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CROSS-OVER EXPONENTS FOR ANISOTROPIC QUADRATIC PERTURBATIONS

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Наттерманн Т. **E17 - 10316**

Кроссовер-экспоненты анизотропных квадратичных возмущений

Рассматривается влияние возмущений, квадратичных относительно n -компонентного параметра порядка, на систему, испытывающую фазовый переход второго рода. Оказывается, что только лишь для соответствующим образом подобранных возмущений поведение может быть описано единственной кроссовер-экспонентой ϕ_i. Эти экспоненты получены путем расчёта скелетных диаграмм в низшем порядке для двух моделей размерностью $d = 4 - \epsilon$: для модели с кубической анизотропностью и модели со слабым диполярным взаимодействием. Кратко обсуждается связь этих экспонент с экспериментально наблюдаемыми величинами.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований

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Cross-Over Exponents for Anisotropic Quadratic Perturbations

The influence of anisotropic perturbations quadratic in the n-component order parameter in a system undergoing a second order phase transition is considered. It turns out that only for appropriate chosen perturbations the behaviour can be described by a unique cross-over exponent ϕ_i. These exponents are calculated by a lowest order skeleton-graph approach for two $d=4-\epsilon$-dimensional models: a model with cubic anisotropy and a model with weak dipolar interaction. The relation of the exponents ϕ_i to experimental observable quantities is briefly discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. In connection with the discussion of critical properties of a system undergoing a second order phase transition the question arises whether an external perturbation described by a Hamoltonian gH_a leads to a new type of critical behaviour. In the scaling theory of critical phenomena one assumes that the singular part of the free energy $F(T,g)$ can be written as

$$F(T,g) = |t|^{2-\alpha} f(g|t|^{-\phi}) , \tag{1}$$

where $t=(T-T_c)/T_c$, $T_c=T_c(g=0)$ and $f(x)$ has a finite limit for $x\to 0$ /1/. However, for positive values of the cross-over exponent ϕ, $g|t|^{-\phi}$ increases if $t\to 0$ and a new type of critical behaviour is expected.

Recently, it has been suggested /2-4/ that the cross-over exponents ϕ_i for anisotropic quadratic perturbations

$$g_i H_a(\lambda^i_{\alpha\beta}) = \frac{1}{2} g_i \sum_{\alpha\beta} \lambda^i_{\alpha\beta} \int d^d x \Phi_\alpha(x)\Phi_\beta(x) \tag{2}$$

in a system described by an n-component order parameter $\Phi_\alpha(x)$ ($\alpha = 1,\ldots, n$) should be observable directly in experiment. Here $\lambda^i_{\alpha\beta}$ has to be chosen in an appropriate way as will be discussed below (see Eqs. (6)-(9)). Indeed, a perturbation of the form (2) is obtained for example from a Hamiltonian $H(Q_i, \Phi_\alpha)$ if one integrates the field $Q_i(x)$ out of the partition function. Here it is assumed that H is only quadratic in the non-critical variable $Q_i(x)$ and the coupling Hamiltonian of both the fields reads

$$H_{int}(Q_i, \Phi_\alpha) = \frac{1}{2}\sum_{\alpha,\beta} \lambda^i_{\alpha\beta} \int d^d x\, Q_i(x)\, \Phi_\alpha(x)\Phi_\beta(x). \qquad (3)$$

The g_i denotes the thermodynamical conjugate variable to the field $Q_i(x)$. Examples for g_i are uniaxial pressure in ferromagnets and systems undergoing structural transitions /2/, the square of a homogeneous magnetic field in uniaxial antiferromagnets /3/, and the electric field in improper ferroelectrics /5/. We remind of three possibilities to determine the cross-over exponents ϕ_i corresponding to perturbations (2):

(i) The phase boundary $T_c(g_i)$ varies as $T_c(g_i) - T_c(0) \propto g_i^{1/\phi_i}$.

(ii) The static non-ordering susceptibility $\chi_{g_i} = \partial^2 F/\partial g_i^2$ diverges as $|t|^{-\alpha_i}$ for $g_i = 0$, $t \to 0$ where $\alpha_i = 2\phi_i - \nu d$.

(iii) The imaginary part $\Omega\Gamma_Q(k,\Omega)$ of the self-energy of the field Q_i behaves as $\lim_{\Omega\to 0} \Gamma_Q(k<\xi^{-1},\Omega) \propto |t|^{-\rho_i}$, where $\rho_i = 2\phi_i + \nu(z-d)$ /4/ z is the dynamical critical exponent /6/ and $\xi \propto |t|^{-\nu}$ denotes the correlation length. In deriving this result it has been assumed that the leading contributions to $\Gamma_Q(k,\Omega)$ arise due to order parameter fluctuations (but not due to energy fluctuations) and that the order parameter obeys the dynamical scaling /4, 6/.

Finally we want to stress that the coupling to a non-critical variable can lead to a first order transition in some cases. However, if this transition is close to a second order, one expects to observe the above-mentioned power laws in a broad region around the transition point /7/.

So far, the cross-over exponents for anisotropic quadratic perturbations have been calculated using renormalization group techniques close to 4 dimensions ($d = 4-\epsilon$) for an isotropic /1,8/ and cubic /4,9/ models with short range interaction and an isotropic model with strong dipolar interaction /10/. These models are limiting cases of more general (and realistic) ones which will

be considered in this paper, namely, (i) a model with cubic anisotropy both in the dispersion relation and in the interaction part /11, 12/ and (ii) a model with weak dipolar interaction /13/. Both models exhibit interesting effects as has been discussed in previous papers /11 - 13/. For brevity, we will use the results of these paper without a new derivation throughout this work. For the calculation we use the skeleton-graph approach /14/ in the main logarithmic accuracy for a $d = 4-\epsilon$-dimensional system, where ϵ is assumed to be small. Due to the complicated form of the propagators for the models considered here, higher order calculations become much harder than for the models considered in /4, 8 -10/. Finally we note that the exponents we derive are effective ones, i.e., they change into those obtained earlier if one approaches sufficiently close to the critical point. However, as has been discussed in detail in /11 - 13/, this asymptotical critical region may be inaccesible to experiment.

2. We start with the general Ginzburg-Landau-Wilson functional $H = \int d^d x (\frac{1}{2} r_0 \vec{\Phi}^2 + \sum_{\alpha,\beta,\gamma,\delta} u_{\alpha\beta\gamma\delta} \Phi_\alpha \Phi_\beta \Phi_\gamma \Phi_\delta +$ gradient terms), where we have only one quadratic invariant in H since all components should become critical simultanously ($r_0 = r_0'(T - T_{c0})$). The form of the gradient terms will be specified later. It is convenient to calculate the cross-over exponents ϕ_i from the relation (see Eq. (1))
over exponents ϕ_i from the relation (see Eq. (1))

$$\partial\chi^{-1}/\partial g_i|_{g_i=0} \propto |t|^{\gamma-\phi_i}, \qquad (4)$$

where $\chi_{\alpha\beta}(k) = = <\Phi^\alpha_k \Phi^\beta_{-k}>$ denotes the susceptibility and Φ^α_k is the Fourier transform of $\Phi_\alpha(x)$. First of all we consider the response of $\chi^{-1}_{\alpha\beta}$ to an *arbitrary* quadratic perturbation $\tilde{g}_i H_a(\tilde{\lambda}^i_{\alpha\beta})$:

$$\partial\chi^{-1}_{\alpha\beta}(k)/\partial\tilde{g}_i|_{\tilde{g}_i=0} \equiv \tilde{\Lambda}^i_{\alpha\beta}(k) =$$

$$= -\sum_{\nu\mu} \chi^{-1}_{\alpha\nu}(k)\chi^{-1}_{\beta\mu}(k) <\phi^\nu_k \phi^\mu_{-k} H_a(\tilde{\lambda}^i_{\gamma\delta})>_{c,\tilde{g}_i=0}$$

$\tilde{\Lambda}^{i}_{\alpha\beta}(k)$ obeys a Ward-identity

$$\tilde{\Lambda}^{i}_{\alpha\beta}(k) = \tilde{\lambda}^{i}_{\alpha\beta} - \sum_{\gamma,\delta,\nu,\mu} \int \frac{d^d p}{(2\pi)^d} \Gamma_{\alpha\beta\gamma\delta}(k,k,p,p)\chi_{\gamma\nu}(p)\chi_{\delta\mu}(p)\tilde{\lambda}^{i}_{\nu\mu},$$

where $\Gamma_{\alpha\beta\gamma\delta}(k,p)$ denotes the renormalized four-point interation with different external momenta k, p. However, it is easier to calculate $\Gamma_{\alpha\beta\gamma\delta}$ if all external momenta are of the same order of magnitude

$$K_d^{-1} = \lambda^{(d-1)} \pi^{d/2} \Gamma(\frac{d}{2}),$$

$$\Gamma_{\alpha\beta\gamma\delta}(k) = (12K_d)^{-1} k^{\epsilon-2\eta} \gamma_{\alpha\beta\gamma\delta}(k/\Lambda),$$

$$\Gamma_{\alpha\beta\gamma\delta}(\Lambda) = u_{\alpha\beta\gamma\delta},$$

Λ denotes the cut-off momentum, $\Gamma_{\alpha\beta\gamma\delta}$ exhibits scaling behaviour if there is a stable solution (fixed point) of the equation

$$d\gamma_{\alpha\beta\gamma\delta}(k/\Lambda)/d\ln k = 0, \quad (5)$$

where the stability is concluded from a linearization of (5) around the solutions $\gamma^*_{\alpha\beta\gamma\delta}$. Then, in the framework of the skeleton graph approach we can express $\tilde{\Lambda}^{i}_{\alpha\beta}$ by a matrix $\bar{\gamma}_{\alpha\beta\gamma\delta}(k/\Lambda)$ and $\tilde{\Lambda}^{j}_{\alpha\beta}$, arising from perturbations $\tilde{g}_j H_a(\tilde{\lambda}^{j}_{\alpha\beta})$:

$$d\tilde{\Lambda}^{i}_{\alpha\beta}/d\ln k = \sum_{j,\gamma,\delta} a_{ij}\bar{\gamma}_{\alpha\beta\gamma\delta}(k/\Lambda)\tilde{\Lambda}^{j}_{\gamma\delta}(k), \quad (6)$$

$\bar{\gamma}_{\alpha\beta\gamma\delta}$ is given by a series of diagrams (see e.g. /14/). To the lowest order (main logarithmic accuracy) we find

$$\bar{\gamma}_{\alpha\beta\gamma\delta}(k/\Lambda) = \sum_{\mu,\nu} \gamma_{\alpha\beta\mu\nu}(k/\Lambda)\rho_{\mu\nu\gamma\delta}, \quad (7)$$

where

$$\rho_{\mu\nu\gamma\delta} = (\epsilon - 2\eta)k^{\epsilon-2\eta} K_d^{-1} \int \frac{d^d p}{(2\pi)^d} \chi_{\mu\gamma}(p+k)\chi_{\nu\delta}(p).$$

First, let us consider the case, where $a_{ij} \propto \delta_{ij}$ in (6). Denoting the corresponding solutions by $\Lambda^{i}_{\alpha\beta}(k)$ we find in the asymptotical region $k/\Lambda \to 0$

$$\Lambda^{i}_{\alpha\beta}(k) = \lambda^{i}_{\alpha\beta}(k/\Lambda)^{\omega_i}, \quad (8)$$

ω_i and $\lambda^{i}_{\alpha\beta}$ are the eigenvalues and eigenvectors of the matrix $\bar{\gamma}^*_{\alpha\beta\gamma\delta} = \bar{\gamma}_{\alpha\beta\gamma\delta}(\gamma^*_{\alpha\beta\gamma\delta})$. In general, there are several eigenvectors $\lambda^{i}_{\alpha\beta,l}$ $(l = 1,\dots,f_i)$ to a eigenvalue ω_i, which we choose to be orthogonal $\sum_{\alpha,\beta}\lambda^{i}_{\alpha\beta,l}\cdot\lambda^{i'}_{\alpha\beta,l'} = \delta_{ii'}\delta_{ll'}$.

We emphasize, that only the perturbations $g_i H_a(\lambda^{i}_{\alpha\beta})$ lead to a single power law behaviour of $\partial\chi^{-1}/\partial g_i$ near to the critical point. For an arbitrary $\tilde{\Lambda}_{\alpha\beta}$ we obtain

$$\tilde{\Lambda}_{\alpha\beta} = \sum_{i,l} a^{i}_{l}\Lambda^{i}_{\alpha\beta,l}, \quad a^{i}_{l} = \sum_{\alpha,\beta}\lambda^{i}_{\alpha\beta,l}\tilde{\lambda}_{\alpha\beta}. \quad (9)$$

Taking into account that for $k \to 0$ k changes into $\xi^{-1} \propto |t|^{\nu}$ in all expressions, we find from (4) and (8) $\nu\omega_i = \gamma - \phi_i$. Physically, the quantities $\Lambda^{i}_{\alpha\beta}(k)$ correspond to the renormalized coupling constants in the interaction Hamiltonian (3) if the three external momenta are of the same order of magnitude. Since we consider a vector model where all n components become critical simultaneously, the trace condition $\sum_{\mu}\bar{\gamma}^*_{\alpha\beta\mu\mu} = \bar{\gamma}^*\delta_{\alpha\beta}$ must be fulfilled /15/. Then $\frac{1}{\sqrt{n}}\delta_{\alpha\beta} = \lambda^{o}_{\alpha\beta}$ is an eigenvector of $\bar{\gamma}^*_{\alpha\beta\gamma\delta}$ and $\omega_0 = \bar{\gamma}^*$ is the corresponding eigenvalue. If we put $g_0 \propto t$ we obtain $\gamma - 1 = \nu\bar{\gamma}^*$ and hence $\phi_i = 1 + \nu(\omega_0 - \omega_i)$ with $\nu = \gamma/(2-\eta)$ The exponent η can be obtained separately and vanishes to order ϵ /12,14/.

3. Let us now consider two examples. For this aim we restrict ourselves to the cases where the Hamiltonian includes only two fourth order invariants, i.e.,

$$u_{\alpha\beta\gamma\delta} = u_1 \frac{1}{3}(\delta_{\alpha\beta}\delta_{\gamma\delta} + 2\,\text{perm.}) + u_2 g_{\alpha\beta\gamma\delta}\,,$$

where $g_{\alpha\beta\gamma\delta} = 1$ if $\alpha=\beta=\gamma=\delta$ and 0 otherwise. This corresponds to the case of cubic anisotropy and is the most general one for $n \le 3$ /15/. However, we note that for $n > 3$ the fixed point Hamiltonian exhibits this symmetry in many cases even if there are more bare fourth order couplings /16/. (We will denote the fixed point values of the corresponding coupling constants by γ_1^* and γ_2^*, respectively). Under this condition we find there, in general, different eigenvalues $\vec{\omega}_0, \omega_1, \omega_2$. The corresponding perturbations are Φ^2, $\Phi_\alpha\Phi_\beta$ and $\Phi_\alpha^2 - \Phi_\beta^2$, $(\alpha \neq \beta)$. These perturbations are still not orthogonal. However, this can be achieved easily in the usual way. For $n=3$ we find, e.g.,

$$\lambda^{\circ}_{\alpha\beta} = \frac{1}{\sqrt{3}}\delta_{\alpha\beta}\,;\lambda^1_{\alpha\beta,1} = \frac{1}{\sqrt{2}}(\delta_{\alpha 1}\delta_{\beta 2} + \delta_{\alpha 2}\delta_{\beta 1})\,,$$

$$\lambda^1_{\alpha\beta,2} = \frac{1}{\sqrt{2}}(\delta_{\alpha 2}\delta_{\beta 3} + \delta_{\alpha 3}\delta_{\beta 1})\,,$$

$$\lambda^1_{\alpha\beta,3} = \frac{1}{\sqrt{2}}(\delta_{\alpha 3}\delta_{\beta 1} + \delta_{\alpha 1}\delta_{\beta 3})\,;$$

$$\lambda^2_{\alpha\beta,1} = \frac{1}{\sqrt{2}}(\delta_{\alpha 1}\delta_{\beta 1} - \delta_{\alpha 2}\delta_{\beta 2})\,,$$

$$\lambda^2_{\alpha\beta,2} = \frac{1}{\sqrt{6}}(2\delta_{\alpha 3}\delta_{\beta 3} - \delta_{\alpha 1}\delta_{\beta 1} - \delta_{\alpha 2}\delta_{\beta 2})\,.$$

(i) First we consider a model with cubic anisotropy in the dispersion relation. The quadratic part $H^{(2)}$ of the Hamiltonian reads /11/:

$$H^{(2)} = \sum_\alpha \int \frac{d^d k}{(2\pi)^d}\frac{1}{2}(r_0 + k^2 - fk_\alpha^2)\Phi_k^\alpha\Phi_{-k}^\alpha$$

which leads to $\rho_{\alpha\beta\gamma\delta} = \delta_{\alpha\gamma}\delta_{\beta\delta}(1 + \frac{s}{4}(\delta_{\alpha\beta}-1))(1-f)^{-1/2}$, where $s=s(f)$ is a measure of the cubic anisotropy in the dispersion relation ($s(0) = 0$, $s(1) = 4$, see Eq. (A.4) of /11/). We find for the three eigenvalues:

$$\omega_0 = \frac{n+2}{3}\gamma_1^* + \gamma_2^*\,,\ \omega_1 = \frac{2}{3}\gamma_1^*(1-\frac{s}{4})\,,\ \omega_2 = \frac{2}{3}\gamma_1^* + \gamma_2^*\,, \tag{10}$$

$\gamma_1^*(n,f)$, $\gamma_2^*(n,f)$ are calculated in /11/ Eq. (A.7) and /12/ Eq. (3.11a) (note, however, that in these papers slightly different definitions have been used for the quantities denoted by the symbols γ_i^*). For $f=0$ then we get if $n \le 4$ (Heisenberg fixed point stable)

$$\gamma^{-1} = 1 - \frac{n+2}{n+8}\frac{\epsilon}{2}\,,\quad \phi_1 = \phi_2 = \gamma(1 - \frac{\epsilon}{n+8}) \tag{11a}$$

if $n \ge 4$ (cubic fixed point stable)

$$\gamma^{-1} = 1 - \frac{n-1}{3n}\frac{\epsilon}{2}\,,\ \phi = \gamma(1 - \frac{\epsilon}{3n})\,,\ \phi = (1 - \frac{n-2}{3n}\frac{\epsilon}{2})\,. \tag{11b}$$

The ϵ -expanded values of (11a,b) agree to order ϵ with those derived earlier /4, 8,9/. For $f \neq 0$ the expressions become more complicated but can easily be derived from (10). At $f = f_c(n)$ the transition changes from the second to the first order /11/. For $f = f_c(n)$ can be excluded from the final expressions for γ, ϕ_i:

$$\gamma^{-1} = 1 - ((\frac{n-1}{3})^{-1/2} + 1)\frac{\epsilon}{8}\,,$$

$$\phi_1 = (1 - (1 + \frac{2-n}{2(3(n-1))^{1/2}}) \frac{\epsilon}{8}),$$
$$\phi_2 = (1 - (1 - \frac{1}{(3(n-1))^{1/2}}) \frac{\epsilon}{8}). \tag{12}$$

For large values of f there is still a region where the system exhibits an approximate scaling behaviour and critical exponents can be estimated as discussed in /11,12/. Using the appropriate values of γ_1^*, γ_2^* (see the remark after Eq. (10)), one can show for *all* f that $\phi_1 < \phi_2$ for $n < 4$ and $\phi_1 > \phi_2$ for $n > 4$. The numerical values for γ, ϕ_1 and ϕ_2 for three different values of $\epsilon = 1$ are displayed in the *table.*

Table

	n = 2			n = 3		
	f = 0	$f = f_c = 0.74$	f = 0.95	f = 0	$f = f_c = 0.42$	f = 0.95
γ	1.25	1.24	1.26	1.30	1.29	1.33
ϕ_1	1.$\overset{1}{\cancel{2}}$3	1.09	1.14	1.18	1.16	1.24
ϕ_2	1.13	1.17	1.26	1.18	1.20	1.29

As usual, one expects shifts to larger values from the second order calculation (which is much harder than the first order one for $f \neq 0$), but the general picture should be the same. Using f = 0.95 as a typical value for $SrTiO_3$ we find for the exponents ρ_i and a_i, describing the sound attenuation and propagation in this material, respectively, $\rho_0 = 1.33$, $\rho_1 = 1.82$, $\rho_2 = 1.91$ and $a_0 \approx 0$, $a_1 = 0.48$, $a_2 = 0.58$. Here z = 2 has been assumed. We note, that the influence of f has the opposite tendency as conjectured by Murata /4/. A possible explanation for the deviations of the experimental values, which exhibit an extremely wide spread from $\rho = 1.1$ to $\rho = 3.95$ /17/, from those found here is the existence of systematic residuals strains /2/, which reduce the number of strongly fluctuating components.

(ii) As a second example we consider an isotropic system (i.e., $\gamma_2^* = 0$) with pweak dipolar interaction $g_0 \ll \Lambda^2$ /13/. Then

$$H^{(2)} = \sum_{\alpha,\beta=1}^{n \leq d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{2} (r_0 + k^2 + g_0 \frac{k_\alpha k_\beta}{k^2}) \phi_k^\alpha \phi_{-k}^\beta .$$

Since the system is isotropic we have $\omega_1 = \omega_2$, $\phi_1 = \phi_2 = \phi$. It is covenient to introduce the ratio $\lambda = g_0/r = g/|t|^\gamma$, where r is the renormalized inverse susceptibility. The limits $\lambda \to 0$ and $\lambda \to \infty$ correspond to the cases of isotropic short range (see Eq. (11a)) and dominating dipolar interaction, respectively. Performing the same calculations as in the previous case we find for $\lambda \to \infty$ $(1 < n \leq d)$

$$\gamma^{-1} = 1 - \frac{(n+2)^2}{(n^2 + 10n + 12)} \frac{\epsilon}{2}, \quad \phi = \gamma (1 - \frac{n^2 - 2}{(n^2 + 10n + 12)(n-1)} \epsilon). \tag{13}$$

Their ϵ-expanded values agree with those found in /10/. However, restricting for simplicity to the case n = d, it is possible to calculate the effective critical exponents $\gamma(\lambda)$ and $\phi(\lambda)$ for all λ. Indeed, if $\Lambda \to \infty$ we find for $\rho_{\alpha\beta\gamma\delta}$

$$n(n+2)\rho_{\alpha\beta\gamma\delta} = (n^2 - 4)\delta_{\alpha\gamma}\delta_{\beta\delta} + (1 + a(\lambda)) S_{\alpha\beta\gamma\delta} +$$
$$+ 2((n+2)\delta_{\alpha\gamma}\delta_{\beta\delta} - S_{\alpha\beta\gamma\delta}) b(\lambda),$$
$$S_{\alpha\beta\gamma\delta} = (\delta_{\alpha\gamma}\delta_{\beta\delta} + 2 \text{ perm.}), \quad a(\lambda) = (1+\lambda)^{-\epsilon/2},$$
$$b(\lambda) = ((1+\lambda)^{1-\epsilon/2} - 1)/(1 - \epsilon/2)\lambda$$

and hence

$$\gamma^{-1}(\lambda) = 1 - \frac{N_1(\lambda)}{D(\lambda)} \epsilon, \quad \phi(\lambda) = \gamma(\lambda) (1 - \frac{N_2(\lambda)}{D(\lambda)} \epsilon),$$

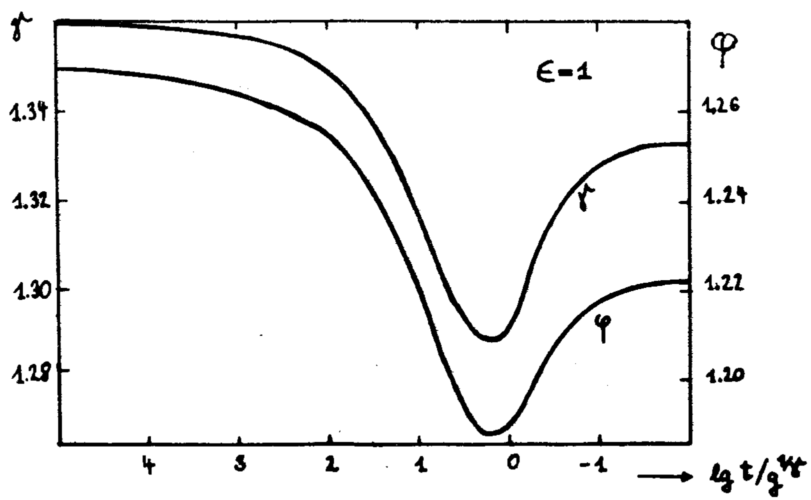
$$D(\lambda) = c_1 + c_2 a(\lambda) + c_3 b(\lambda), \quad N_1(\lambda) = c_4 + c_5 a(\lambda),$$

$$N_2(\lambda) = c_6 + c_7 a(\lambda) + c_8 b(\lambda),$$

$$c_1 = 2(n-1)(n^2 + 10n + 12), \quad c_2 = 2(n^2 + 6n + 20), \tag{14}$$

$$c_3 = 16(n-1), \quad c_4 = (n-1)(n+2)^2, \quad c_5 = (n+2)^2,$$

$$c_6 = 2(n^2 - 2), \quad c_7 = 4, \quad c_8 = 4n.$$



The effective critical exponents $\gamma(\lambda)$ *and* $\phi(\lambda)$ *versus* $-\frac{1}{\gamma}\lg\lambda = \lg(t/g^{1/\gamma})$.

$\gamma(\lambda)$ and $\phi(\lambda)$ are depicted in the figure. Both $\gamma(\lambda)$ and $\phi(\lambda)$ exhibit a pronounced minimum for $\lambda \approx 2.2$. The result for γ has been found already in [13] using a slightly different way. As discussed there, the position of the minimum is in good agreement with the experimental result for EuO. The prediction for ϕ could be tested from the anomaly in the sound propagation of shear waves [18], although the corresponding magnetostrictive coupling constant is small [19] in this substance. We remark finally, that the method presented in this paper can be used for the calculation of cross-over exponents both in a higher order of accuracy and for more complicated models.

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