}(\omega) = P\hat{L}_{int}\sum_{n=1}^{\infty}\left[Q\frac{1}{\omega - \hat{L}_o - \hat{L}_{med}}Q\hat{L}_{int}\right]^n P. \qquad (15)$$

Let's suppose, that the characteristic frequency of medium relaxation ω_{med} , which is determined by

Hamiltonian H_{med}, is considerably larger than the oscillator relaxation frequency ω_{int}, determined by H_{int}, i.e.,

$$\omega_{med} \gg \omega_{int} . \qquad (16)$$

While fulfilling the condition (16), which is quite realizable for weakly bound particle, it is possible to consider the particle (in zero approximation) in the mean field, arising from averaged over medium states interaction with medium. It is consistent with the Hamiltonian definition H_o in formulas (2) or (3). Besides, in this case the expansion (15) for $\hat{M}(\omega)$ may be considered as a perturbation series over $\hat{L}_{int}$ $(i.e.\ H_{int})$ in a further considered frequency range ω near oscillator frequences, determined by the expression (14), and it is possible to calculate the "mass" superoperator with any degree of accuracy in interaction.

Let's restrict further consideration by second order perturbation theory for the "mass" superoperator (n=1 in (15)). In this approximation "mass" superoperator $\hat{M}(\omega+i\varepsilon)$ in time representation may be given as

$$\hat{M}(t) = \frac{2\pi}{i}\theta(t)\,P\hat{L}_{int}(0)\,e^{-i\hat{L}_o t}\,\hat{L}_{int}(-t)P, \qquad (17)$$

where $\theta(t)=1$ for $t>0$, $\theta(t)=0$ for $t<0$, $\hat{L}_{int}(-t)=e^{-i\hat{L}_{med}t}\hat{L}_{int}$.
Calculating $\hat{M}(t)$ by (17), using rule (6) and H_{int} (2), and then turning to the Fourier representation, we get

$$\hat{M}(\omega\pm i\varepsilon) = \int_{-\infty}^{\infty} d\omega' P(\hat{e}^{\vec{u}_o\vec{\nabla}_o}-1)_{\omega'}\cdot \qquad (18)$$

$$\cdot\frac{1}{\omega-\hat{L}_o-\omega'\pm i\varepsilon}(\hat{e}^{\vec{u}_o\vec{\nabla}_o}-1)_o\,\varphi^o_\omega(\omega')\,P.$$

The rule of action of the superoperator $(\hat{e}^{\vec{u}_o\vec{\nabla}_o}-1)_\omega$ on arbitrary operator A is defined in the following way

$$(\hat{e}^{\vec{u}_o\vec{\nabla}_o}-1)_\omega A = (e^{\vec{u}_o\vec{\nabla}_o}-1)A - e^{-\lambda\omega}A(e^{\vec{u}_o\vec{\nabla}_o}-1). \qquad (19)$$

As it is seen from equation (18) "mass" superoperator $\hat{M}(\omega)$ for the correlation function of the singled out and weakly bound particle is defined through function $\varphi^o_\omega(\omega)$, which is the Fourier component of the time correlation function $\varphi^o(t)$ for the medium potential energy

$$\varphi^o(t) = \langle \tilde{U}(\vec{\ell}_o,\vec{R}_1(t),\dots,\vec{R}_N(t))\,\tilde{U}(\vec{\ell}_o,\vec{R}_1(0),\dots,\vec{R}_N(0))\rangle_{med}, \qquad (20)$$

where $\tilde{U}(\vec{\ell}_o,\vec{R}_1,\dots,\vec{R}_N) = U(\vec{\ell}_o,\vec{R}_1,\dots,\vec{R}_N) - \langle U(\vec{\ell}_o,\vec{R}_1,\dots,\vec{R}_N)\rangle_{med}$, $\vec{R}_n(t) = exp(i\hat{L}_{med}t)\vec{R}_n$. Gradient operator $\vec{\nabla}_o$, entering into one of the superoperators $(\hat{e}^{\vec{u}_o\vec{\nabla}_o}-1)_{\omega'}$, acts on one of the functions in (20), and $\vec{\nabla}_o$ in other superoperator acts on the other function $\tilde{U}(\vec{\ell}_o,\dots)$.
As it is seen from the corresponding equations, the superoperators (14) and (18) act only on the operators of the considered oscillator.

In the case of the pairing forces an analogous consideration with the use of the Hamiltonian (3) leads to the equation for $\hat{M}(\omega)$ the same as (18) but with the

Fourier component of the function

$$\varphi_2^o(t) = \sum_{\ell,\ell'\neq 0} < \tilde{\varphi}(\vec{\ell}_o, \vec{R}_\ell(t)) \tilde{\varphi}(\vec{\ell}_o, \vec{R}_{\ell'}(0)) >_{med} \tag{21}$$

as the substitution for $\varphi_\omega^o(\omega')$,
where $\tilde{\varphi}(\vec{\ell}_o, \vec{R}_\ell) = \varphi(\vec{\ell}_o, \vec{R}_\ell) - < \varphi(\vec{\ell}_o, \vec{R}_\ell) >_{med}$.

Extraction of real $\hat{P}(\omega)$ and imaginary $\hat{\Gamma}(\omega)$ parts of "mass" superoperator $\hat{M}(\omega \pm i\varepsilon)$ is easily performed by means of the known identity

$$\frac{1}{\omega \pm i\varepsilon} = \left(\frac{1}{\omega}\right)_{pr} \mp i\pi\delta(\omega), \tag{22}$$

where $(\ldots)_{pr}$ denotes the principal value of integral.

Formulas (11)-(15),(18)-(21) give the complete and strict (under conditions (9), (16)) solution to the problem on spectrum of the singled out particle, weakly interacting with arbitrary medium by means of interaction with any power of displacement operator $\vec{u}_o$.

As it is seen from equation (11), to find the oscillator displacement correlation function it is necessary to calculate trace over oscillator states. For this purpose it is necessary to have superoperator $\bar{\hat{G}}(\omega)$ matrix elements. The superoperator matrix element has four indices unlike the usual operator. It follows from the definition (6) according to which the superoperator acts on the usual operator (and not on the state vector, as the operator) giving the operator, as a result. Thus, if $C = \hat{L}A$ (C, A are operators), then we have for matrix elements

$$C_K^\ell = \sum_{K'\ell'} \hat{L}_{K\ell}^{K'\ell'} A_{K'}^{\ell'} . \tag{23}$$

To find superoperator $\bar{\hat{G}}(\omega)$ matrix elements it is sufficient to have matrix elements of the superoperator $\hat{G}^{-1}(\omega) = \omega - P\hat{L}P - \hat{M}(\omega)$, that is inverse to $\bar{\hat{G}}(\omega)$. Then matrix elements of the $\bar{\hat{G}}(\omega)$ are defined from the system of equations

$$\sum_{n_1 m_1} (\hat{G}^{-1})_{nm}^{n_1 m_1} \bar{\hat{G}}_{n_1 m_1}^{n'm'} = \delta_{nn'}\delta_{mm'}, \tag{24}$$

where n, m denote some complete set of functions corresponding to Hamiltonian H_0.

By multiplying both the parts of the equality (24) by matrix element $B_{n'}^{m'}$ of some operator B acting in space of functions n, m, we receive after summation of (24) over n', m' the following (convenient for the further) relation

$$\sum_{n_1 m_1} (\hat{G}^{-1})_{nm}^{n_1 m_1} \sum_{n'm'} \bar{\hat{G}}_{n_1 m_1}^{n'm'} B_{n'}^{m'} = B_n^m . \tag{25}$$

Superoperator $\hat{G}^{-1}(\omega)$ matrix elements may be defined by formulas (6), (14), (18), (19), (23).

Let's consider now the time correlation function for oscillator displacement

$$< u_o^\alpha(t) u_o^\beta(0) > = < u_o^\alpha e^{-i\hat{L}t} u_o^\beta > .$$

Taking into consideration (10) we get

$$< u_o^\alpha(t) u_o^\beta(0) > = < u_o^\alpha P e^{-i\hat{L}t} P u_o^\beta >_o . \tag{26}$$

It is easy to see that the averaged Green superoperator $\bar{\hat{G}}(\omega+i\varepsilon)$, defining the Fourier representation of the correlation function (26) (see (11)), is the Fourier component of the superoperator

$$\bar{\hat{G}}(t)=\frac{2\pi}{i}\theta(t)\,P e^{-i\hat{L}t}P . \qquad (27)$$

Let's introduce the function

$$g(n,m;t)=\sum_{n'm'}\left(P e^{-i\hat{L}t}P\right)_{nm}^{n'm'}B_{n'}^{m'}, \qquad (28)$$

which according to equation (26) defines the time correlation function for displacment with $B=u_o^{\beta}$.

Then from equation (27) for the Fourier component of the $\bar{\hat{G}}(\omega+i\varepsilon)$ we have

$$\theta(t)g(n,m;t)=\frac{i}{2\pi}\int_{-\infty}^{\infty}e^{-i\omega t}\sum_{n'm'}\left[\bar{\hat{G}}(\omega+i\varepsilon)\right]_{nm}^{n'm'}B_{n'}^{m'}d\omega . \qquad (29)$$

Differentiating eq. (29) with respect to time and taking into consideration the relation (25), we receive the equation for function $g(n,m;t)$ for large times $t\sim\omega_{int}^{-1}\gg\omega_{med}^{-1}$

$$i\frac{\partial}{\partial t}g(n,m;t)=\sum_{n_1m_1}\left[P\hat{L}_oP+\hat{M}(\omega_i+i\varepsilon)\right]_{nm}^{n_1m_1}g(n_1,m_1;t). \qquad (30)$$

Here "mass" superoperator $\hat{M}(\omega+i\varepsilon)$ is taken on one of the peak frequencies ω_i of spectral distribution $\langle A(t)B(o)\rangle_\omega$ for the operators of the considered oscillator,

as far as at large times $t\gg\omega_{med}^{-1}$ the correlation function $\langle A(t)\,B(o)\rangle$ describes peak region of the spectral distribution. Peak frequencies are determined by the matrix element of the superoperator $P\hat{L}_oP$ and it is supposed that these frequencies do not coincide and aren't close to each other.

While receiving equation (3o) it was taken into consideration the initial condition

$$g(n,m;o)=B_n^m , \qquad (31)$$

which follows from the definition (28).

Equation (30) plays the role of a quantum kinetic equation and defines the oscillator correlation function (26) in general case for the system described by Hamiltonian (2). "Mass" superoperator $\hat{M}(\omega+i\varepsilon)$ is defined by the expansion (15) in any perturbation order over interaction Hamiltonian in case of the fast medium relaxation relative to oscillator relaxation frequency. In the second order on H_{int} $\hat{M}(\omega+i\varepsilon)$ (in 30)) is defined by formula (18).

3. Some general results for the spectrum of singled out oscillator.

Let's consider the case, when Hamiltonian H_o has the inversion centre. Let's expand the displacement of particle $\vec{u}_o$ in normal coordinates of singled out oscillators, which diagonalise quadratic in $\vec{u}_o$ part of the

Hamiltonian H_o (together with the term $\frac{P_o^2}{2M_o}$), by the formula

$$\vec{U}_o = \sum_{\varkappa} (\vec{\alpha}_{\varkappa} b_{\varkappa} + \vec{\alpha}^*_{\varkappa} b^+_{\varkappa}), \qquad (32)$$

where $\varkappa$ numbers singled out oscillators, and $b_{\varkappa}, b^+_{\varkappa}$ obey the Bose commutation rules and are annihilation and creation operators for $\varkappa$ oscillation. The operator of momentum $\vec{P}_o$ is expanded in $b_{\varkappa}, b^+_{\varkappa}$ analogically.

Then, omitting "non-resonance" terms of $b^+_{\varkappa} b^3_{\varkappa}$ type and so on (containing unequal number of $b_{\varkappa}$ and $b^+_{\varkappa}$ operators), which lead to higher-order corrections, we receive the following form for H_o

$$H_o = \sum_{\varkappa} \omega_{\varkappa} \hat{n}_{\varkappa} + \frac{1}{2} \sum_{\varkappa\varkappa'} V_{\varkappa\varkappa'} \hat{n}_{\varkappa} \hat{n}_{\varkappa'} + \sum_{\varkappa\varkappa'\varkappa''} V_{\varkappa\varkappa'\varkappa''} \hat{n}_{\varkappa} \hat{n}_{\varkappa'} \hat{n}_{\varkappa''} + \ldots \qquad (33)$$

Here $\omega_{\varkappa}$ is $\varkappa$ oscillator frequency, $\hat{n}_{\varkappa} = b^+_{\varkappa} b_{\varkappa}$; $V_{\varkappa\varkappa'}, V_{\varkappa\varkappa'\varkappa''}$ are nonlinear constants (for example $V_{\varkappa\varkappa'} = |\vec{\alpha}_{\varkappa}\vec{\nabla}_o|^2 |\vec{\alpha}_{\varkappa'}\vec{\nabla}_o|^2 \langle U(\vec{R}_o, \vec{R}_1, \ldots, \vec{R}_N)\rangle_{med}$). Let's note, that later on it is not important to what nonlinearity order the series (33) is considered, but let's assume nonlinearity to be small in comparison with $\omega_{\varkappa}$.

Let's consider the correlation function $\langle b_{\varkappa}(t) b^+_{\varkappa}(0)\rangle_{\omega}$ for creation and annihilation operators of singled out oscillator $\varkappa$. Taking into consideration (11) and (32) this function may be given as

$$\langle b_{\varkappa}(t) b^+_{\varkappa}(0)\rangle_{\omega} = -\frac{1}{\pi} \frac{1-\eta e^{-\lambda\omega_{\varkappa}}}{1-\eta e^{-\lambda\omega}} \cdot \mathrm{Im} \langle b_{\varkappa} \bar{\hat{G}}(\omega+i\varepsilon) b^+_{\varkappa} \rangle_o . \qquad (34)$$

When receiving equation (34) it was taken into consideration, that for Hamiltonian (33)

$$b_{\varkappa} e^{-\lambda H_o} = e^{-\lambda\hat{\omega}_{\varkappa}} e^{-\lambda H_o} b_{\varkappa},$$

where $\hat{\omega}_{\varkappa} = \omega_{\varkappa} + \frac{1}{2} V_{\varkappa\varkappa} + \sum_{\varkappa'} V_{\varkappa\varkappa'} \hat{n}_{\varkappa'} + \ldots,$ and then nonlinearity terms are omitted from $\hat{\omega}_{\varkappa}$ because they are small, i.e., it is put $\hat{\omega}_{\varkappa} = \omega_{\varkappa}$.

The trace in formula (34) will be calculated with the eigenfunctions of the Hamiltonian H_o , which look like $\prod_{\varkappa} |n_{\varkappa}\rangle$, where $|n_{\varkappa}\rangle$ are eigenfunctions of operator $\hat{n}_{\varkappa}$ corresponding to eigenvalues $n_{\varkappa}$ $(n_{\varkappa} = 0, 1, \ldots)$. In this representation the correlation function (34) may be written in the following way

$$\langle b_{\varkappa}(t) b^+_{\varkappa}(0)\rangle_{\omega} = -\frac{1}{\pi} \frac{1-\eta e^{-\lambda\omega_{\varkappa}}}{1-\eta e^{-\lambda\omega}} \mathrm{Im} \sum_{m,m'} (\rho_o)^m_m \cdot (b_{\varkappa})^{m+\delta_{\varkappa\varkappa'}}_m \left[\bar{\hat{G}}(\omega+i\varepsilon)\right]^{m'+\delta_{\varkappa\varkappa',m'}}_{m+\delta_{\varkappa\varkappa',m}} (b^+_{\varkappa})^{m'}_{m'+\delta_{\varkappa\varkappa'}} . \qquad (35)$$

Here for short m and m' denote certain sets of numbers $n_{\varkappa}$, corresponding to eigenvalues of H_o , and summation runs over all such sets of $n_{\varkappa}$. The notation of $m \pm \delta_{\varkappa\varkappa'}$ type means a certain set of numbers $n_{\varkappa}$, as a set m , but in which the occupation number for oscillator $\varkappa$ is one unit larger or lesser in comparison with set m . While deriving equation (35), the known forms for matrix elements of the operators $b_{\varkappa}, b^+_{\varkappa}$ were also taken into account.

Let's consider now superoperator matrix elements, entering into $\hat{G}^{-1}(\omega)$, which determine superoperator matrix elements of $\hat{\bar{\bar{G}}}(\omega+i\varepsilon)$ according to relations (24), (25). The superoperator matrix element of $\hat{L}_o$, which determines the energy excitation of the oscillator (see (14)), in the chosen representation has the form

$$\hat{L}_{o\,nm}^{\;n'm'} = (\omega_n - \omega_m)\,\delta_{nn'}\,\delta_{mm'}\,; \tag{36}$$

$$\omega_n = \sum_{\varkappa} \omega_\varkappa n_\varkappa + \frac{1}{2}\sum_{\varkappa\varkappa'} V_{\varkappa\varkappa'} n_\varkappa n_{\varkappa'} + \sum_{\varkappa\varkappa'\varkappa''} V_{\varkappa\varkappa'\varkappa''} \cdot n_\varkappa n_{\varkappa'} n_{\varkappa''} + \dots$$

where n and m denote different sets of numbers $n_\varkappa$. Thus, $\hat{L}_o$ is diagonal in the given representation.

If we neglect "mass" superoperator $\hat{M}(\omega)$ in the equation (13) for $\hat{\bar{\bar{G}}}(\omega)$ (zero approximation over interaction H_{int}), then superoperator $\hat{G}^{-1}(\omega)$ is diagonal (taking into consideration that ω and $\hat{L}_o$ are diagonal). Then in this limiting case from equations (35), (25)(in (25) it is necessary to put $n = m + \delta_{\varkappa\varkappa'}$, $n' = m' + \delta_{\varkappa\varkappa'}$ and replace $B_{n'}^{m'}$ $(b_\varkappa^+)_{m'+\delta_{\varkappa\varkappa'}}^{m'} = \sqrt{m'_\varkappa + 1}$) and (36) we obtain the following equation for the correlation function

$$\langle b_\varkappa(t) b_\varkappa^+(0)\rangle_\omega = \frac{1}{Z_o}\sum_m e^{-\lambda\sum_{\varkappa'}\omega_{\varkappa'} m_{\varkappa'}} (1+m_\varkappa)\cdot$$
$$\cdot\,\delta(\omega - \omega_\varkappa(m))\,; \tag{37}$$
$$\omega_\varkappa(m) \equiv \omega_{m+\delta_{\varkappa\varkappa'}} - \omega_m = \omega_\varkappa + \frac{1}{2}V_{\varkappa\varkappa} + \sum_{\varkappa'} V_{\varkappa\varkappa'} m_{\varkappa'} + \dots$$

Nonlinear corrections to ω_m in ρ_o are omitted here and $Z_o = Sp\, e^{-\lambda H_o} = \prod_\varkappa (\bar{n}_\varkappa + 1)$, $\bar{n}_\varkappa = (e^{\lambda\omega_\varkappa} - 1)^{-1}$ is the mean occupation number of oscillator $\varkappa$.

It follows that in the considered limiting case the $\varkappa$-oscillator spectrum consists of a series of fine structure lines, which is produced by the oscillator nonlinearity. In case, when there is only one oscillator $\varkappa$, the spectrum consists of a set of line corresponding to different $m_\varkappa = 0, 1, \dots$. If there are several interacting singled out oscillations, then every line with a certain $m_\varkappa$ splits into a series of lines, corresponding to different $m_{\varkappa'}$. At low temperatures $(k_B T \ll \omega_{\varkappa'})$ there is practically only one line in the spectrum, corresponding $m_{\varkappa'} = 0$. With growing temperature the other spectrum lines begin to appear.

For linear oscillator $(V_{\varkappa\varkappa'}, V_{\varkappa\varkappa'\varkappa''}, \dots = 0)$ as it follows from equation (37), we have one spectrum line with $\omega = \omega_\varkappa$, that corresponds to equidistant energy levels of the linear oscillator (full spectrum degeneration).

Now we shall take into consideration $\hat{M}(\omega)$, describing the interaction of oscillator with medium fluctuations. Consideration of interaction H_{int} leads to broadening and shift of spectrum fine structure lines of the nonlinear oscillator. If broadening becomes larger (for example, when the temperature grows), then spectrum lines begin to overlap and fine structure will gradually disappear. At rather high temperature there may appear a unified spectrum distribution, in general case, of a complicated form, instead of fine structure.

So far as expression (18), for example, for $\hat{M}(\omega+i\varepsilon)$ may be given as

$$\hat{M}(\omega+i\varepsilon)=\frac{1}{i}\int_{-\infty}^{\infty}d\omega'\,P(\hat{e}^{\vec{u}_0\vec{\nabla}_0}-1)_{\omega'}\int_{0}^{\infty}e^{i(\omega-\hat{L}_0-\omega'+i\varepsilon)t}dt\cdot$$
$$\cdot(\hat{e}^{\vec{u}_0\vec{\nabla}_0}-1)_0\,\varphi^0_{\omega}(\omega')P, \tag{38}$$

then matrix elements of $\hat{M}(\omega)$ are defined by superoperator matrix elements of $\hat{K}=(\hat{e}^{\vec{u}_0\vec{\nabla}_0}-1)_{\omega'}e^{-i\hat{L}_0 t}(\hat{e}^{\vec{u}_0\vec{\nabla}_0}-1)_0$.

According to rules (7), (19), (23) matrix elements of $\hat{K}$ are defined by the equation

$$\hat{K}^{n'm'}_{nm}A^{m'}_{n'}=\left[(e^{\vec{u}_0\vec{\nabla}_0}-1)e^{-iH_0t}(e^{\vec{u}_0\vec{\nabla}_0}-1)\right]^{n_1}_{n}A^{m}_{n_1}(e^{iH_0t})^{m}_{m}+$$
$$+e^{-i\omega'}(e^{-iH_0t})^{n}_{n}A^{n_1}_{n}\left[(e^{\vec{u}_0\vec{\nabla}_0}-1)e^{iH_0t}(e^{\vec{u}_0\vec{\nabla}_0}-1)\right]^{m}_{n_1}-$$
$$-(e^{\vec{u}_0\vec{\nabla}_0}-1)^{n_1}_{n}(e^{-iH_0t})^{n_1}_{n_1}A^{m_1}_{n_1}(e^{\vec{u}_0\vec{\nabla}_0}-1)^{m}_{m_1}(e^{iH_0t})^{m}_{m}-$$
$$-e^{-i\omega'}(e^{-iH_0t})^{n}_{n}(e^{\vec{u}_0\vec{\nabla}_0}-1)^{n_1}_{n}A^{m_1}_{n_1}(e^{iH_0t})^{m_1}_{m_1}(e^{\vec{u}_0\vec{\nabla}_0}-1)^{m}_{m_1}. \tag{39}$$

Hereafter summation is meant over repeating indices n', m', n_1, m_1, and diagonality of the H_0 in the considered representation is taken into account (n, m are eigenfunctions of H_0).

As it was said above, the case is considered when the frequencies of the singled out oscillations do not coincide and aren't close $(\omega_{\varkappa}\neq\omega_{\varkappa'})$. Such a situation may take place, for example, in case of the law symmetry H_0 .

Let the different resonance situations between oscillations, when some combinations of frequencies $\omega_{\varkappa}, \omega_{\varkappa'},\ldots$ coincide, also do not realize. Correlation function $\langle b_{\varkappa}(t)b^+_{\varkappa}(0)\rangle_{\omega}$ will be considered in actual frequency range $\omega\approx\omega_{\varkappa}$.

It is not difficult to verify, that changing in eq. (39) the displacement operators $\vec{u}_0$ to the operators $b_{\varkappa}, b^+_{\varkappa}$ by formula (32), it is necessary to keep only terms, containing the equal number of creation and annihilation operators for every oscillation $\varkappa$ in any order in $\vec{u}_0$ while expanding $e^{\vec{u}_0\vec{\nabla}_0}$ into series. Terms, containing unequal number of creation and annihilation operators for every oscillator $\varkappa$, give a contribution of $\Gamma_{\varkappa}/\omega_{\varkappa}$ or $P_{\varkappa}/\omega_{\varkappa}$ order ($\Gamma_{\varkappa}, P_{\varkappa}$ are of an order of the line broadening and oscillation $\varkappa$ frequency shift, respectively, resulting from the interaction H_{int}) into correlation function $\langle b_{\varkappa}(t)b^+_{\varkappa}(0)\rangle_{\omega}$. Consequently, these terms may be omitted because the interaction Hamiltonian H_{int} is small.

Thus, in the first two summands of (39) for $(e^{\vec{u}_0\vec{\nabla}_0}-1)\cdot e^{\pm iH_0t}(e^{\vec{u}_0\vec{\nabla}_0}-1)$ it is necessary to keep only diagonal terms with $n_1=n$ and $n_1=m$. Taking into consideration all the above said the superoperator matrix element of $\hat{M}(\omega+i\varepsilon)$ may be represented via formula (38) as

$$\left[\hat{M}(\omega+i\varepsilon)\right]^{n'm'}_{nm}=\int_{-\infty}^{\infty}d\omega'\left\{\left[\frac{(e^{\vec{u}_0\vec{\nabla}_0}-1)^{n_1}_{n}(e^{\vec{u}_0\vec{\nabla}_0}-1)^{n}_{n_1}}{\omega-\omega_{n_1}+\omega_m-\omega'+i\varepsilon}+\right.\right.$$
$$\left.+\frac{(e^{\vec{u}_0\vec{\nabla}_0}-1)^{m_1}_{m}(e^{\vec{u}_0\vec{\nabla}_0}-1)^{m}_{m_1}}{\omega-\omega_n+\omega_{m_1}+\omega'+i\varepsilon}\right]\delta_{nn'}\delta_{mm'}-(e^{\vec{u}_0\vec{\nabla}_0}-1)^{n'}_{n}\cdot \tag{40}$$
$$\left.\cdot(e^{\vec{u}_0\vec{\nabla}_0}-1)^{m}_{m'}\left(\frac{1}{\omega-\omega_{n'}+\omega_m-\omega'+i\varepsilon}+\frac{1}{\omega-\omega_n+\omega_{m'}+\omega'+i\varepsilon}\right)\right\}\varphi^0_{\omega}(\omega').$$

When deriving the equation (40) the well-known relation for correlation functions is used:

$$e^{-\lambda\omega}\varphi^{\circ}_{\omega}(\omega)=\varphi^{\circ}_{\omega}(-\omega).$$

Equations (35), (24), (25), (36), (40) give a possibility to carry out a full and strict exploration of the oscillator $\varkappa$ spectral distribution.

Let's consider the case, when widths of fine structure lines $\Gamma_{\varkappa}(m)$ of the singled out oscillator, arising due to interaction H_{int}, are considerably lesser, than distances between lines, i.e.,

$$\Gamma_{\varkappa}(m) \ll \left|\tilde{\omega}_{\varkappa}(m+\delta_{\varkappa_1\varkappa'})-\tilde{\omega}_{\varkappa}(m)\right| \equiv \Delta_{\varkappa}(m), \qquad (41)$$

where $\tilde{\omega}_{\varkappa}(m)$ are renormalized by real part of $\hat{M}(\omega+i\varepsilon)$ frequencies $\omega_{\varkappa}(m)$, and $\varkappa_1$ may corresponds to any singled out oscillation.

Let's write equation (25) in the considered representation (for $B=b^{+}_{\varkappa}$), separating the diagonal term, as

$$(\hat{G}^{-1})^{m+\delta_{\varkappa\varkappa'},m}_{m+\delta_{\varkappa\varkappa'},m}\sum_{m'}\hat{\bar{G}}^{m'+\delta_{\varkappa\varkappa'},m'}_{m+\delta_{\varkappa\varkappa'},m}\sqrt{m'_{\varkappa}+1}=$$
$$=\sqrt{m_{\varkappa}-1}-\sum_{\substack{m_1\neq m\\ n_1\neq m+\delta_{\varkappa\varkappa'}}}(\hat{G}^{-1})^{n_1 m_1}_{m+\delta_{\varkappa\varkappa'},m}\sum_{m'}\hat{\bar{G}}^{m'+\delta_{\varkappa\varkappa'},m'}_{n_1 m_1}\cdot \qquad (42)$$
$$\cdot\sqrt{m'_{\varkappa}+1}.$$

It isn't difficult to see, that non-diagonal terms $(\hat{G}^{-1})^{n_1 m_1}_{m+\delta_{\varkappa\varkappa'},m}$ in the right-hand side of (42) are proportional to $\Gamma_{\varkappa}(m)$, because ω and $\hat{L}_0$ are diagonal. On the other hand, the denominators of functions $\hat{\bar{G}}^{m'+\delta_{\varkappa\varkappa'},m'}_{n_1 m_1}(\omega)$ contain the determinant of system (24), which should be solved for finding $\hat{\bar{G}}^{m'+\delta_{\varkappa\varkappa'},m'}_{m+\delta_{\varkappa\varkappa'},m}(\omega+i\varepsilon)$. Roots of this determinant define fine structure frequencies $\tilde{\omega}_{\varkappa}(m)$ if the inequality (41) is satisfied. Thus, if we consider frequencies $\omega\approx\tilde{\omega}_{\varkappa}(m)$, then functions $\hat{\bar{G}}^{m'+\delta_{\varkappa\varkappa'},m'}_{n_1 m_1}$ in the right-hand side of (42) will be of order $\frac{1}{\Delta_{\varkappa}(m)}$. It follows, that sums in the right-hand side of (42) contain small parameter $\Gamma_{\varkappa}(m)/\Delta_{\varkappa}(m)$ and this system of equations may be solved by the iteration method. In zero approximation it may be written as

$$\sum_{m'}\hat{\bar{G}}^{m'+\delta_{\varkappa\varkappa'},m'}_{m+\delta_{\varkappa\varkappa'},m}(\omega+i\varepsilon)\sqrt{m'_{\varkappa}+1}=\frac{\sqrt{m_{\varkappa}+1}}{\left[\hat{G}^{-1}(\omega+i\varepsilon)\right]^{m+\delta_{\varkappa\varkappa'},m}_{m+\delta_{\varkappa\varkappa'},m}}. \qquad (43)$$

Substituting (43) into (35) and using (36), (40), (22), we obtain in the actual range of frequencies $\omega\approx\omega_{\varkappa}$

$$\langle b_{\varkappa}(t)b^{+}_{\varkappa}(0)\rangle_{\omega}=\frac{1}{\pi Z_0}\sum_{m}\exp\{-\lambda\sum_{\varkappa'}\omega_{\varkappa'}m_{\varkappa'}\}\cdot \qquad (44)$$
$$\cdot(1+m_{\varkappa})\frac{\Gamma_{\varkappa}(m)}{[\omega-\omega_{\varkappa}(m)-P_{\varkappa}(m)]^2+\Gamma^2_{\varkappa}(m)}$$

with notation

$$P_{\varkappa}(m)=\int_{-\infty}^{\infty}d\omega'\Big[\frac{(e^{\vec{u}_0\vec{\nabla}_0}-1)^{n_1}_{m+\delta_{\varkappa\varkappa'}}(e^{\vec{u}_0\vec{\nabla}_0}-1)^{m+\delta_{\varkappa\varkappa'}}_{n_1}}{\omega_{\varkappa}-\omega_{n_1}+\omega_{m}-\omega'}+$$

$$+\frac{(e^{\vec{u}_0\vec{\nabla}_0}-1)_{m}^{m_1}(e^{\vec{u}_0\vec{\nabla}_0}-1)_{m_1}^{m}}{\omega_{\varkappa}-\omega_{m+\delta_{\varkappa\varkappa'}}+\omega_{m_1}+\omega'}\Big]\,\varphi_\omega^0(\omega');$$

$$\Gamma_{\varkappa}(m)=\pi\Big[(e^{\vec{u}_0\vec{\nabla}_0}-1)_{m+\delta_{\varkappa\varkappa'}}^{n_1}(e^{\vec{u}_0\vec{\nabla}_0}-1)_{n_1}^{m+\delta_{\varkappa\varkappa'}}\cdot$$

$$\cdot\,\varphi_\omega^0(\omega_{\varkappa}-\omega_{n_1}+\omega_m)+(e^{\vec{u}_0\vec{\nabla}_0}-1)_{m}^{m_1}(e^{\vec{u}_0\vec{\nabla}_0}-1)_{m_1}^{m}\cdot$$

$$\cdot\,\varphi_\omega^0(\omega_{m+\delta_{\varkappa\varkappa'}}-\omega_{m_1}-\omega_{\varkappa})-2(e^{\vec{u}_0\vec{\nabla}_0}-1)_{m+\delta_{\varkappa\varkappa'}}^{m+\delta_{\varkappa\varkappa'}}\cdot$$

$$\cdot\,(e^{\vec{u}_0\vec{\nabla}_0}-1)_{m}^{m}\,\varphi_\omega^0(0)\Big], \qquad (45)$$

where $⨍$ denotes the principal value of integral. Unessential nonlinear corrections to frequency $\omega_{\varkappa}$ are omitted in formulas (45) (frequencies ω_n must be also taken here neglecting nonlinearity).

Thus in the considered limiting case, when the condition (41) is fulfilled, the oscillation $\varkappa$ spectral distribution consists of a series of Lorentzian lines with widths $\Gamma_{\varkappa}(m)$ whose maxima are shifted relative to frequencies $\omega=\omega_{\varkappa}(m)$ by $P_{\varkappa}(m)$. Broadening of lines is caused by transitions between levels of oscillators and also by oscillator frequency modulation, appearing due to medium fluctuations. The modulation broadening is described by terms, containing the medium correlation function in zero frequency $\varphi_\omega^0(0)$ and arises, as is seen from equation (45) for $\Gamma_{\varkappa}(m)$.

only in the interaction with medium, containing even powers of expansion $e^{\vec{u}_0\vec{\nabla}_0}$ in $\vec{u}_0$.

Let's note, that the obtained result (44) is of a general character, as it is true for the nonlinear oscillator with an arbitrary (in power of $\vec{u}_0$) interaction with medium.

In general case, when the condition (41) is not satisfied, it is necessary, as it will be done in a subsequent work, to consider a specific (in power of $\vec{u}_0$) type of the oscillator interaction with medium fluctuations and to look a solution to system of eqs. (24) or (25).

Let's give one more expression for the correlation function of occupation numbers, which may be obtained in the same way as (35).

Putting in this case $\eta=-1$, we have

$$\langle\hat{n}_{\varkappa}(t)\hat{\tilde{n}}_{\varkappa}(0)\rangle_\omega=-\frac{2}{\pi}\left(1+e^{-\lambda\omega}\right)^{-1}\cdot$$

$$\cdot\,\mathrm{Im}\sum_{m,m'}(\rho_0)_m^m\,m_{\varkappa}\left[\hat{\bar{G}}(\omega+i\varepsilon)\right]_{mm}^{m'm'}(m'_{\varkappa}-\bar{n}_{\varkappa}). \qquad (46)$$

Equation (46) may be calculated by formulas for super-operator matrix elements obtained earlier. Instead of system (42) for calculating correlation function (46) it is necessary to use the system

$$\sum_{n_1m_1}(\hat{G}^{-1})_{mm}^{n_1m_1}\sum_{m'}\hat{\bar{G}}_{n_1m_1}^{m'm'}(m'_{\varkappa}-\bar{n}_{\varkappa})=m_{\varkappa}-\bar{n}_{\varkappa}, \qquad (47)$$

which follows from (25) in the same way as (42).

The quantum kinetic equation for the function

$$g_{\varkappa}(m+\delta_{\varkappa\varkappa'}, m; t) = \sum_{m'} \left(Pe^{-i\hat{L}t}P\right)^{m'+\delta_{\varkappa\varkappa'}, m'}_{m+\delta_{\varkappa\varkappa'}, m} \sqrt{1+m'_{\varkappa}}\,,$$

determining correlation function $\langle b_{\varkappa}(t)\, b^{+}_{\varkappa}(0)\rangle$,

looks like equation (30), in which it is necessary to put

$$n = m+\delta_{\varkappa\varkappa'}\,,\ \left(P\hat{L}_{o}P\right)^{n_1 m_1}_{nm} = \omega_{\varkappa}(m)\,\delta_{m+\delta_{\varkappa\varkappa'}, n_1}\,\delta_{m m_1}\,,\ \omega_i = \omega_{\varkappa}.$$

The kinetic equation for the function

$$g'_{\varkappa}(m, m; t) = \sum_{m'} \left(Pe^{-i\hat{L}t}P\right)^{m'm'}_{mm} \left(m'_{\varkappa} - \bar{n}_{\varkappa}\right),$$

which determines correlation function $\langle \hat{n}_{\varkappa}(t)\, \tilde{\hat{n}}_{\varkappa}(0)\rangle$,

coincides with equation (30) for $n = m$ (here the term $P\hat{L}_{o}P$ disappears, as it follows from (36), and thus, $\omega_i = 0$) .

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