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ON THE SURFACE ELECTRICAL RESISTIVITY OF THIN SEMICONDUCTING AND SEMIMETALLIC FILMS

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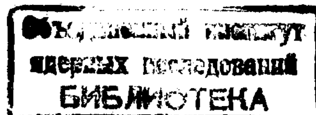

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К теории поверхностного электросопротивления тонких полупроводниковых и полуметаллических пленок

В приближении метода эффективной массы с учетом сил изображения вычислены низкоэнергетические электронные состояния в тонкой кристаллической пленке, а также транспортное время релаксации носителей тока и электросопротивление пленки, когда механизм рассеяния связан с экранированными кулоновскими и электрически-дипольными центрами, хаотически расположенными по ее граничным поверхностям. В работе показано, что в некоторых случаях тонкопленочных материалов силы изображения влияют существенно на их электронные состояния и поверхностное электросопротивление.

Работа выполнена в Лаборатории теоретической физики, ОИЯИ.

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Dimitrov H.D. E17 - 11463

On the Surface Electrical Resistivity of Thin Semiconducting and Semimetallic Films

In the effective-mass approximation taking into account the image forces the low-energy electronic states in thin crystalline films and transport relaxation time of current-carriers and electrical resistivity of the films are calculated when the scattering mechanism is connected with the randomly distributed on their boundary surfaces screened Coulomb and electric dipole impurity centres. It is shown that in some cases of thin solid film materials the image forces influence essentially the electronic states and their surface electrical resistivity.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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I. INTRODUCTION

Recently, considerable attention has been paid to the investigation of electrical, optic, galvano-magnetic and other properties of thin film semiconducting and semimetallic materials, as well as to the surface layers of many semiconductors with size-quantized motion of the current-carriers (see, e.g., surveys [1-3]). This concern is related with the presence of a number of specific quantum size effects and new possibilities for investigating the physical properties of crystals, as well as with the continuosly increasing needs of practice and mostly microelectronics.

The electrical resistivity of thin crystalline films is of special interest, as well as its dependence on their thickness and on the scattering mechanisms of the current-carriers within them. A lot of theoretical [4-10] and experimental [11-18] papers have been devoted to the investigation of these problems (see also the survey [1] and papers, mentioned there) snd, moreover, a considerable part of them have been published quite recently.

As is well known (cf. [1]), under the accomplishment of definite conditions for the transport relaxation time and current-carriers' concentration in some sufficiently thin semiconducting and semimetallic films, including conditions for the temperature and homogeneity of the films along their thickness, one can observe experimentally quantum size effects which are a result from the quantization of the current-carriers transversal motion and are expressed in oscillating dependence of the electrical resistivity and other macroscopic characteristics of the films on their thickness. In the case of some thin film materials other interesting features of their electrical resistivity as a function of their

thickness have been observed[12]. As a rule, the theory of these phenomena is built in the effective-mass approximation with a suitable choice of the current-carriers' potential. The most frequently used model for this potential represents the film (in transversal direction) as a rectangular pit with infinitely high walls[1,4,19,20] and, naturally, the wave function of every current-carrier at the film's boundaries vanishes. The problem of the kind of potential in the film and boundary conditions for the current-carriers' wave functions in the film is discussed in detail in[1,11]. In[21] the potential within the film is assumed to be zero, while on its boundaries it changes in such a way that the normal derivative of the wave functions equals zero. A considerable study of the conditions which must satisfy the envelopes of the electronic wave functions at the crystals boundaries has been recently accomplished in[22]. Besides, note that in all mentioned cases the obtained solutions for the states of current-carriers in thin films are non-self-consistent, which, in its turn, expresses their approximate character.

A number of authors (see[1], see also[8,14,19]) have done calculations of the electrical resistivity of thin film materials or of the transport relaxation time of the current-carriers within them when accounting the electron-phonon interaction or scattering from point-defects in the volume (most often described by δ -like potential). When the scattering mechanism is associated with surface Coulomb and electrical dipole centres, corresponding calculations can be found in[4,7,23-26].

The present paper is concerned with calculations of the low-energy states and transport relaxation time of current-carriers (conductivity electrons and holes) in thin semiconducting and semimetallic films and the electrical resistivity of the latter accounting for the influence of image forces and when the scattering mechanism is due to screened Coulomb and electric dipole centres which are randomly distributed on the boundary surfaces of the films. The investigation is carried out in the effective-mass approximation with isotropic dispersion law of the current-carriers and potential model with infinitely high walls. Moreover, the energy-band bending in the films, due to other reasons besides the image forces, is neglected. It is also assumed that

the average concentration of the scattering centres is not very great so that each of them can practically scatter independent of the others. The scattering amplitudes are calculated in Born approximation.

2. IMAGE FORCES AND LOW-ENERGY ELECTRONIC STATES IN THIN SOLID FILMS

First we will calculate the low-energy electronic states in a thin crystalline film assuming that the image forces have a considerable influence on them. The description will be done with respect to a Cartesian orthogonal co-ordinate system $Oxyz$ with origin at the centre of the film and z axis perpendicular to its surfaces. In the same way as in[27], we will use an approximate, but sufficiently precise, representation of the potential $V(z)$ of the image forces acting on everyone of the current-carriers in the film with electric charge $\pm e$, namely

$$V(z) = \frac{\alpha}{a^2 - z^2} \quad \text{for} \quad z^2 < a^2,$$

where

$$\alpha = \frac{\Gamma a e^2}{8\pi\varkappa} \, \frac{\varkappa - \varkappa_0}{\varkappa + \varkappa_0}. \tag{1}$$

Here Γ is a constant with value close to unity (in[27] it is set $\Gamma = 0,795$), a is the film half-thickness, while $\varkappa$ and $\varkappa_0$ are the permittivities of the film and the surrounding medium, respectively. Note that in expression (1) $\varkappa$ and $\varkappa_0$ have to represent the high-frequency values of the permittivities of the corresponding media[28,29]. It can be seen from the same expression that the influence of the image forces will be considerable when $\varkappa$ is small, and $\varkappa_0$ is large, or when $\varkappa \cong 2,41\varkappa_0$. Following what was said in the Introduction, we will consider that outside the film the potential has an infinitely large value.

So, the stationary states of the current-carriers (conductivity electrons or holes) in the film will be defined by the wave equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + \frac{\alpha}{a^2 - z^2} - E\right)\Psi(\vec{r}) = 0 \tag{2}$$

and the boundary conditions

$$\Psi(x,y,\pm a) = 0; \tag{3}$$

m is the effective mass of the considered current-carrier. Besides, further in our paper, the energy of the conductivity electrons and holes will be accounted from the low and upper end, respectively, of their energy bands in specimens of the considered material in the absence of quantum size effects. We will conditionally assume that the form of the film in the plane Oxy is rectangular with dimensions L_1 and L_2 along the axes x and y, respectively, and $2a \ll L_1, L_2$, the film volume V being $V = 2aL_1L_2$. Then, setting

$$\Psi(\vec{r}) = \frac{1}{\sqrt{L_1L_2}} e^{i(k_1x+k_2y)}\chi(z) \tag{4}$$

and substituting it in (2) and (3), we obtain

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + \frac{\alpha}{a^2-z^2} - \epsilon\right)\chi = 0, \quad z^2 < a^2 \tag{5}$$

for

$$\chi(\pm a) = 0, \quad \epsilon = E - \frac{\hbar^2}{2m}(k_1^2+k_2^2). \tag{6}$$

The equation (5) under the boundary conditions in (6) has a solution for χ only for discrete values of ϵ. With the help of the variation method we will find approximate solutions for χ, corresponding to the first two smallest values of ϵ and denoting them by ϵ_0 and ϵ_1. This will be done first for the function of the ground state $\chi_0(z)$, choosing it of the form, as follows

$$\chi_0(z) = \frac{C_0}{\sqrt{a}}(1+\beta z^2)\cos\left(\frac{\pi z}{2a}\right), \tag{7}$$

where β is a variational parameter, and the constant C_0 is determined by the normalizing condition

$$\int_{-a}^{+a} dz\, |\chi_0(z)|^2 = 1. \tag{8}$$

From (8) (after approximate calculation of definite numerical coefficients containing the number π */) we obtain

*/ Similar calculations will be done further in the paper without mentioning them.

$$C_0 = \left(1 + 0,261\beta_0 + 0,041\beta_0^2\right)^{-1/2} \tag{9}$$

for

$$\beta_0 = a^2\beta. \tag{10}$$

For the quantity ϵ_0 from (5)-(7) we find

$$\epsilon_0 = \frac{\hbar^2 C_0^2}{2ma^2}\Big[2,467 + 2,438\alpha_0 + (0,645 + 0,875\alpha_0)\beta_0 + (0,624 + 0,176\alpha_0)\beta_0^2\Big], \tag{11}$$

where

$$\alpha_0 = \frac{m\alpha}{\hbar^2} \tag{12}$$

and (from the condition ϵ_0 to have minimal value for (7) and (8))

$$\beta_0 = \frac{\left[(1,250 + 0,139\alpha_0)^2 - 0,079\alpha_0\right]^{1/2} - (1,250 + 0,182\alpha_0)}{0,327 + 0,024\alpha_0}. \tag{13}$$

In the same way we determine the function $\chi_1(z)$ and its corresponding eigenvalue ϵ_1. For this reason, set

$$\chi_1(z) = \frac{C_1}{\sqrt{a}}(1+\gamma z^2)\sin\left(\frac{\pi z}{a}\right), \tag{14}$$

where γ is a variational parameter, and C_1 is a normalizing constant. From the normalizing condition for $\chi_1(z)$ and from (5), (6) and (14) we obtain

$$C_1 = \left(1 + 0,565\gamma_0 + 0,114\gamma_0^2\right)^{-1/2}, \tag{15}$$

$$\gamma_0 = a^2\gamma,$$

$$\epsilon_1 = \frac{\hbar^2 C_1^2}{2ma^2}\Big[9,870 + 3,114\alpha_0 + (5,580 + 2,229\alpha_0)\gamma_0 + (2,257 + 0,549\alpha_0)\gamma_0^2\Big] \tag{16}$$

for

$$\gamma_0 = \frac{\left[(0,565 + 0,053\alpha_0)^2 - 0,025\alpha_0\right]^{1/2} - (0,565 + 0,097\alpha_0)}{0,320 + 0,028\alpha_0}. \tag{17}$$

In such a way, for the energy in the first two sub-bands (corresponding to the ground and first excited state of the transversal motion of the current-carriers), according to (6), we

will have

$$E_s(k_1,k_2) = \epsilon_s + \frac{\hbar^2}{2m}(k_1^2+k_2^2), \quad s = 0, 1, \tag{18}$$

where ϵ_0 and ϵ_1 are defined by (9)-(13) and (15)-(17).

Further, we will assume that practically there are current-carriers only in the first energy sub-band ($s = 0$), i.e., we will consider that the absolute temperature T of the film satisfies the condition

$$kT < \epsilon_1 - \epsilon_0 \tag{19}$$

(k is the Boltzmann constant) and, naturally, the Fermi energy ϵ_F has a value, smaller than that of ϵ_1,

$$\epsilon_F < \epsilon_1 . \tag{20}$$

For example, for $\Gamma = 0{,}795$, $\varkappa = 2{,}4$, $\varkappa_0 = 1$, $a = 10^{-6}$ cm and $m = 0{,}1 m_0$ (m_0 is the free electron mass), from the formulae obtained above, we find

$$\epsilon_0 = 1{,}035.10^{-2} \text{ eV}, \quad \epsilon_1 = 3{,}881.10^{-2} \text{ eV}, \quad T < 330^\circ\text{K},$$

and for $m = m_0$ and under the same values of the other parameters

$$\epsilon_0 = 0{,}188.10^{-2} \text{ eV}, \quad \epsilon_1 = 0{,}496.10^{-2} \text{ eV}, \quad T < 35{,}7^\circ\text{K}.$$

Since the scattering of current-carriers from defects situated on the film boundary surfaces will be more substantial for $\varkappa < \varkappa_0$, then we will give as an illustration the values of ϵ_0 and ϵ_1 and the temperature condition for $\Gamma = 0{,}795$, $\varkappa = 2$, $\varkappa_0 = 16$, $a = 10^{-6}$ cm and $m = 0{,}1 m_0$, which follow from (1),(9),(11)-(13),(15),(16),(17) and (19),

$$\epsilon_0 = 0{,}723.10^{-2} \text{ eV}, \quad \epsilon_1 = 3{,}482.10^{-2} \text{ eV}, \quad T < 320^\circ\text{K}$$

while for $m = m_0$ and when the other parameters rest the same

$$\epsilon_0 = -\,0{,}129.10^{-2} \text{ eV}, \quad \epsilon_1 = 0{,}089.10^{-2} \text{ eV}, \quad T < 25{,}3^\circ\text{K}.$$

Finally, note that the image forces in a number of cases of thin crystalline films have a considerable influence on the energy spectrum and wave functions of the current-carriers within them, which is the reason, in particular, for them to contribute to the change of the energy gap and the film electrical resistivity.

3. TRANSPORT RELAXATION TIME OF CURRENT-CARRIERS

Now, using the results obtained above, we will calculate the transport relaxation time τ of the current-carriers in the first energy sub-band, assuming that of essential importance is only the scattering mechanism connected with screened Coulomb or electric dipole centres distributed on the boundary surfaces of the considered thin film. We will also suppose that scattering from centres on the one surface of the film practically takes place without the influence of the ones on its other surface. Then, it will be sufficient to consider the case when scattering centres with average concentration N can be found only on the one surface of the film, and this will be done further in the paper, and we will consider this surface to be $z + a = 0$. Under these assumptions and the ones mentioned in the Introduction, for the inverse transport relaxation time $1/\tau(\vec{K}_\parallel)$ of a current-carrier with two-dimensional quasi-wave vector $\vec{K}_\parallel = (k_1, k_2, 0)$ (parallel to the plane Oxy) and energy in the framework of the first sub-band ($s = 0$) according to Baskin and Entin [4] (see also [7]), we have

$$\frac{1}{\tau(\vec{K}_\parallel)} = \frac{N}{4\pi\hbar}\int d^2K'_\parallel \left\langle U_{00}(\vec{K}_\parallel - \vec{K}'_\parallel)\, U_{00}(\vec{K}'_\parallel - \vec{K}_\parallel)\right\rangle_{ct} \delta\left(E_0(\vec{K}_\parallel) - E_0(\vec{K}'_\parallel)\right) \tag{21}$$

Here $\langle \ldots \rangle_{ct}$ means averaging on the orientation of the scattering centres, $\delta(x)$ is the Dirac function and

$$U_{00}(\vec{K}_\parallel - \vec{K}'_\parallel) = \int_V d^3r\, \psi^*_{0\vec{K}_\parallel}(\vec{r})\, U(\vec{r})\, \psi_{0\vec{K}'_\parallel}(\vec{r}). \tag{22}$$

where $U(\vec{r})$ is the operator of the interaction energy of the current-carrier with one scattering centre (choosen to lie on the z axis), and the functions $\psi_{0\vec{K}_\parallel}(\vec{r})$ are defined by (1),(4),(7),(9),(10),(12) and (13).

In the considered cases of Coulomb and electric dipole centres (when their screening radius ρ_0 is smaller than the film thickness $2a$), we can approximately write for the first ones

$$U_c(\vec{r}) = \pm \frac{Ze^2}{\varkappa r} e^{-\frac{r}{\rho_0}} \tag{23}$$

and for the second ones[30]

$$U_d(\vec{r}) = -\frac{e(\vec{q}\cdot\vec{r})}{\varkappa r^3}\left(1+\frac{r}{\rho_0}\right)e^{-\frac{r}{\rho_0}}, \tag{24}$$

where

$$r = \left[x^2+y^2+(z+a)^2\right]^{1/2}, \quad z^2 < a^2$$

and Ze and $\vec{q}$ are the magnitude of the electric charge of the Coulomb centre and the electric dipole moment of the dipole centre, respectively.

Consider first the case of scattering from Coulomb centres. Then, by substituting (4),(7) and (23) in (22) and integrating along the coordinates, we obtain

$$U_{cc}^{c}(\vec{P}) = \pm\frac{Ze^2C_0^2}{2\pi\varkappa a}\frac{e^{-aW}}{W}\left(1+\beta\frac{\partial^2}{\partial W^2}\right)^2 J(W), \tag{25}$$

where

$$\vec{P} = \vec{K}_{\parallel}-\vec{K}_{\parallel}', \quad W=\sqrt{S_0^2+P^2}, \quad S_0 = 1/\rho_0, \quad P=|\vec{P}| \tag{26}$$

and

$$J(W) = \frac{sh(aW)}{\left(\frac{a^2}{\pi^2}W^2+1\right)W}. \tag{27}$$

Now, substituting (25), for (26) and (27), in (21) and passing to polar coordinates $K_{\parallel}'$ and φ',

$$K_{\parallel}' = \sqrt{K_1'^2+K_2'^2}, \quad \varphi' = \arccos\left(\frac{\vec{K}_{\parallel}\cdot\vec{K}_{\parallel}'}{K_{\parallel}K_{\parallel}'}\right), \quad K_{\parallel}=|\vec{K}_{\parallel}|, \tag{28}$$

when integrating along K_1' and K_2', and then integrate along $K_{\parallel}'$, we find

$$\frac{1}{\tau^c(K_{\parallel})} = \frac{mNZ^2e^4C_0^4}{8\pi^2\hbar^3\varkappa^2a^2}\,Q^c(K_{\parallel}), \tag{29}$$

where

$$Q^c(K_{\parallel}) = \frac{1}{\pi}\int_0^{\pi}d\varphi'\,\frac{e^{-2aW_0}}{W_0^2}\left[\left(1+\beta\frac{\partial^2}{\partial W_0^2}\right)^2 J(W_0)\right]^2 \tag{30}$$

for

$$W_0 = \sqrt{S_0^2+4K_{\parallel}^2\sin^2\frac{\varphi'}{2}}. \tag{31}$$

Obviously, when the condition

$$K_c^2\rho_0^2 \ll 1 \tag{32}$$

holds, where K_c is a characteristic value of $K_{\parallel}$ (for instance, for the strongly degenerated statistics of the current-carriers $\hbar K_c$ will be the Fermi quasi-momentum), according to (26) and (31), instead of (30) we will approximately have

$$Q^c(K_{\parallel}) \cong \rho_0^2\,G_c \tag{33}$$

for

$$G_c = e^{-2\frac{a}{\rho_0}}\left[\left(1+\beta\frac{\partial^2}{\partial S_0^2}\right)^2 J(S_0)\right]^2. \tag{34}$$

The calculations are carried out in an analogical way for the scattering from electric dipole centres. Then, from (4),(7),(22) and (24) we find

$$U_{cc}^{D}(\vec{P}) = \frac{-eC_0^2}{2\pi\varkappa a}\,e^{-aW}\left[q_3 - i\frac{(\vec{q}_{\parallel}\cdot\vec{P})}{W}\right]\left(1+\beta\frac{\partial^2}{\partial W^2}\right)^2 J(W), \tag{35}$$

where q_j, j = 1,2,3, are the Cartesian coordinates of the vector $\vec{q}$, and we have set $\vec{q}_{\parallel} = (q_1, q_2, 0)$, while the quantities $\vec{P}$, W, S_0 and $J(W)$ are determined by the corresponding formulae in (26) and (27). Further we will assume that the longitudinal (i.e., parallel to the film's surfaces) component $\vec{q}_{\parallel}$ of the electric dipole moment $\vec{q}$ of the scattering centres has random distribution of orientation, and the average values of $q_{\parallel}^2$ and q_3^2 are $\langle q_{\parallel}^2\rangle$ and $\langle q_3^2\rangle$, respectively, and they will be considered as known values. Under these conditions, by substituting (35) in (21) and taking into account (26) and (27), by averaging along all possible orientations of $\vec{q}_{\parallel}$ and along all possible values of $q_{\parallel}^2$ and q_3^2, and after that using the polar coordinates $K_{\parallel}'$ and φ', according to (28), when integrating along K_1' and K_2', and integrate along $K_{\parallel}'$, we obtain

$$\frac{1}{\tau^{D}(K_{\parallel})} = \frac{me^2NC_0^4}{8\pi^2\hbar^3\varkappa^2a^2}\left[\langle q_3^2\rangle\,Q_{\perp}^{D}(K_{\parallel}) + 2\langle q_{\parallel}^2\rangle K_{\parallel}^2\,Q_{\parallel}^{D}(K_{\parallel})\right] \tag{36}$$

for

$$Q_{\perp}^{D}(K_{\parallel}) = \frac{1}{\pi}\int_0^{\pi}d\varphi'\,e^{-2aW_0}\left[\left(1+\beta\frac{\partial^2}{\partial W_0^2}\right)^2 J(W_0)\right]^2, \tag{37}$$

$$Q_{\parallel}^{d}(K_{\parallel}) = \frac{1}{\pi}\int_0^{\pi} d\varphi' \sin^2\frac{\varphi'}{2}\,\frac{e^{-2aW_0}}{W_0^2}\left[\left(1+3\frac{\ell}{aW_0^2}\right)^2 J(W_0)\right]^2, \quad (38)$$

where W_c is given by (3I) and, as in (30), the magnitude $J(W_c)$ is obtained from (27) by the change of W with W_0. When condition (32) holds, instead of (37) and (38) we will approximately have

$$Q_{\perp}^{d}(K_{\parallel}) \cong Q_0, \quad Q_{\parallel}^{d}(K_{\parallel}) \cong \frac{1}{2}\rho_c^2\, Q_0, \quad (39)$$

where Q_0 is given by (34).

Here we will note that the conditions for validity of the consideration of electric dipole centres as point (ideal) dipoles and of the first Born approximation for the scattering, might be written in the following way, respectively:

$$K_0 d < 1, \quad 1 \ll \frac{2\pi}{K_0 r_c},$$

where d is the distance between the centres of the positive and negative electric charge in the dipoles, while the distance r_c for scattering from Coulomb and electric dipole centres is determined by the corresponding equations:

$$r_c\, e^{\frac{r_c}{\rho_c}} = \frac{2mZe^2}{\hbar^2 K_0^2 \varkappa} \quad \text{and} \quad \frac{r_c^2\, e^{\frac{r_c}{\rho_c}}}{1+\frac{r_c}{\rho_c}} = \frac{2meq}{\hbar^2 K_0^2 \varkappa}.$$

For example, for $\Gamma = 0{,}795$, $\varkappa = 2{,}4$, $\varkappa_0 = $ I, $a = 10^{-6}$ cm, $m = 0{,}$I$\,m_e$, $N = $ IO^{13} cm^{-2}, $Z = $ I, $\langle q_{\parallel}^2\rangle = \langle q_3^2\rangle = 9D^2$, $n = $ IO^{17} cm^{-3} (n is the average concentration of current-carriers in the film) and $T \ll 90^{\circ}$K, (32) will hold comparatively well and hence from (29) and (33) and from (36) and (39) we approximately find

$$\tilde{\tau}^{c} = 0{,}569.\text{IO}^{-\text{IO}}\ \text{s}, \quad \tilde{\tau}^{d} = 0{,}886.\text{IO}^{-7}\ \text{s}, \quad (40)$$

while for $\varkappa = 2$, $\varkappa_0 = $ I6, $m = m_0$, $T \ll$ IO$^{\circ}$K, and when the other parameters rest the same

$$\tilde{\tau}^{c} = \text{I},\!02\text{I}.\text{IO}^{-7}\ \text{s}, \quad \tilde{\tau}^{d} = \text{I},\!498.\text{IO}^{-6}\ \text{s}. \quad (4\text{I})$$

If the image forces are not accounted ($\ell = 0$), then, instead of (40) and (4I) we will have $\tilde{\tau}^{c} = 0{,}562.IO^{-\text{IO}}$ s, $\tilde{\tau}^{d} = 0{,}874.$IO^{-7} s and $\tilde{\tau}^{c} = 0{,}3$II.IO^{-7} s, $\tilde{\tau}^{d} = 4{,}548.$IO^{-6} s.

We will make a further remark that the presence of electric dipole centres as defects in a close proximity to the boundary surfaces of the crystals may be due to different reasons, such as association of oppositely charged electric centres (ions), adsorption of dipole molecules, induction of dipole moments in physically adsorbed atoms and molecules on the surface of solids, etc.(see, for example,[3I-35]).

4. ELECTRICAL RESISTIVITY

Under the conditions mentioned in the Introduction and when the inequalities (I9) and (20) hold for the surface electrical resistivity ρ of the film (for an applied constant and homogeneous electric field parallel to its surfaces), we have approximatively[4,7]*)

$$\rho = -\frac{16\pi^3 m^2}{\hbar^2 e^2}\left[\int d^2K_{\parallel}\, K_{\parallel}^2\, \tau(\vec{K}_{\parallel}) \int dE\, \frac{\partial f_0(E)}{\partial E}\,\delta\!\left(E - E_0(\vec{K}_{\parallel})\right)\right]^{-1}, \quad (42)$$

where $f_0(E)$ is the equilibrium function of Fermi-Dirac distribution,

$$f_0(E) = \left[1 + \exp\left(\frac{E-\zeta}{kT}\right)\right]^{-1}, \quad (43)$$

while $E_0(\vec{K}_{\parallel})$ is determined by (I8) in the considered problem. So, in order to find the surface electrical resistivity of the film for scattering of the current-carriers from screened Coulomb or electric dipole impurity centres, distributed on one of the film's boundary surfaces, we have to substitute (I8) and (43) and (29)-(3I) or (3I),(36)-(38) in (42) and then calculate the integrals along φ', K_1, K_2 and E. However, since this is a difficult problem from the point of view of technique, we will restrict ourselves to the case when condition (32) holds and the current-carriers' statistics is strongly degenerated, i.e., we have

$$1 \ll \frac{\zeta_0 - \epsilon_0}{kT} = \frac{\hbar^3 (3\pi^2 n)^{2/3}}{2mkT} \quad (44)$$

(ζ_0 is the value of the chemical potential ζ for $T = 0$). Then, using (33) and (39) instead of (30), (37) and (38), for scattering from screened Coulomb and electric dipole centres, we

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*) Here, for simplicity, the electric conductivity of the film is assumed to be unipolar.

obtain, respectively,

$$\rho \cong \rho_c^0 = \frac{mNZ^2e^2\rho_0^2 C_0^4}{2\hbar \varkappa^2 a^2(\zeta_0-\epsilon_0)} \, Q_0 \tag{45}$$

and

$$\rho \cong \rho_d^c = \frac{mNC_c^4}{2\hbar \varkappa^2 a^2(\zeta_0-\epsilon_0)} \left[\frac{2m\rho_0^2}{\hbar^2}(\zeta_0-\epsilon_0)\langle q_\parallel^2\rangle + \langle q_3^2\rangle\right] Q_0, \tag{46}$$

where Q_0 is given by (34). Moreover, as is known, because of (44), we have for the screening radius ρ_0:

$$\rho_0^2 = \frac{\hbar^2\varkappa}{4me^2}\left(\frac{\pi}{3n}\right)^{1/3}. \tag{47}$$

For example, under the same conditions for which the results (40) and (4I) were obtained, from (45)-(47) we find, respectively, $\rho_c^0 = 2{,}66.10^{-10}$, $\rho_d^0 = 1{,}71.10^{-13}$ and $\rho_c^c = 14{,}82.10^{-12}$, $\rho_d^c = 1{,}01.10^{-13}$ (in units of s/cm). If the image forces are not taken into account, instead of these values, we get: $\rho_c^c = 2{,}69.10^{-10}$, $\rho_d^c = 1{,}73.10^{-13}$ and $\rho_c^0 = 4{,}86.10^{-12}$, $\rho_d^0 = 0{,}33.10^{-13}$.

Practically, of importance is the case of a thin film with bipolar electric conductivity (intrinsic semiconductor or semimetal) with scattering impurity centres on its two boundary surfaces. If for such a film the conditions for the problem considered up to now hold in every other respect, then it is clear that in the general case the inverse transport relaxation time of any current-carrier with a quasi-wave vector $\vec{K}_\parallel$ will equal the sum of inverse transport relaxation times of the same current-carrier under its scattering separately from the unitype (screened Coulomb or electric dipole) impurity centres on the one and the other boundary surfaces of the film. These inverse transport relaxation times are determined by the obtained formulae (29)-(3I) or (3I) and (36)-(38) after substituting the current-carrier' effective mass and the corresponding characteristics of the considered type of scattering impurity centres (Z or $\langle q_\parallel^2\rangle$ and $\langle q_3^2\rangle$, N) for each one of the two boundary surfaces separately. Besides, the screening radius ρ_0 (in Debye-Hückel approximation) will be determined by the formula /30/

$$\frac{1}{\rho_0^2} = \frac{4\pi e^2}{\varkappa}\left(\frac{dn_e}{d\zeta_e} + \frac{dn_h}{d\zeta_h}\right),$$

where n_e, ζ_e and n_h, ζ_h are the average concentrations and chemical potentials of the conductivity electrons and holes,

respectively, in the film in the absence of scattering centres. By determining in this way the transport relaxation time $\tau^{(e)}(\vec{K}_\parallel)$ of the conductivity electrons and that $\tau^{(h)}(\vec{K}_\parallel)$ of holes, we can use them one by one together with the corresponding Fermi-Dirac distribution function and energy $E_c(\vec{K}_\parallel)$ (according to (I8)) to calculate with the help of(42)the surface electrical resistivity ρ_e and ρ_h, the first one being connected only with the conductivity electrons, and the second, only with the holes. Then the surface electrical resistivity ρ of the film will be

$$\rho = \frac{\rho_e \rho_h}{\rho_e + \rho_h}.$$

Naturally, for the validity of the final formulae for electrical resistivity ρ in the problem, it is further necessary for every type of current-carriers λ (λ = e,h) to fulfill the condition

$$2\hbar e^2 a \frac{n_\lambda \rho_\lambda}{m_\lambda} \ll \epsilon_1^{(\lambda)} - \epsilon_0^{(\lambda)},$$

where m_λ is the effective mass of a current-carrier of the type λ, while $\epsilon_c^{(\lambda)}$ and $\epsilon_1^{(\lambda)}$ are determined from (II)-(I3) and (I2), (I6) and (I7), respectively, for $m = m_\lambda$.

5. CONCLUSIONS

The calculations carried out in this paper show that in a number of cases of thin film crystalline materials the image forces can considerably influence the low-energy states of the current-carriers within them and their low-temperature electrical resistivity when the latter is due to scattering mechanisms related to their boundary surfaces. Relatively, the influence of these forces is substantial when the thickness of the film is sufficiently small and for a not very small effective mass of the current-carriers, and for one and the same scattering centres it leads to a greater electrical resistivity when the permittivity of the thin film material is smaller than that of the surrounding medium compared to the electrical resistivity in the inverse case. The obtained results for the transport relaxation time of the current-carriers and for the electrical resistivity when the scattering mechanism is connected with the screened Coulomb and electric dipole centres are of interest both in the presence or

absence of image forces. The applicability of the results is restricted by the requirement for the screening radius to be smaller than the thickness of the film material, but this does not concern the formulae for the electronic states in the absence of scattering centres. Note that the inaccuracy of these formulae because of the non-self-consistency of the electronic states, is expected to be relatively smaller when the thin film materials are intrinsic semiconductors or semimetals with close in magnitude effective masses of the conductivity electrons and holes. Moreover, the image forces cause an additional change of the energy gap of the film material, which could play a considerable role, for instance, in the case of low temperatures and semiconducting materials with a sufficiently small energy gap.

The influence of the image forces on the optic, galvano-magnetic and other quantum size effects in thin film materials will be analogical to the one considered in this paper.

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