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PARAMETRIC EXCITATION IN KANE'S SEMICONDUCTORS

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Параметрическое возбуждение в кейновских полупроводниках

Исследуется параметрическое возбуждение в полупроводниках с с узкой щелью с непараболическим законом дисперсии. Для изучения проблемы используется квантовомеханический формализм, развитый в [1]. Численные оценки, приведенные для типичного образца n - InSb для различных случаев, показывают, что отклонение от параболичности спектра приводит к уменьшению на порядок точки бифуркации спектра.

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Parametric Excitation in Kane's Semiconductors

The parametric excitation of electromagnetic vibrations in narrow gap semiconductors with nonparabolic energy dispersion defined by the Kane's model is investigated using the quantum formalism proposed in [1]. The numerical estimates performed for the typical sample of n-InSb crystal in different cases show that the deviation from parabolicity makes the instability threshold field values lowered by about one order of magnitude.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

As is well known, the parametric excitation in solid state plasmas is intensively studied in the last decade.

In this phenomenon the energy of external ectromagnetic wave field is transferred to the system under consideration by a resonant mechanism that takes place when the field amplitude is large enough to cause the vibration (with the external field frequency) of certain physical parameters of the system. In the case of relativistic systems such a vibrating parameter is the mass of particles that depends upon their velocity in radiation field. This effect was first considered by N.L.Tsintsadze [2]. Later it was pointed out [3] that in the plasma of narrow gap semiconductors in which the electron energy dispersion is expressed in the Kane's model [4] by the pseudorelativistic formula

$$\mathcal{E}(\vec{p}) = \sqrt{(mc^{*2})^2 - p^2 c^{*2}}, \qquad (1)$$

where m and $\vec{p}$ are the particle's effective mass and canonical momentum, respectively, $c^* = (E_g/2m)^{1/2}$ with E_g being the energy gap value, there realises the same excitation mechanism. The authors of [2,3] have investigated this case for a single electron system is the hydrodynamical approximation without taking into account the thermal motion effects.

The purpose of this paper is to make an account of the effect of nonparabolicity of the type

(1) in the parametric excitation process in interacting electron-phonon systems. The calculations will be carried out in the framework of the quantum formalism proposed in (1). Analytical results for instability growth rates will be presented for interacting electron-phonon (Sec. 3) as well as for single electron (Sec. 4) systems. For comparison with the results presented in Ref. [1,3,5] the numerical estimates for the threshold field values have been performed for a sample of n-InSb crystal.

2. EQUATIONS FOR POTENTIALS OF ELECTROMAGNETIC PERTURBATION FIELD

Let us consider an interacting electron-phonon system placed under the action of a strong radiation field that can be presented in the well-known approximation by an oscillatory electric field

$$\vec{E}_0(t) = -\frac{1}{c}\frac{\partial \vec{A}_0(t)}{\partial t} = \vec{E}_0 \sin\omega_0 t. \tag{2}$$

For arbitrary electron energy dispersion $\mathcal{E}(p - \frac{e}{c}\vec{A}_0(t))$ and in the presence of the external field $\vec{E}_0(t)$, the Hamiltonian of such a system introduced in (1) takes the form:

$$\begin{aligned} H = {} & \sum_{\vec{p}} \mathcal{E}(\vec{p} - \frac{e}{c}\vec{A}_0(t))\, a^+_{\vec{p}} a_{\vec{p}} + \sum_{i,\vec{q}} \omega_{i\vec{q}}\, b^+_{i,\vec{q}} b_{i,\vec{q}} - \\ & - \sum_{\vec{p},\vec{q}} \{ e\phi(\vec{q},t) - \frac{e}{2c}[\vec{v}(\vec{p} - \frac{e}{c}\vec{A}_0(t) + \vec{q}) + \\ & + \vec{v}(\vec{p} - \frac{e}{c}\vec{A}_0(t))]\vec{A}(\vec{q},t)\}\, a^+_{\vec{p}+\vec{q}} a_{\vec{p}} + \\ & + \sum_{i,\vec{p},\vec{q}} C^i_{\vec{q}} (b_{i,\vec{q}} + b^+_{i,-\vec{q}})\, a^+_{\vec{p}+\vec{q}} a_{\vec{p}}. \end{aligned} \tag{3}$$

Here $\vec{v}(\vec{p}) = \frac{\partial \mathcal{E}(\vec{p})}{\partial \vec{p}}$ is the electron velocity. All other notations are the same as in [1].

Introducing the "quantum distribution function" in the form

$$\begin{aligned} & \langle a^+_{\vec{p}} a_{\vec{p}+\vec{q}} \rangle = n_{\vec{p}}(t)\,\delta_{\vec{q},0} + f(\vec{p}+\vec{q},\vec{p},t), \\ & n_p(t) \equiv \langle a^+_{\vec{p}} a_{\vec{p}} \rangle_t , \end{aligned} \tag{4}$$

one can obtain, using the Hamiltonian (3), the linearized equation of motion for $f(\vec{p}+\vec{q},\vec{p},t)$ as:

$$\begin{aligned} i\frac{\partial}{\partial t} f(\vec{p}+\vec{q},\vec{p},t) = {} & \{\mathcal{E}(p - \frac{e}{c}\vec{A}_0(t) + q) - \\ & - \mathcal{E}(p - \frac{e}{c}\vec{A}_0(t))\} f(\vec{p}+\vec{q},\vec{p},t) + \\ & + \{ e\phi(\vec{q},t) - \frac{e}{2c}[\vec{v}(\vec{p} - \frac{e}{c}\vec{A}_0(t) + \vec{q}) + \vec{v}(\vec{p} - \frac{e}{c}\vec{A}_0(t))]\vec{A}(\vec{q},t)\} \times \\ & \times [n_{\vec{p}}(t) - n_{\vec{p}+\vec{q}}(t)]. \end{aligned} \tag{5}$$

Following the work [3], we shall consider $n_{\vec{p}}(t)$ as an equlibrium Fermi distribution function.

Now we must solve the complete system of Maxwell equations with charge density ρ and current density $\vec{j}$ expressed through the function f as follows:

$$\rho(\vec{q},t) = e\sum_{\vec{p}} f(\vec{p}+\vec{q},\vec{p},t), \tag{6a}$$

$$\begin{aligned} \vec{j}(q,t) = \frac{e}{m}\sum_{\vec{p}} \{ & \vec{p} + \frac{\vec{q}}{2} - \frac{e}{c}\vec{A}_0(t) - \\ & - \frac{1}{2mE_g}[(\vec{p}+\vec{q} - \frac{e}{c}\vec{A}_0(t))^2(\vec{p}+\vec{q} - \frac{e}{c}\vec{A}_0(t)) + \\ & + (\vec{p} - \frac{e}{c}\vec{A}_0(t))^2(\vec{p} - \frac{e}{c}\vec{A}_0(t))]\} f(\vec{p}+\vec{q},\vec{p},t) - \end{aligned}$$

$$- \frac{e^2}{mc} \sum_{\vec p} n_{\vec p} \{ \vec A(\vec q, t) - \frac{1}{mE_g} [(\vec p - \frac{e}{c}\vec A_0(t))^2 \vec A(\vec q, t) + + 2((\vec p - \frac{e}{c}\vec A_0(t)) \cdot \vec A(\vec q, t))(\vec p - \frac{e}{c}\vec A_0(t))] \} . \tag{6b}$$

Introducing the transformation

$$X \to \tilde X = X \cdot \exp\{-i(\lambda \sin\omega_0 t + \eta \sin 3\omega_0 t)\},$$

where

$$\lambda \equiv \frac{e}{m\omega_0^2} [1 - \frac{3}{8}(\frac{V_E}{c^*})^2] (\vec q \cdot \vec E_0),$$

$$\eta \equiv -\frac{1}{24} \frac{e}{m\omega_0^2}(\frac{V_E}{c^*})^2 (\vec q \cdot \vec E_0),$$

$$\vec V_E \equiv \frac{e\vec E_0}{m\omega_0},$$

for the quantities f, $\vec A$, ϕ, ρ, $\vec j$, one obtains the system of equations for the Fourier components of $\tilde{\vec A}$ and $\tilde\phi$:

$$\sum_L \{ \vec R_L(\omega)\tilde\phi(\omega + L\omega_0) - \hat Q_L(\omega)\tilde{\vec A}(\omega + L\omega_0) +$$

$$+ \beta^2 \hat Q_L(\omega) [\tilde{\vec A}(\omega + (L+2)\omega_0) + \tilde{\vec A}(\omega + (L-2)\omega_0)] \} -$$

$$- \mu \cdot \vec e_0 \sum_{L, m' = \pm 1} P_L(\omega + m'\omega_0)\tilde\phi(\omega + (L+m')\omega_0) -$$

$$- \beta^2 \sum_{L, m' = \pm 2} \{ \frac{3\vec q}{2mc} P_L(\omega + m'\omega_0)\tilde\phi(\omega + (L+m')\omega_0) -$$

$$- \hat Q_L(\omega + m'\omega_0)\tilde{\vec A}(\omega + (L+m')\omega_0) \} -$$

$$- \frac{\omega_p^2}{q^2c^2} \{ [1 - 2\beta^2 - \frac{P_F^2}{mE_g}] \tilde{\vec A}(\omega) - \beta^2 [\tilde{\vec A}(\omega + 2\omega_0) + \tilde{\vec A}(\omega - 2\omega_0)] \} =$$

$$= \sum_{\substack{n,n' \\ \ell,\ell'}} J_n(\lambda) J_{n'}(\eta) J_{n+\ell}(\lambda) J_{n'+\ell'}(\eta) \{ \Delta(\omega - (n+3n')\omega_0) \tilde{\vec A}(\omega + (\ell+3\ell')\omega_0) + \tag{7a}$$

$$+ \vec q_0 [\frac{\omega - (n+3n')\omega_0}{qc} - \frac{q}{2mc}] \epsilon(\omega - (n+3n')\omega_0) \phi(\omega + (\ell+3\ell')\omega_0) \}$$

for $\vec q \,||\, \vec E_0$ and

$$\sum_L \{ \vec R_L(\omega)\phi(\omega + L\omega_0) - \hat Q_L(\omega)\vec A(\omega + L\omega_0) +$$

$$+ \beta^2 [\hat Q_L(\omega) + 2(\hat Q_L(\omega) \cdot \vec e_0) \vec e_0] [\vec A(\omega + (L+2)\omega_0) + \vec A(\omega + (L-2)\omega_0)] \} -$$

$$- \mu \cdot \vec e_0 \sum_{L, m' = \pm 1} P_L(\omega + m'\omega_0)\phi(\omega + (L+m')\omega_0) -$$

$$- \beta^2 \sum_{L, m' = \pm 2} \{ \frac{\vec q}{2mc} P_L(\omega + m'\omega_0)\phi(\omega + (L+m')\omega_0) -$$

$$- [\hat Q_L(\omega + m'\omega_0) - 2\vec e_0 (\hat Q_L(\omega + m'\omega_0) \cdot \vec e_0)] \vec A(\omega + (L+m')\omega_0) \} = \tag{7b}$$

$$= [\Delta(\omega) + \frac{\omega_p^2}{q^2c^2}(1 - 2\beta^2 - \frac{P_F^2}{mE_g})] \vec A(\omega) + \vec q_0 [\frac{\omega}{qc} - \frac{q}{2mc}] \epsilon(\omega)\phi(\omega) -$$

$$- \beta^2 \frac{\omega_p^2}{q^2c^2} \{ \vec A(\omega + 2\omega_0) + \vec A(\omega - 2\omega_0) +$$

$$+ 2\vec e_0 \cdot [2\vec e_0 \cdot \vec A(\omega) + \vec e_0 \cdot (\vec A(\omega + 2\omega_0) + \vec A(\omega - 2\omega_0))] \}$$

for $\vec q \perp \vec E_0$,

where the following notations have been introduced:

$$X \equiv X^{\ell,\ell'}_{\ell_1,\ell'_1} ,$$

$$\omega + L\omega_0 \equiv \omega + (\ell - \ell' + 2\ell_1 - 2\ell'_1)\omega_0 ,$$

$$P_L(\omega) \equiv \frac{4\pi e^2}{q^2} \sum_{\vec p} J_\ell(\eta_1) J_{\ell'}(\eta_1) J_{\ell_1}(\eta_2) J_{\ell'_1}(\eta_2) \times \frac{(n_{\vec p+\vec q} - n_{\vec p})}{\tilde E_{\vec p+\vec q} - \tilde E_{\vec p} - \omega - (\ell + 2\ell_1)\omega_0 - i0} ,$$

$$\vec R_L(\omega) = \frac{4\pi e^2}{mcq^2} \sum_{\vec p} J_\ell(\eta_1) J_{\ell'}(\eta_1) J_{\ell_1}(\eta_2) J_{\ell'_1}(\eta_2) \times \frac{(n_{\vec p+\vec q} - n_{\vec p})\vec p}{\tilde E_{\vec p+\vec q} - \tilde E_{\vec p} - \omega - (\ell + 2\ell_1)\omega_0 - i0} ,$$

$$\hat Q_L(\omega) \equiv Q^{ij}_L(\omega) \equiv \frac{4\pi e^2}{(mcq)^2} \sum_{\vec p} J_\ell(\eta_1) J_{\ell'_1}(\eta_2) J_\ell(\eta\ ) J_{\ell'_1}(\eta_2) \times \frac{(n_{\vec p+\vec q} - n_{\vec p}) p_i p_j}{\tilde E_{\vec p+\vec q} - \tilde E_{\vec p} - \omega - (\ell + 2\ell_1)\omega_0 - i0} ,$$

$i, j = x, y, z$,

$$\tilde E_{\vec p} \equiv \mathcal{E}(\vec p) + \frac{e^2 E_0^2}{4m\omega_0^2}\left[1 - \frac{p^2 + 2(\vec e_0 \cdot \vec p)^2}{mE_g} - \frac{3}{8}\frac{e^2 E_0^2}{mE_g \omega_0^2}\right],$$

$$\eta_1 = \eta_1(\vec p) \equiv \frac{eE_0}{m^2 E_g \omega_0^2}\left[p^2(\vec e_0 \cdot \vec p) - (\vec p + \vec q)^2(\vec e_0 \cdot (\vec p + \vec q))\right],$$

$$\eta_2 = \eta_2(\vec p) = \frac{e^2 E_0^2}{8m^2 E_g \omega_0^3}\{2[(\vec e_0 \cdot (\vec p + \vec q))^2 - (\vec e_0 \cdot \vec p)^2] + (\vec p + \vec q)^2 - p^2\},$$

$$\Lambda(\omega) = 1 - \frac{\omega^2}{q^2 c^2}\epsilon(\omega), \quad \omega_p^2 = \frac{4\pi e^2 n_0}{m} \quad (n_0 = \sum_{\vec p} n_{\vec p}) ,$$

$$\mu = \frac{eE_0}{2mc\omega_0} , \quad \beta^2 = \frac{1}{8}\left(\frac{V_E}{c^*}\right)^2,$$

$$(\hat Q \cdot \vec a)_i = Q^{ij} a_j, \quad (\vec a \vec b) = a_i b_j , \quad \vec e_0 = \frac{\vec E_0}{E_0} , \quad \vec q_0 = \frac{\vec q}{q} ,$$

$p_F = mv_F$ is the electron Fermi momentum, J_ℓ is the Bessel function of the first kind; n , n' , ℓ , ℓ' , ℓ_1 , ℓ'_1 are integers.

In obtaining (7a,b) we have set $\mu \ll 1$, $\beta^2 \ll 1$ and also taken into account the fact that quantities $|R_L| \ll 1$, $|Q^{ij}| \ll 1$ when $i \neq j$ as they represent the small contribution of nonparabolicity effects. In agreement with this only the terms linear with respect to the mentioned quantities have been retained.

3. INSTABILITY ANALYSIS FOR COUPLED ELECTRON-PHONON SYSTEMS

In this section, proceeding from Eqs. (7a,b) we obtain the dispersion relations for electrostatic and electromagnetic waves in an electron-phonon system and solve for possible instability growth rates of these waves in different cases.

We shall work in the rectangular system of coordinates with z-axis being parallel to the wave vector $\vec q$ and the electric field vector $\vec E_0$ lying in xz -plane. As the parameters λ, η, η_1, η_2 involved in Eqs. (7a,b) are much smaller than 1, in further calculations only terms linear in them will be retained and in accordance with this one will have $J_0(\lambda) = J_0(\eta) \approx 1$.

The analysis will be done in the high-frequency long wave-length limit ($qv_F \ll \omega$, $q < q_{FT} = \omega_p / v_F$) and in the so-called "two-mode approximation" (see ref. /1/).

For zero external field ($E_0 = 0$) one can obtain from the system (7a,b) the expression for the frequencies $\omega_{1,2}$ of the two longitudinal (electrostatic) and $\omega_{1,2}(\vec{q})$ of the two transverse (electromagnetic) eigenmodes involving the terms due to nonparabolicity effect. Thus, one has:

$$\omega^2_{1,2} = \frac{1}{2}\{(\tilde{\omega}^2_p + \omega^2_L) \pm (\tilde{\omega}^2_p - \omega^2_L)[1 + \frac{4\tilde{\omega}^2_p \omega^2_L (1 - \epsilon_\infty/\epsilon_0)}{(\tilde{\omega}^2_p - \omega^2_L)^2}]^{1/2}\}, \tag{8}$$

where ω_L is the longitudinal optical phonon frequency and

$$\tilde{\omega}^2_p = \frac{\omega^2_p}{\epsilon_\infty}(1 - \frac{p^2_F}{mE_g}).$$

The expression for $\omega^2_{1,2}(\vec{q})$ has the same form as (8) with

$$\tilde{\omega}^2_p \to \omega^2_{pc} = \frac{1}{\epsilon_\infty}[q^2c^2 + \omega^2_p(1 - \frac{p^2_F}{mE_g})].$$

For $E_0 \neq 0$, $\vec{q} \parallel \vec{E}_0$ the system (7) leads to uncoupled dispersion equations describing longitudinal and transverse waves. The equation for longitudinal modes is:

$$\in(\omega) \in(\omega - \omega_0) = \{J_1(\lambda)[\epsilon(\omega - \omega_0) - \epsilon(\omega)] + P^{1,0}_{0,0}(\omega - 2\omega_0) - P^{1,0}_{0,0}(\omega - \omega_0)\}^2, \tag{9}$$

$$\in(\omega) \equiv \epsilon(\omega) - P^{0,0}_{0,0}(\omega).$$

$\epsilon(\omega)$ is the lattice dielectric function that yields the growth rate γ (the resonant coupling modes are: $\omega = \omega_2$, $\omega_0 - \omega = \omega_1$ and so $\omega_0 = \omega_1 + \omega_2$, $\omega_1 < \omega_2$) in the form

$$\gamma = \frac{eqE_0}{4m\epsilon_\infty} \cdot \frac{\omega^2_p}{(\omega_1 + \omega_2)^2 \omega_1 \omega_2}[\frac{(\omega^2_T - \omega^2_1)(\omega^2_2 - \omega^2_T)}{\omega_1\omega_2}]^{1/2} \times$$

$$\times |1 - 9\beta^2 - \frac{q^2 + (16/5)p^2_F}{mE_g} + \frac{3\omega_1\omega_2}{E_g(\omega_2 - \omega_1)}|. \tag{10}$$

For transverse modes ($\omega = \omega_1(\vec{q})$, $\omega_0 - \omega = \omega_2(\vec{q})$) we have the dispersion relation

$$\Lambda^*(\omega)\Lambda^*(\omega - \omega_0) = \{J_1(\lambda)[\Lambda(\omega) - \Lambda(\omega - \omega_0)] + Q^{1,0}_{0,0}(\omega - 2\omega_0) - Q^{1,0}_{0,0}(\omega - \omega_0)\}^2, \tag{11}$$

where

$$\Lambda^*(\omega) \equiv \Lambda(\omega) + Q(\omega) + \frac{\omega^2_p}{q^2c^2}(1 - \frac{p^2_F}{mE_g}),$$

$$Q(\omega) \equiv Q^{0,0}_{0,0}(\omega)$$

(note that $Q^{xx}_L = Q^{yy}_L \equiv Q_L$ for $\vec{q} \parallel \vec{E}_0$). The growth rate in this case is expressed by the formula

$$\gamma = \frac{eqE_0}{20m\epsilon_\infty} \cdot \frac{q^2p^2_F}{m^2} \frac{\omega^2_p}{\omega^2_1(\vec{q})\omega^2_2(\vec{q})[\omega_1(\vec{q}) + \omega_2(\vec{q})]^2} \times$$

$$\times\{\frac{[\omega^2_T - \omega^2_1(\vec{q})][\omega^2_2(\vec{q}) - \omega^2_T]}{\omega_1(\vec{q})\omega_2(\vec{q})}\}^{1/2} \cdot |1 - 9\beta^2 + \frac{6}{7}\frac{p^2_F}{mE_g} + \frac{3\omega_1(\vec{q})\omega_2(\vec{q})}{E_g[\omega_2(\vec{q}) - \omega_1(\vec{q})]}|. \tag{12}$$

As is seen, in the case of parabolic energy dispersion the growth rates (10) and (12) coincide with those calculated in [1].

We now turn to the case of $\vec{q} \perp \vec{E}_0$. Eqs. (7) show that there exist the purely electromagnetic (transverse) perturbations polarized perpendicularly to $\vec{E}_0 (\vec{E} \perp \vec{E}_0)$ and the coupled electrostatic (with $\vec{E} \perp \vec{E}_0$)-electromagnetic (with $\vec{E} \parallel \vec{E}_0$) ones with "mixed" polarization. The usual procedure leads to the following growth rate formula for the purely transverse modes (with $\omega = \omega_2(\vec{q})$, $2\omega_0 - \omega = \omega_1(\vec{q})$):

$$\gamma = \frac{1}{2}\beta^2 \frac{\omega_p^2}{\epsilon_\infty [\omega_2^2(\vec{q}) - \omega_1^2(\vec{q})]} \left\{ \frac{[\omega_2^2(\vec{q}) - \omega_T^2][\omega_T^2 - \omega_1^2(\vec{q})]}{\omega_1(\vec{q})\,\omega_2(\vec{q})} \right\}^{1/2} \qquad (13)$$

(13) shows that the excitation in this case is caused essentially by the nonparabolicity effect. For the coupled plasmon-polariton modes ($\omega = \omega_2$, $\omega_0 - \omega = \omega_1(\vec{q})$) the growth rate γ has the form

$$\gamma = \frac{e q E_0}{4 m \epsilon_\infty} \frac{\omega_p^2}{\omega_2 [\omega_1(\vec{q}) + \omega_2]} \left\{ \frac{(\omega_2^2 - \omega_T^2)[\omega_T^2 - \omega_1^2(\vec{q})]}{\omega_1(\vec{q})\omega_2(\omega_2^2 - \omega_1^2)[\omega_2^2(\vec{q}) - \omega_1^2(\vec{q})]} \right\}^{1/2} \qquad (14)$$

$$\times \left\{ 1 - 2\beta^2 - \frac{p_F^2}{m E_g}\left(1 - \frac{2}{5}\frac{\omega_2}{\omega_1(\vec{q})}\right) \right\}.$$

The numerical values of the threshold fields are estimated in the same way as is shown in ref. [1]. For this purpose the sample of n-InSb crystal was taken. This is a typical narrow gap semiconductor that may be described by the electron energy dispersion formula (1). The results are:

1) for the longitudinal modes with $\vec{q} \parallel \vec{E}$:

$$E_{oth} \approx 3{,}5 \cdot 10^3 \ \frac{V}{cm} \quad \text{for} \quad q = 5 \cdot 10^5 \ cm^{-1}, \qquad (15)$$

2) for the coupled electrostatic-electromagnetic modes with $\vec{q} \perp \vec{E}_0$

$$E_{oth} \approx 10^6 \ \frac{V}{cm} \quad \text{for} \quad q = 5 \cdot 10^3 \ cm^{-1}. \qquad (16)$$

Here the following data have been employed (see refs. [1,5]): $n = 10\ cm$, $m = 0{,}01\, m$, $B = 0{,}234\, eV$, $\epsilon_\infty = 9$, $\epsilon_0 = 27$, $\omega_L = 10^{13} sec^{-1}$, $\omega_p = 2 \cdot 10^{13} sec^{-1}$ the damping constants: $T_{ph} = 7 \cdot 10^{10} sec^{-1}$ for phonons and $\Gamma_{pl} = 1{,}7 \cdot 10^{12}\ sec^{-1}$ for plasmons.

Comparison of (15)-(16) with the results obtained in refs. [1,5] shows that the nonparabolicity effect makes the threshold field values lowered by several times in all cases.

4. SINGLE ELECTRON SYSTEMS

The problem of parametric excitation in single electron plasma can be treated using Eqs. (7) with $\epsilon(\omega) = 1$.

The plasmon frequency ω_p^* in the absence of any external influence is then obtained from $\epsilon(\omega) = 0$ and has the form

$$\omega_p^{*2} = \omega_p^2 \left(1 - \frac{p_F^2}{m E_g}\right), \qquad (17a)$$

while the frequencies of electromagnetic modes are determined by $\Lambda^*(\omega) = 0$ and are

$$\omega_\perp^2(\vec{q}) = \omega^{*2} + q^2 c^2 . \qquad (17b)$$

The instability analysis in the presence of the radiation field $\vec{E}_0(t)$ is carried out analogously to the case of electron-phonon systems. The results for instability growth rates and threshold fields are ($n = 10^{17} cm^{-3}$, $\omega_p = 2{,}7 \cdot 10^{14}\ sec^{-1}$, and the effective collision frequency of electrons $\gamma_{eff} = 6 \cdot 10^{11}\ sec^{-1}$ have been taken [3]):

1) for the plasmon-mode $\omega = \omega_p^*$ with $\vec{q} \parallel \vec{E}_0$ when $\omega_0 = \omega_p^*$:

$$\gamma = \frac{3}{2}\beta^2\omega_p\left(1 - \frac{p_F^2}{mE_g}\right)^{-1/2}, \quad (18)$$

$$E_{oth} \approx 9\cdot 10^3 \frac{V}{cm};$$

2) for the purely electromagnetic mode $\omega_\perp(\vec{q})$ with $\vec{q}\,||\,\vec{E}_0$ when

$$\gamma = \frac{1}{2}\beta^2\frac{\omega_p^2}{\omega_\perp(\vec{q})}\left(1 - \frac{1}{5}\frac{q^2p_F^2}{m^2\omega_\perp^2(\vec{q})}\right), \quad (19)$$

$$E_{oth} \approx 5\cdot 10^4 \frac{V}{cm} \text{ for } q = 5\cdot 10^3 \text{ cm}^{-1};$$

3) for the electromagnetic mode $\omega_\perp(\vec{q})$ with $\vec{q}\perp\vec{E}_0$, $\vec{E}\perp\vec{E}$ $(\omega_0 = \omega_\perp(\vec{q}))$:

$$\gamma = \frac{1}{2}\beta^2\frac{\omega_p^2}{\omega_\perp(\vec{q})}\left(1 + \frac{1}{5}\frac{q^2p_F^2}{m^2\omega_\perp^2(\vec{q})}\right), \quad (20)$$

$$E_{oth} \approx 5\cdot 10^4 \frac{V}{cm} \text{ for } q = 5\cdot 10^3 \text{ cm}^{-1};$$

4) for the coupled electrostatic-electromagnetic mode ω_p^* with $\vec{q}\perp\vec{E}$ $(\omega_0 = \omega_p^*)$, the growth rate γ and the threshold field E_{oth} are determined by (18).

As is seen from (18)-(20), the results for the growth rate expressions coincide with those calculated in ref. /3/ is the thermal motion effect is neglected. So it is clear that this effect has made the threshold field values somewhat lower than ones obtained in ref. /3/.

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