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METHOD OF INVESTIGATION OF NUCLEAR REACTIONS IN CHARGE-NONSYMMETRICAL COMPLEXES

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1 Introduction

At the present time interest in the study of charge-nonsymmetrical muonic complexes like HμZ (H$\equiv p, d, t$; Z$\equiv$ ^{3,4}He, ^{6,7}Li, ^{7}Be,...) [1–10] has significantly risen. It is connected with appearance of real possibility of measuring the characteristics of fusion reactions in the ultra-low ($\sim$ keV) energy range of relative motion of nuclei. This will allow one to solve such problems as problems existing in astrophysics [11,12], verification of the hypothesis concerning charge symmetry in strong interactions for the energy range mentioned [1]. Thus, application of muon catalyzed fusion to investigations of nuclear reactions in the keV energy region seems to be more promising than the traditional method. Due to very small cross-sections for the processes under investigation ($\sim 10^{-40} - 10^{-45}$ cm^2) and insufficient beam intensity, investigation of nuclear reactions in single collisions is not practical.

The present work is devoted to description of the rate of nuclear synthesis in $d\mu$He complex as well as optimization of experiments with a deuterium-helium mixture (D_2+^{3}He). The aim of these experiments is to obtain information about rates of deexcitation and decay of muonic complexes ($d\mu^{3,4}$He) together with partial rates for nuclear fusion inside these complexes [2].

2 Scheme of muonic processes in D_2+He mixture

Figure 1 presents a scheme of muonic processes occurring in the D_2+^{3}He mixture after the negative muon stopped in it. Muon transfer from muonic atoms $d\mu$ to He nuclei occurs via creation of an intermediate μ-molecular $d\mu$He complex in the excited state with the orbital momentum $J = 1$ (term 2pσ). Subsequently, the complex decays to a Heμ atom and a deuteron (predissociation channel) with the rate $\lambda^1_{\rm p}$ or deexcites to the ground state 1sσ with emission of the 6.85 keV X-ray or Auger electron. The rates of the deexcitation process are λ^1_γ and $\lambda^1_{\rm e}$, respectively. The total rate $\lambda^1_{\rm dec} = \lambda^1_{\rm p} + \lambda^1_{\rm e} + \lambda^1_\gamma$ is the decay rate of the $d\mu$He complex in the state with $J = 1$. The complex in the ground state 1sσ decays to hydrogen isotope nucleus and to a muonic atom He in the

[1] In nuclear fusion reactions in charge-nonsymmetrical muonic complexes the astrophysical range of energies ($\sim$ keV) is realized [13,14].

[2] Choice of the $d\mu$He system for experimental investigations is motivated by a large number of theoretical studies on this complex (in comparison with other charge-nonsymmetrical complexes) [3,10,15].

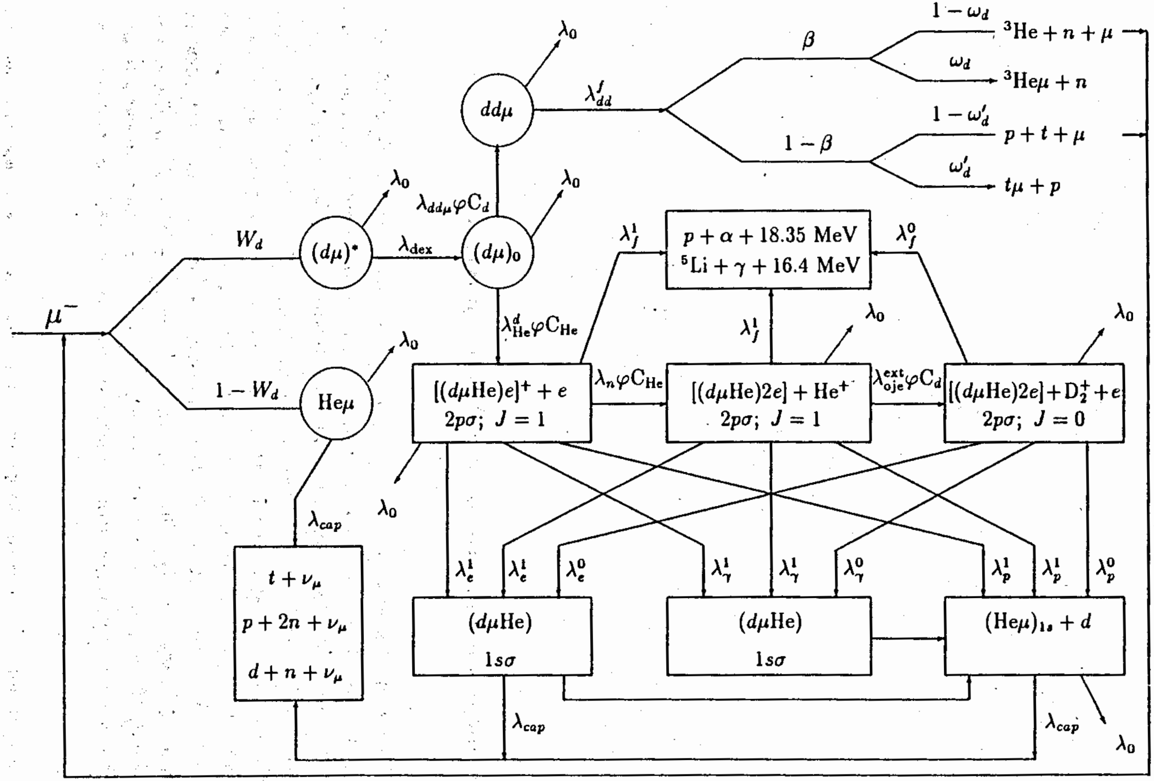


Fig. 1. Scheme of muonic processes in the $D_2 + ^3He$ mixture

ground state 1s (see Fig. 2)[3].
Apart from these decay channels from $2p\sigma$, a rotational transition to the state with $J=0$ or a fusion reaction inside the complex with the rate λ_f^1 are also possible:

$$d\mu^3\text{He} \to p + \alpha + \mu + 18.35 \text{ MeV}, \quad (1)$$
$$\to {}^5\text{Li}\mu + \gamma(16.4 \text{ MeV}), \quad (2)$$
$$d\mu^4\text{He} \to {}^6\text{Li}\mu + \gamma(1.48 \text{ MeV}). \quad (3)$$

In the state with $J=0$ (as in the state with $J=1$) the nuclear fusion reaction with the rate λ_f^0 and the decay process with the rate $\lambda_{dec}^0 = \lambda_p^0 + \lambda_\gamma^0 + \lambda_e^0$ in the above-mentioned channels are also possible.
Because the transition between the states with $J=1$ and $J=0$ occurs via collision of the complex with molecules (atoms) of the mixture, the population of the $d\mu^3$He complex in the state with $J=0$ is a function of the mixture density.
Papers [8,9] present different schemes of the 1-0 transition[4]. According to [8], the 1-0 transition is realized in the form of a two-stage reaction:

i) creation of a neutral quasi-atom of helium, the rate of this process is $\lambda_n = 2 \cdot 10^{13}$ s^{-1} (normalized to liquid hydrogen density (LHD), $n_0 = 4.25 \cdot 10^{22}$ cm^{-3}):

$$\left[(d\mu\text{He})^{++}_{\substack{2p\sigma\\J=1}} e\right]^+ + \text{He} \to \left[(d\mu\text{He})^{++}_{\substack{2p\sigma\\J=1}} 2e\right] + \text{He}^+; \quad (4)$$

ii) an external Auger effect with the rate $\lambda_{\text{Aug}}^{\text{ext}} = 8.5 \cdot 10^{11}$ s^{-1}:

$$\left[(d\mu\text{He})^{++}_{\substack{2p\sigma\\J=1}} 2e\right] + \text{D}(\text{D}_2) \to \left[(d\mu\text{He})^{++}_{\substack{2p\sigma\\J=0}} 2e\right] + \text{D}(\text{D}_2)^+ + e. \quad (5)$$

Paper [9] (in its authors' opinion) presents a full scheme of the 1-0 transition (see Fig. 3):

$$(\text{M}_1 e)^+ + \text{D}_2 \to (\text{M}_0 e)^+ + \text{D}_2^+ + e, \quad (6)$$
$$(\text{M}_1 e)^+ + \text{D}_2 + \text{X} \to (\text{M}_1 e\text{D}_2)^+ + \text{X}, \quad (7)$$
$$(\text{M}_1 e)^+ + \text{He} + \text{X} \to (\text{M}_1 e\text{He})^+ + \text{X}, \quad (8)$$
$$(\text{M}_1 e)^+ + \text{He} \to (\text{M}_1 ee)^+ + \text{He}^+, \quad (9)$$

[3] A more detailed description of muonic processes occurring in the D_2+^{3}He mixture can be found in [3,4,7,9,10,15] and references therein.
[4] The processes determining the 1-0 transition in [8] are a part of the branched scheme of all possible processes [9] in the rotational 1-0 transition.

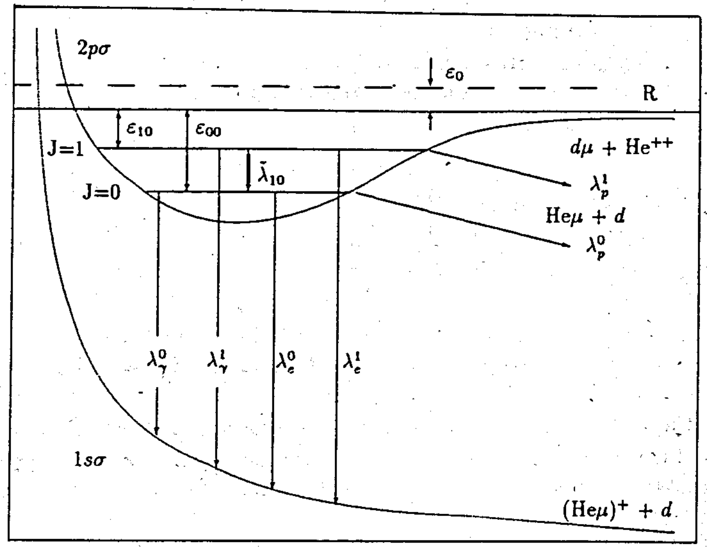


Fig. 2. Scheme of molecular charge transfer of $d\mu$-atoms on helium nuclei

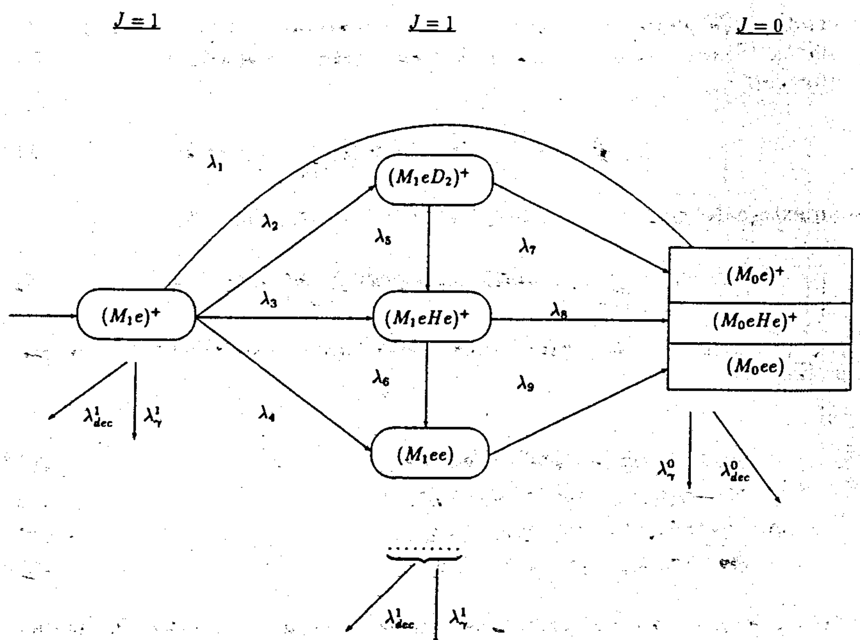


Fig. 3. Full scheme of the 1-0 transition processes in the $d\mu^3$He complex between the states with $J = 1$ and $J = 0$

$$(\mathrm{M_1}e\mathrm{D_2})^+ + \mathrm{He} \to (\mathrm{M_1}e\mathrm{He})^+ + \mathrm{D_2}, \tag{10}$$
$$(\mathrm{M_1}e\mathrm{He})^+ + \mathrm{He} \to (\mathrm{M_1}ee) + \mathrm{He}^+, \tag{11}$$
$$(\mathrm{M_1}e\mathrm{D_2})^+ \to (\mathrm{M_0}e) + \mathrm{D_2^+} + e, \tag{12}$$
$$(\mathrm{M_1}e\mathrm{He})^+ + \mathrm{D_2} \to (\mathrm{M_0}e\mathrm{He})^+ + \mathrm{D_2^+} + e, \tag{13}$$
$$(\mathrm{M_1}ee) + \mathrm{D_2} \to (\mathrm{M_0}ee) + \mathrm{D_2^+} + e, \tag{14}$$

where $\mathrm{M_1} = (d\mu\mathrm{He})_{J=1}$; $\mathrm{M_0} = (d\mu\mathrm{He})_{J=0}$; $X \equiv \mathrm{D_2}$, He.
All the following considerations will be connected to these two variants of the 1-0 transition scheme:

a) two-stage transition 1-0 [8] (processes (9) and (14)),
b) all processes in the 1-0 transition [9].

Because the nuclear reaction can proceed in the states with $J = 1$ and $J = 0$, the yield of this reaction will be determined by the partial rates from these states and by the population of these states while the reaction takes place[5]. The yield of the X-rays will be determined by the partial rates of the complex radiative decay from the states $J = 0, 1$ (λ^0_γ, λ^1_γ), and the effective rate of the 1-0 transition ($\tilde{\lambda}_{10}$).
This means that the variables λ^0_f, λ^1_f, $\tilde{\lambda}_{10}$ can be found by joint analysis of the experimentally measured yields and time distribution of the reaction (1)-(3) products and the X-ray. The following formulae are used:

$$\frac{dN_{p(\gamma)}}{dt} = \frac{dN^1_{p(\gamma)}}{dt} + \frac{dN^0_{p(\gamma)}}{dt} = = n_\mu T q_{1s} W_d \varepsilon_{p(\gamma)} \frac{\lambda_{form}}{\lambda_4}\left(\lambda^1_{f,p(\gamma)} + \frac{\tilde{\lambda}_{10}\lambda^0_{f,p(\gamma)}}{\lambda^0_{dec}}\right)\cdot e^{-\lambda_1 t}; \tag{15}$$

$$\frac{dN_x}{dt} = \frac{dN^1_x}{dt} + \frac{dN^0_x}{dt} = = n_\mu T q_{1s} W_d \varepsilon_x \frac{\lambda_{form}}{\lambda_4}\left(\lambda^1_\gamma + \frac{\tilde{\lambda}_{10}\lambda^0_\gamma}{\lambda^0_{dec}}\right)\cdot e^{-\lambda_1 t}; \tag{16}$$

$$N_{p(\gamma)} = N^1_{p(\gamma)} + N^0_{p(\gamma)} = n_{p(\gamma)} \cdot T; \tag{17}$$
$$N_x = N^1_x + N^0_x = n_x \cdot T; \tag{18}$$
$$n_{p(\gamma)} = n_\mu q_{1s} W_d \varepsilon_{p(\gamma)} \frac{\lambda_{form}}{\lambda_1\lambda_4}\left(\lambda^1_{f,p(\gamma)} + \frac{\tilde{\lambda}_{10}\lambda^0_{f,p(\gamma)}}{\lambda^0_{dec}}\right); \tag{19}$$

[5] According to theoretical estimations, $\lambda^1_{f,p(\gamma)}/\lambda^0_{f,p(\gamma)} \approx 10^{-2} \div 10^{-3}$ [9].

$$n_x = n_\mu q_{1s} W_d \varepsilon_x \frac{\lambda_{form}}{\lambda_1 \lambda_4}\left(\lambda_\gamma^1 + \frac{\tilde\lambda_{10}\lambda_\gamma^0}{\lambda_{dec}^0}\right); \qquad (20)$$

$$\tilde\lambda_{10} = \frac{\lambda_n \lambda_{Aug}^{ext}\varphi^2 C_d C_{He}}{(\lambda_{dec}^1 + \lambda_{Aug}^{ext}\varphi C_d + \lambda_n \varphi C_{He})} \quad \text{(according to [8]);} \qquad (21)$$

$$\tilde\lambda_{10} = \frac{\lambda_{cl}\lambda_{Aug}^{int}\varphi C_d}{(\lambda_{dec}^1 + \lambda_{Aug}^{int} + \lambda_{cl}\varphi C_d)} \quad \text{(according to [9]);} \qquad (22)$$

$$\lambda_1 = \lambda_{form} + \lambda_0 + \lambda_{dd\mu}\varphi\omega_d\beta C_d; \qquad (23)$$

$$\lambda_{form} = \lambda_{^3He}^d \varphi C_{He}; \qquad (24)$$

$$\lambda_4 = \lambda_{dec}^1 + \tilde\lambda_{10}, \qquad (25)$$

where n_μ is the μ-stop intensity in the mixture, λ_0 is the free muon decay rate ($\lambda_0 = 0.455 \cdot 10^6$ s^{-1}), λ_{He}^d is the rate of muon transition between the $d\mu$ atom and the He nucleus, C_{He} and C_d are the helium and deuterium atomic concentrations in the mixture, $\varepsilon_{p(\gamma)}$ is the registration efficiency for protons (γ quanta) from reaction (1), φ is the mixture density, $\tilde\lambda_{10}$ is effective rate of the 1-0 transition in the $d\mu$He system, $\lambda_{f,p(\gamma)}^{1(0)}$ are partial rates of nuclear fusion in the complex in the states with $J = 1$ and $J = 0$ in the proton and γ channels respectively (for the $d\mu^3$He system – reactions (1) and (2), for $d\mu^4$He – reaction 3)), q_{1s} is the probability that the $d\mu$ atom created in the excited states reaches the ground level, W_d is the probability of direct capture of a muon by a deuterium atom in the mixture ($W_d = (1 + AC_{He}/(1 - C_{He}))^{-1}$, where $A = 1.7 \pm 0.2$ [16] is the fraction of probabilities of direct capture by a deuterium atom and helium), ε_x is the X-ray registration efficiency, $\lambda_{dd\mu}$ is the rate of the $dd\mu$ molecule formation, β is the relative probability of muon sticking to helium nuclei from the dd-reaction, T is the total exposure time.
Formulae (15)-(20) were obtained under the assumption that $\lambda_{dec}^{1(0)}$, $\tilde\lambda_{10} \gg \lambda_{form}$, λ_0, $\lambda_f^{1(0)}$, $\lambda_{dd\mu}$ and correspond to times $t \gg 1/\lambda_{dec}^{1(0)}$. All the following considerations and calculations will concern processes in D_2+^{3}He mixture.
The formulae for proton and X-ray yields (normalized to one complex) have the form:

$$Y_{p(\gamma)} = \frac{1}{\lambda_4}\left(\lambda_{f,p(\gamma)}^1 + \frac{\tilde\lambda_{10}\lambda_{f,p(\gamma)}^0}{\lambda_{dec}^0}\right); \qquad (26)$$

$$Y_x = \frac{1}{\lambda_4}\left(\lambda_\gamma^1 + \frac{\tilde\lambda_{10}\lambda_\gamma^0}{\lambda_{dec}^0}\right). \qquad (27)$$

3 Results of the optimization

One can see from (16)-(19) that information about the parameters $\lambda_{f,p(\gamma)}^0$, $\lambda_{f,p(\gamma)}^1$, $\tilde\lambda_{10}$ can be obtained only under the assumption that the values of W_d, ω_d, q_{1s}, $\lambda_{dec}^{1(0)}$, n_μ, $\varepsilon_{p(\gamma)}$, ε_x, λ_{form}, λ_1 are known [6].
Figures 4a-b show calculated dependencies of: a) the proton yield from reaction (1) and b) the X-ray (equations (26),(27), as a function of the mixture density. Figures 4c-d show analogous dependencies for one muon stopped in the D_2+^{3}He mixture (equations (17)-(20)).
Figure 5 shows dependence of the effective rate of the 1-0 transition on density for two schemes of the 1-0 transition.
The results are obtained for the above-mentioned schemes of the 1-0 transition and for the variable values [7]:

$$n_\mu = 2.5 \cdot 10^4\ \mu\text{stop/s}; \quad W_d = 0.92\ [16];$$

$$\lambda_{dec}^1 = 7 \cdot 10^{11}\ \text{s}^{-1}, \quad \lambda_\gamma^1 = 1.52 \cdot 10^{11}\ \text{s}^{-1};$$

$$\lambda_{dec}^0 = 5.9 \cdot 10^{11}\ \text{s}^{-1}; \quad \lambda_\gamma^0 = 1.72 \cdot 10^{11}\ \text{s}^{-1}$$

(the results of averaging some theoretical investigations see [3,10] and references therein);

$$\lambda_n = 2 \cdot 10^{13}\,\text{s}^{-1}\ [8]; \quad \lambda_{Aug}^{ext} = 8.5 \cdot 10^{11}\,\text{s}^{-1}\ [8];$$

$$\lambda_{cl} = 3 \cdot 10^{13}\,\text{s}^{-1}\ [9]; \quad \lambda_{Aug}^{int} = 5 \cdot 10^{11}\,\text{s}^{-1}\ [9];$$

$$\lambda_{^3He}^d = 1.8 \cdot 10^8\,\text{s}^{-1}\ [19]; \quad \lambda_{dd\mu} = 0.04 \cdot 10^6\,\text{s}^{-1}\ [20]$$

(the values of $\lambda_{^3He}^d$ and $\lambda_{dd\mu}$ for the density region φ from 0 to 1 were treated as constant and equal to the corresponding values for 30 K) [8];

$$\omega_d = 0.122\,[21]; \quad \beta = 0.58\,[21]; \quad C_{He} = 0.05; \ C_d = 0.95;$$

[6] The variables λ_1 and λ_{form} are found indirectly from the analysis of the time distribution of muonic X-rays (6.85 keV): $\lambda_{form} = \lambda_1 - \lambda_0 - \lambda_{dd\mu}\varphi\beta C_d\omega_d$.
[7] During the calculations of proton and muonic X-ray yields the q_{1s} values from [17,18] recalculated for our conditions were used.
[8] The experimental programme for investigation of the $d\mu$He molecule properties suggests performance of experiments in such a condition.

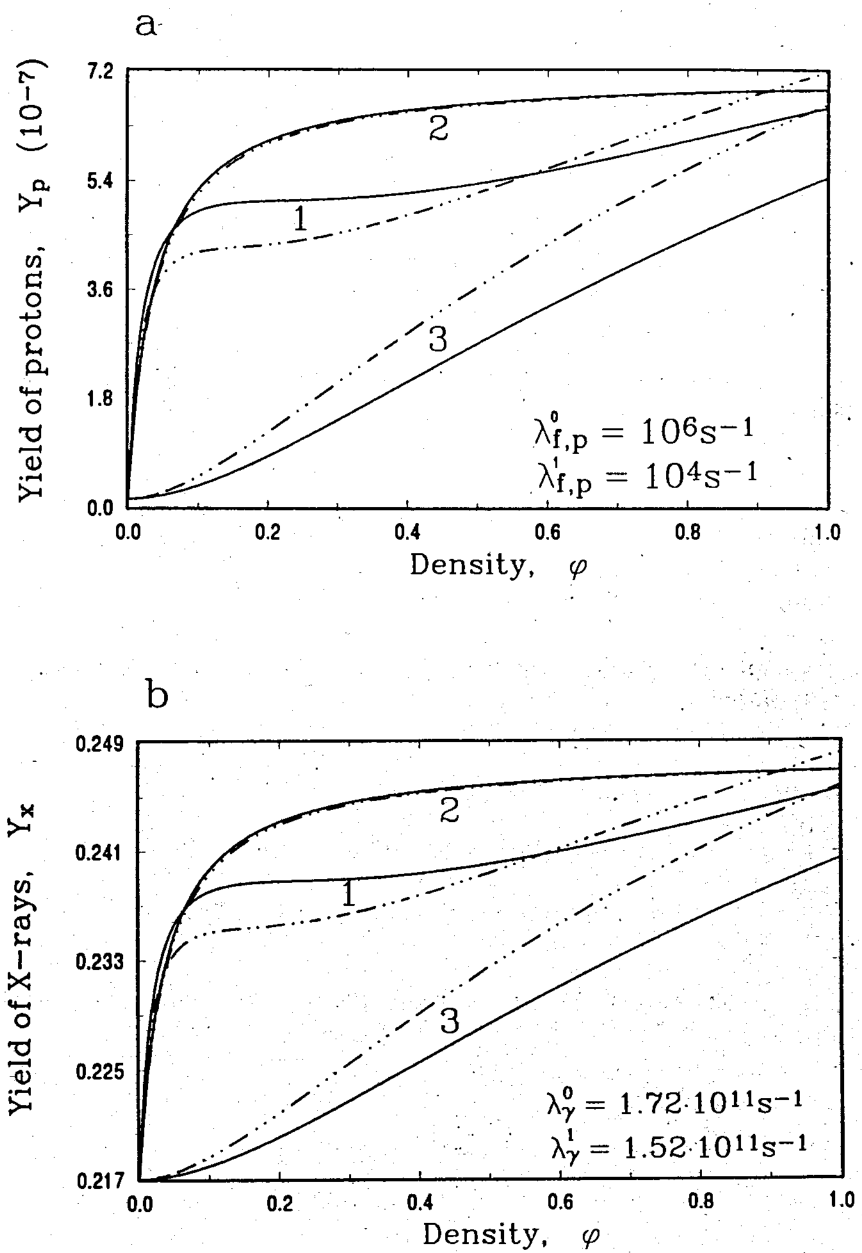


Fig.4. a),b)

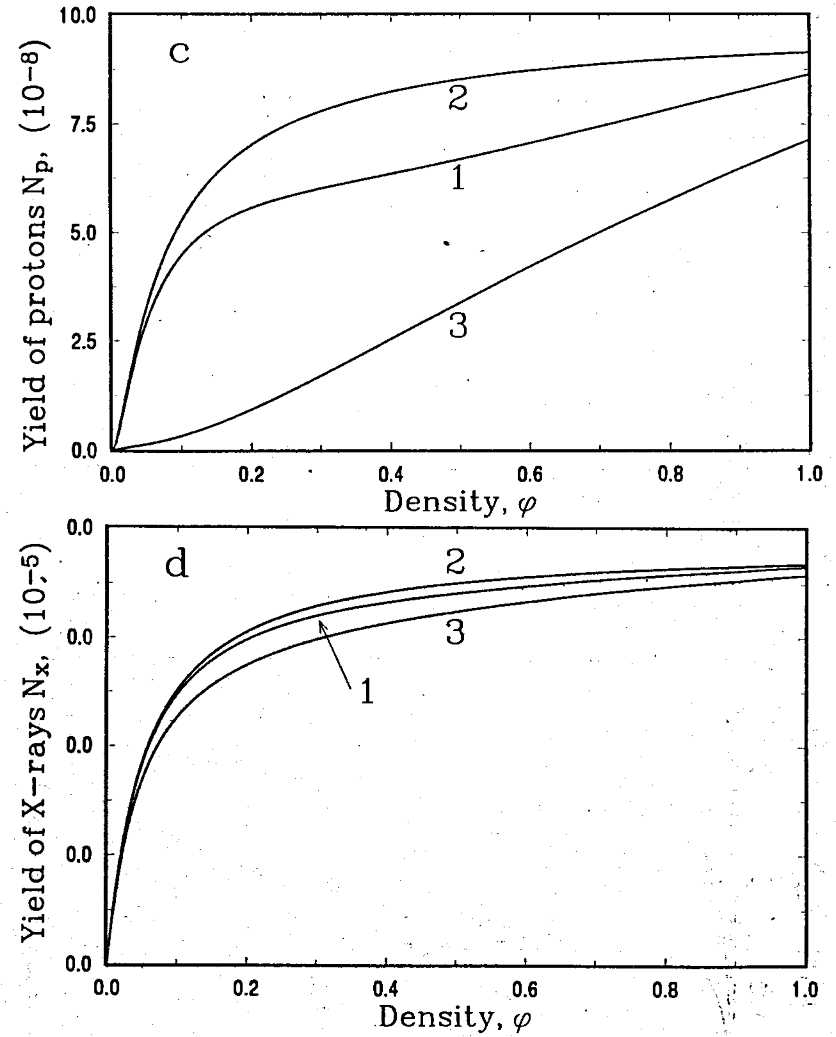


Fig. 4. Calculated dependence of:
a) the yields of the proton from nuclear synthesis in the $d\mu^3$He complex;
b) the yields of the X-ray with an energy of 6.85 keV;
c) the same as a) but for one muon stopped in the mixture;
d) the same as b) but for one muon stopped in the mixture
on the deuterium-helium mixture density.
Curves:
1 — full scheme of the 1-0 transition processes [9];
2 — two-stage 1-0 transition (processes (7) and (12));
3 — two-stage 1-0 transition (processes (9) and (14));
solid lines are for $C_{He} = 0.05$, dotted lines for $C_{He} = 0.1$

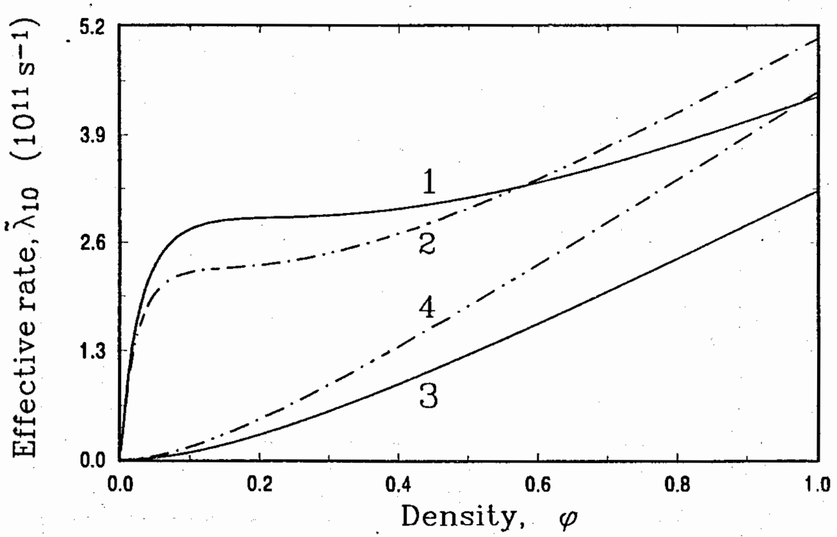


Fig. 5. Full scheme of the 1-0 transition: 1 — $C_{\rm He} = 0.05$; 2 — $C_{\rm He} = 0.1$; two-stage 1-0 transition (processes (9) and (14)): 3 — $C_{\rm He} = 0.05$; 4 — $C_{\rm He} = 0.1$

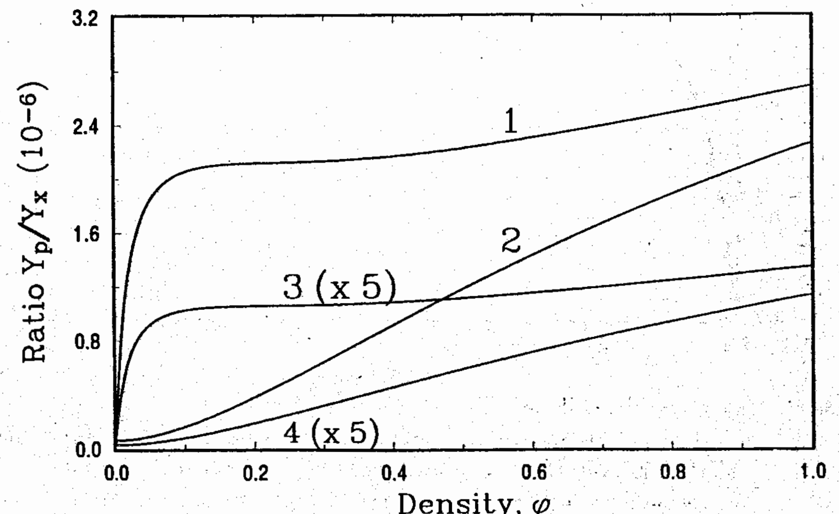


Fig. 6. Dependence of the ratio of the proton to the X-ray on the D_2+^{3}He mixture density.
1, 2 — 1-0 transition [8] and [9], respectively ($\lambda^0_{f,p} = 10^6$ s^{-1}; $\lambda^1_{f,p} = 10^4$ s^{-1});
3, 4 — the same as 1 and 2, but for $\lambda^0_{f,p} = 10^5$ s^{-1}; $\lambda^1_{f,p} = 10^3$ s^{-1}

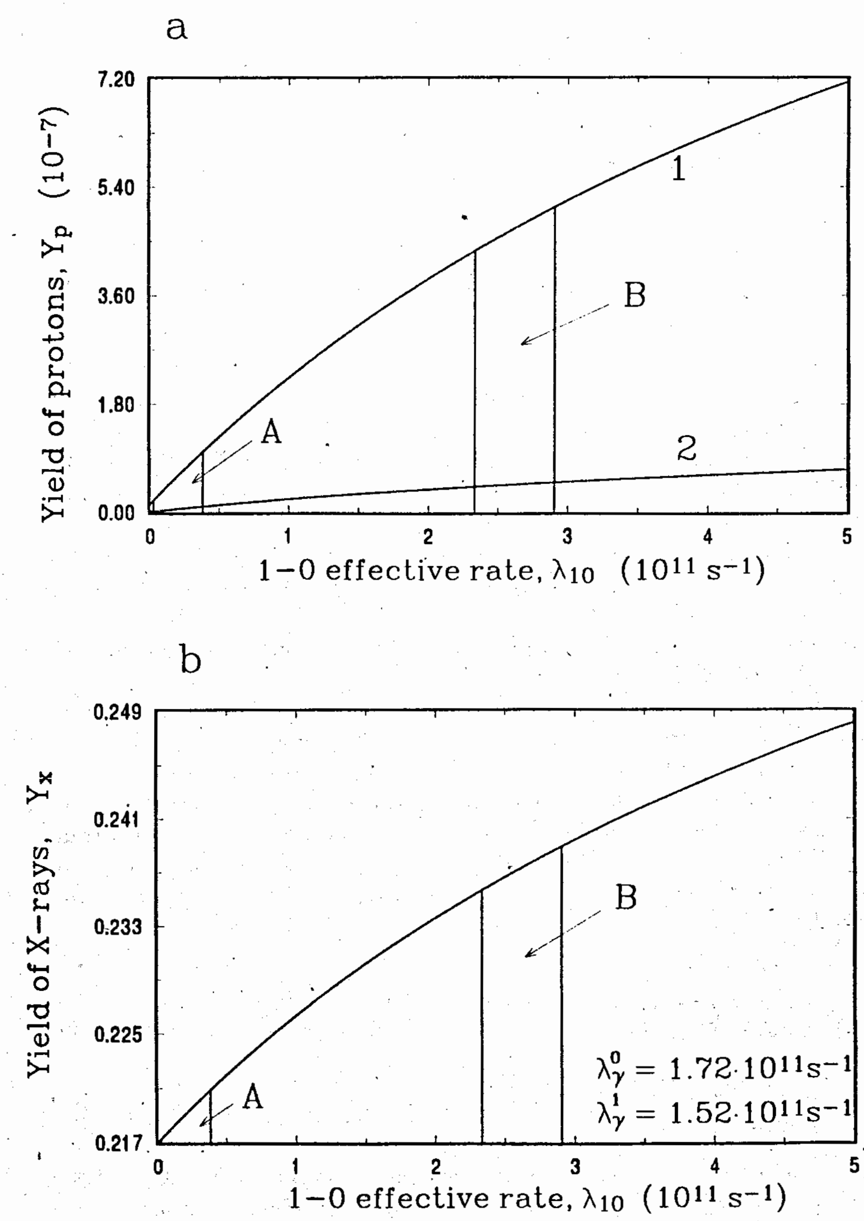


Fig. 7. a) The proton $Y_p(\tilde\lambda_{10})$ and b) the muonic X-rays yield $Y_x(\tilde\lambda_{10})$ as a function of the 1-0 transition rate $\tilde\lambda_{10}$

$$\varepsilon_p = 0.3;\ \varepsilon_x = 5 \cdot 10^{-3}.$$

Besides, the dashed lines in Figures 4a-4 and 5 show an analogous dependence of $Y_p(\varphi)$, $Y_x(\varphi)$ and $\tilde{\lambda}_{10}(\varphi)$, calculated for the helium concentration $C_{He} = 0.1$. Also, figures 4a, 4b show this dependence for a simplified scheme of the 1-0 transition (two-stage processes including (7) and (12))[9]. It is worth mentioning that in the process of analyzing the experimental data a parametrization of the function $\tilde{\lambda}_{10}(\varphi)$ was used:

$$\tilde{\lambda}_{10}(\varphi) = r\varphi^3 + s\varphi^2 + t\varphi. \tag{28}$$

This parametrization plays the role of an interpolation polynomial of the $\tilde{\lambda}_{10}$ function. To obtain this polynomial at least three exposures to a muon beam for different densities φ (and for fixed helium concentration C_{He}) are required. Having found the interpolation polynomial, one can determine the real $\tilde{\lambda}_{10}$ function and by comparison with the above-discussed theoretical predictions identify the 1-0 transition mechanism.

Figure 6 presents the ratio Y_p/Y_x as a function of the mixture density for the mechanisms [8,9] of the 1-0 transition and two sets of $\lambda^0_{f,p}$ and $\lambda^1_{f,p}$ values: a) $\lambda^0_{f,p} = 10^6\ s^{-1}$; $\lambda^1_{f,p} = 10^4\ s^{-1}$; b) $\lambda^0_{f,p} = 10^5\ s^{-1}$; $\lambda^1_{f,p} = 10^3\ s^{-1}$.

As is seen, by measuring the ratio for some values of φ one can clearly identify the mechanism of the 1-0 transition.

The dependence of proton yields from fusion in the $d\mu^3$He system and the muonic X-rays on the effective rate of the 1-0 transition for the two considered mechanisms of the 1-0 transition is shown in figures 7a,b.

Now we briefly discuss the procedure of choosing the optimal conditions for the experiment for each of the above-mentioned mechanisms of the 1-0 transition[10]. The choice of optimal conditions consists in searching for three or more values of the mixture density in the range from 0 to 1 by the least squares method:

$$\chi_1^2 = \sum_{i=1}^{k} \frac{(N_p^i(\exp) - n_p^i(\text{theor})T_i)^2}{(\sigma_p^i)^2}; \tag{29}$$

$$\chi_2^2 = \sum_{i=1}^{k} \frac{(N_x^i(\exp) - n_x^i(\text{theor})T_i)^2}{(\sigma_x^i)^2}, \quad (k \geq 3); \tag{30}$$

[9] According to [9], in the experimental condition $\varphi \sim 0.1$, $C_{He} \leq 0.1$, the 1-0 transition can be considered as a two-stage process. In our opinion, such simplification is valid only for $\varphi \leq 0.05$ and $C_{He} \leq 0.05$.

[10] It is assumed that the experiment will be performed at the meson factory PSI with the intensity of muon stops in the target $n_\mu = 2.5 \cdot 10^4\ s^{-1}$.

where $N^i_p(\exp)$, $N^i_x(\exp)$ are the numbers of registered protons from reaction (1) and 6.85 keV X-rays for exposures with the mixture density $\varphi = \varphi_i$, respectively:

$$n_p^i(\text{theor}) = f_1(\lambda^1_{f,p}, \lambda^0_{f,p}, \tilde{\lambda}^i_{10} = r\varphi_i^3 + s\varphi_i^2 + t\varphi_i, T_i) \tag{31}$$

(see eq. (19));

$$n_x^i(\text{theor}) = f_2(\tilde{\lambda}^i_{10} = r\varphi_i^3 + s\varphi_i^2 + t\varphi_i, T_i) \tag{32}$$

(see eq. (20)).

The values $\sigma^i_p = \sqrt{n^i_p \cdot T_i}$; $\sigma^i_x = \sqrt{n^i_x \cdot T_i}$ are the variances of the calculated normal distribution of registered protons and X-rays respectively:

$$N^i_p(\exp) = n^i_p \cdot T_i + \sigma^i_p \cdot \eta, \quad N^i_x(\exp) = n^i_x \cdot T_i + \sigma^i_x \cdot \eta;$$

T_i is the time of statistics gathering for the exposure with the mixture density φ_i; η is the random number according to normal distribution.

The varied parameters are r, s, t, T_1, T_2, T_3 ($T = T_1 + T_2 + T_3$ is the total time of statistics gathering for the case with three exposures).

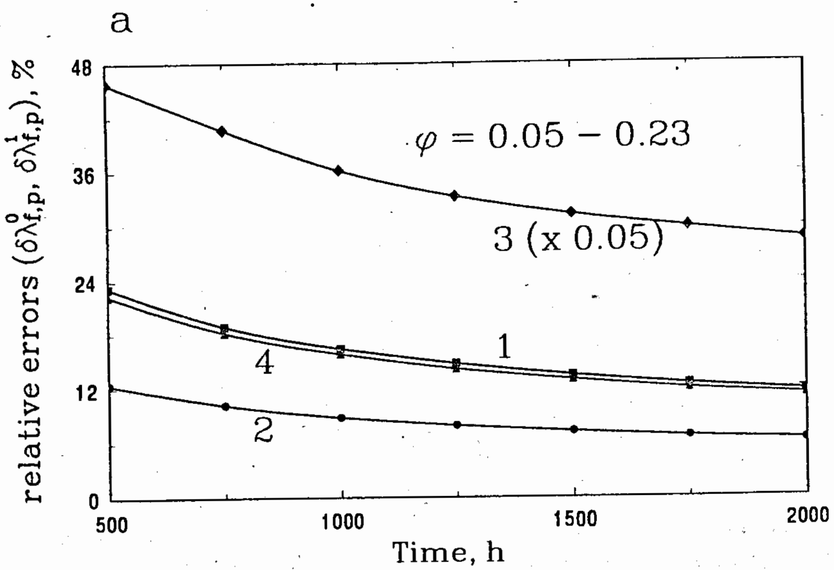
During modelling the experimental conditions it was assumed that the random variables $N^i_p(\exp)$, $N^i_x(\exp)$ are distributed normally with the variances σ^i_p, σ^i_x and the mean values $n^i_p T_i$, $n^i_x T_i$, respectively.

Figures 8a-c present dependence of the relative errors of the parameters $\lambda^0_{f,p}$, $\lambda^1_{f,p}$, $\tilde{\lambda}_{10}$ for the optimal experiment condition on the full statistics gathering time in all exposures (it was assumed that the target constructed by our group [22] and the registering apparatus [3] will be used). The target allows experiments with a deuterium-helium mixture in the mixture density range from 0.05 to 0.23 LHD (the presence of thin kapton windows in the corpus of the target and the parameters of the muon beam in the $\mu E4$ channel at PSI determine the minimum and maximum values of the mixture density).

Registration efficiency for protons from reaction (1) is 0.2 and for X-rays is $5 \cdot 10^{-4}$.

It is seen from the results of optimizations (see Table 1) that the situation is not clear and depends significantly not only on the scheme of the 1-0 transition but also on the absolute values of the nuclear fusion reaction (1) rate in the states with $J = 0$ and $J = 1$.

Table 1 shows results of the optimization for the case of gathering statistics time of 1000 h in the $\mu E4$ channel at PSI and with using registration apparatus with parameters as [2,23]. According to this table one can determine the following characteristics of the muonic complex:



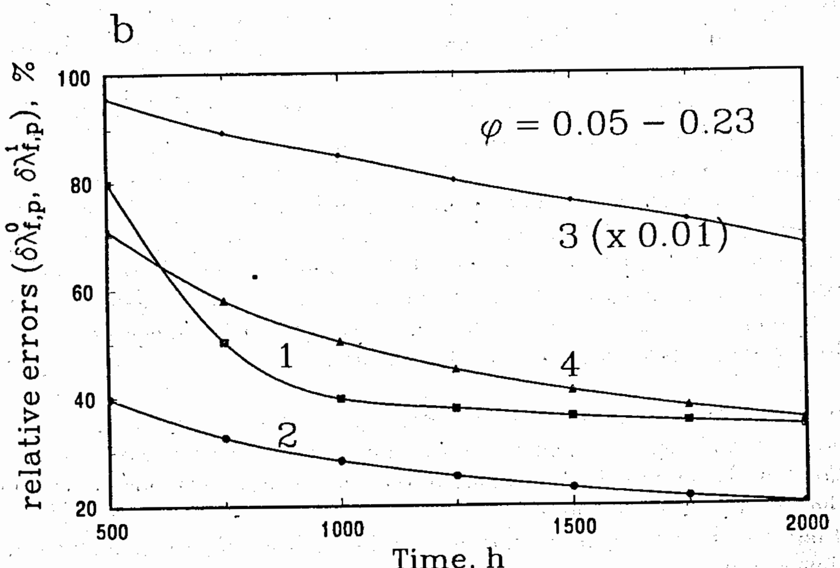


Fig.8. a),b)

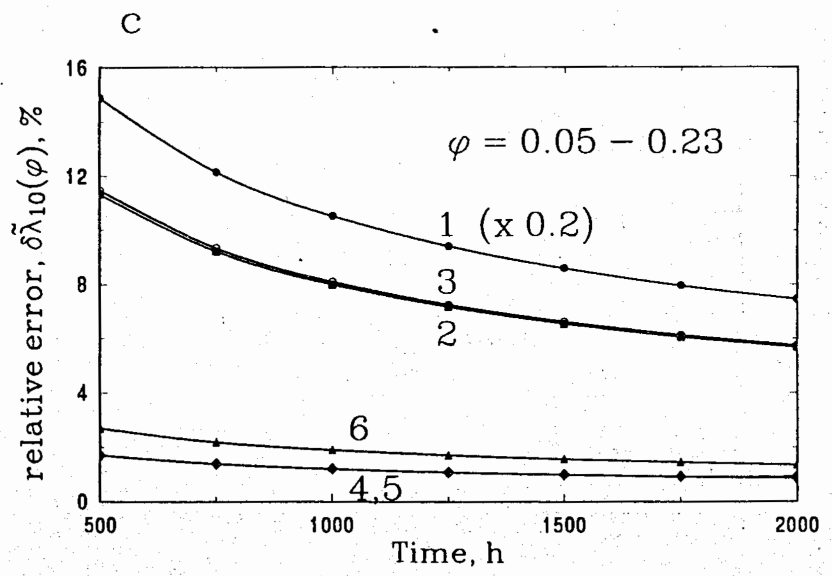
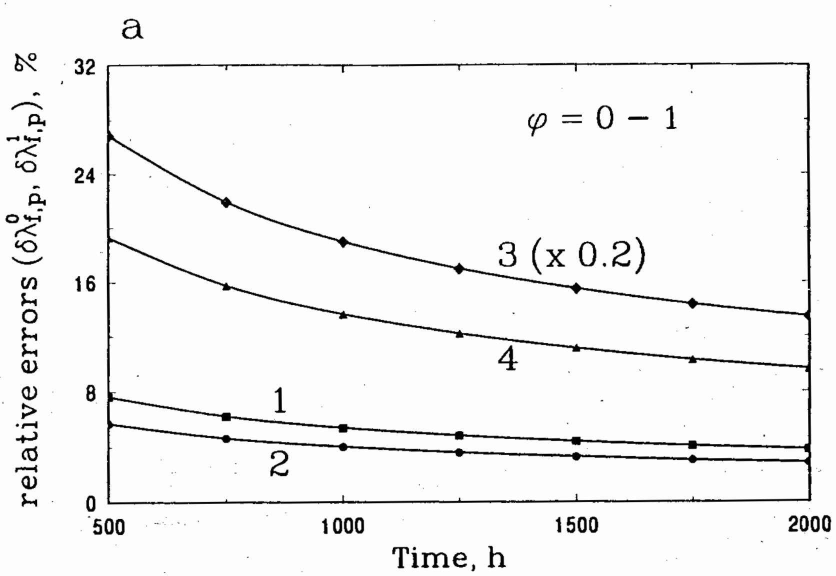


Fig. 8. Dependence of relative errors of the partial rates of nuclear fusion in the $d\mu^3$He complex (a and b) and the effective rate of the 1-0 transition (c) on statistics gathering time ($\varphi = 0.05 - 0.23$) :
a) $\lambda^0_{f,p} = 10^6$ s^{-1}; $\lambda^1_{f,p} = 10^4$ s^{-1}; 1, 2 and 3, 4 — $\delta\lambda^0_{f,p}$ and $\delta\lambda^1_{f,p}$ for the 1-0 transition mechanisms [8] and [9], respectively;
b) $\lambda^0_{f,p} = 10^5$ s^{-1}; $\lambda^1_{f,p} = 10^3$ s^{-1}; description is the same as for Fig 8a;
c) creation of the neutral muonic complex and external Auger effect:
1 — $\varphi = 0.05$, 2 — $\varphi = 0.18$, 3 — $\varphi = 0.23$;
full scheme of the 1-0 transition:
4 — $\varphi = 0.05$, 5 — $\varphi = 0.15$, 6 — $\varphi = 0.23$



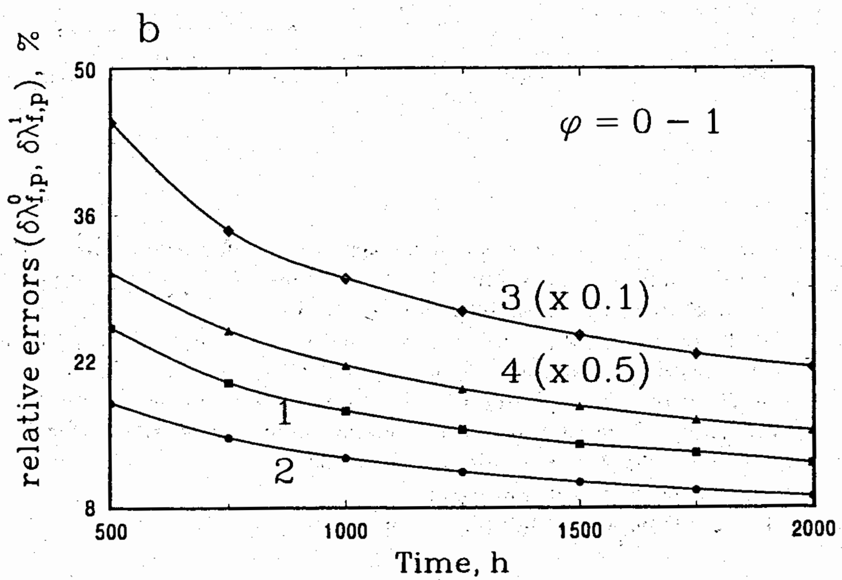


Fig.9. a),b)

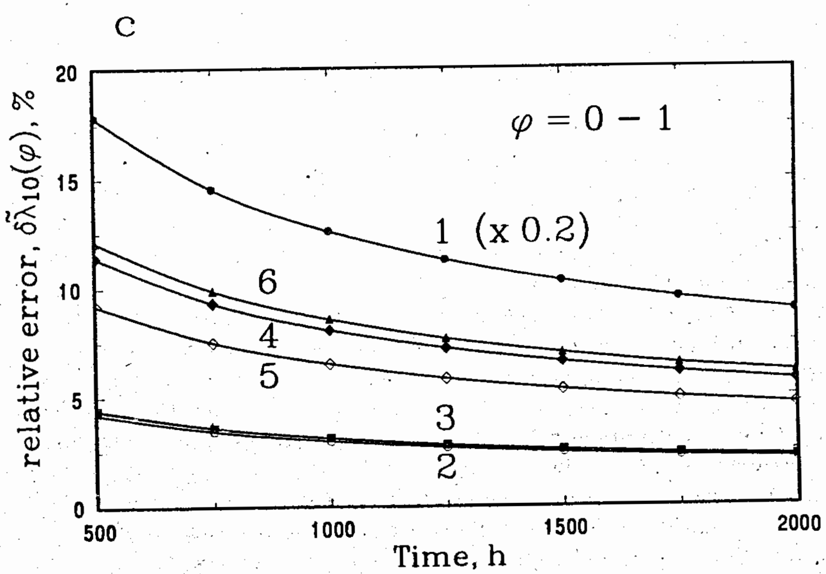


Fig. 9. The same as Fig. 8 but for $\varphi = 0 - 1$

(1) The rate of nuclear fusion in the $d\mu^3$He molecule from $J = 0$ if this rate is larger than $8 \cdot 10^3$ s^{-1} and $2 \cdot 10^4$ s^{-1} for the mechanisms [8] and [9] respectively.
(2) The rate of nuclear fusion $\lambda^1_{f,p}$ if the mechanism [8] is correct and the absolute value of this rate is larger than $3 \cdot 10^2$ s^{-1}. In the other case only the upper limit can be determined.
(3) The upper limit of $\lambda^1_{\rm f,p}$ at the 90% confidence level for the mechanism [9] of the 1-0 transition. This limit is determined not only by the absolute value of $\lambda^1_{\rm f,p}$ but also by the ratio $\lambda^1_{\rm f,p}/\lambda^0_{\rm f,p}$ (see Table 1).

It is worth mentioning that these considerations are only valid if the background is negligible [11]. As to determination of the 1-0 transition mechanism by measuring $\tilde{\lambda}_{10}$, the situation is the same.
One can see from figures 7a-b that the intervals of measuring $Y_p(\tilde{\lambda}_{10})$ and $Y_{\rm x}(\tilde{\lambda}_{10})$ correspond to the 1-0 transfer mechanisms [8],[9] and the same region of density variations do not overlap, which means that it is possible to determine the real model of the 1-0 transition. Information about the variables r,s,t (see eq. (28)) is that additional information which helps reveal the physical meaning of the 1-0 transition.
For understanding the potential possibilities of reducing the lower limit of nuclear fusion cross-section measurements in the complex we optimized the experiments for the mixture density range from 0 to 1. We do not list all technical problems concerning this investigation in the density range 0-1 with the registration apparatus with parameters analogous to [3,22] [12]. Figures 9a-c show dependencies of relative errors of the parameters $\lambda^0_{\rm f,p}$, $\lambda^1_{\rm f,p}$ and λ_{10} on the total statistics gathering time for the considered experimental set-up.
Tables 1 and 2 show that sensibility of further investigations of the muonic complexes in the D_2+^{3}He mixture will be determined by two factors, first - importance of physical problem and second - cost of creation of the experimental set-up working in such a wide range of mixture density.

[11] The background level measured in the test experiments [4] in μE4 channel at PSI shows that after some modification of the registering system [3] it is possible to reduce the background to a negligibly small value.
[12] Creation of universal experimental set-up working in such a wide range of mixture density ($0.001 \ll \varphi \ll 1$) is a complex task. For example, work at low densities (~ 0.001) of the deuterium helium mixture requires using the mixture in the form of magnetic bottle.

Table 1: Results of the projected experiments optimization – our cryogenic target: $\varphi = 0.05 \div 0.23$ (T = 1000h) (* — two stage 1-0 transition: formation of a neutral helium quasi-atom and external Auger effect; ** — full scheme of the 1-0 transition processes)

$\lambda^0_{f,p}$; $\lambda^1_{f,p}$ s^{-1}	$\delta\lambda^0_{f,p}$, %	$\delta\lambda^1_{f,p}$, %	$\delta\tilde{\lambda}_{10}(\varphi_1)$, %	$\delta\tilde{\lambda}_{10}(\varphi_2)$, %	$\delta\tilde{\lambda}_{10}(\varphi_3)$, %	φ_1	φ_2	φ_3	1-0 transit.	upper limit, 90% CL	
										$\lambda^0_{f,p}$, s^{-1}	$\lambda^1_{f,p}$, s^{-1}
10^6; 10^4	8.8	15.8	52.6	8.1	8.0	0.05	0.18	0.23	*		
	16.4	>100	1.2	1.2	1.9	0.05	0.15	0.23	**		$1.1 \cdot 10^5$
10^5; 10^3	27.9	50.0	52.6	8.1	8.0	0.05	0.18	0.23	*		
	39.4	>100	1.2	1.2	1.9	0.05	0.15	0.23	**		$1.1 \cdot 10^5$
$5 \cdot 10^4$; $5 \cdot 10^2$	39.4	70.6	52.6	8.1	8.0	0.05	0.18	0.23	*		
	71.8	>100	1.2	1.2	1.9	0.05	0.15	0.23	**		$1.1 \cdot 10^5$
10^4; 10^2	88.2	>100	52.6	8.1	8.0	0.05	0.18	0.23	*		$3.0 \cdot 10^2$
	>100	>100	1.2	1.2	1.9	0.05	0.15	0.23	**	$3.1 \cdot 10^4$	$1.1 \cdot 10^5$
$5 \cdot 10^3$; $5 \cdot 10^1$	>100	>100	52.6	8.1	8.0	0.05	0.18	0.23	*	$1.3 \cdot 10^4$	$2.0 \cdot 10^2$
	>100	>100	1.2	1.2	1.9	0.05	0.15	0.23	**	$1.8 \cdot 10^4$	$1.1 \cdot 10^5$
$2 \cdot 10^3$; $2 \cdot 10^1$	>100	>100	52.6	8.1	8.0	0.05	0.18	0.23	*	$7.0 \cdot 10^3$	$1.1 \cdot 10^2$
	>100	>100	1.2	1.2	1.9	0.05	0.15	0.23	**	$1.1 \cdot 10^4$	$1.1 \cdot 10^5$

Table 2: Results of the $D_2 +^3 He$ experiment optimization — general case: $\varphi = 0 \div 1$ (T = 1000 h) (* — two stage 1-0 transition: formation of a neutral helium quasi-atom and external Auger effect; ** — full scheme of the 1-0 transition processes)

$\lambda^0_{f,p}$; $\lambda^0_{f,p}$ s^{-1}	$\delta\lambda^0_{f,p}$, %	$\delta\lambda^1_{f,p}$, %	$\delta\tilde{\lambda}_{10}(\varphi_1)$, %	$\delta\tilde{\lambda}_{10}(\varphi_2)$, %	$\delta\tilde{\lambda}_{10}(\varphi_3)$, %	φ_1	φ_2	φ_3	1-0 transit.	upper limit, 90% CL	
										$(\lambda^0_{f,p})$, s^{-1}	$(\lambda^1_{f,p})$, s^{-1}
10^6; 10^4	4.0	13.6	63.4	2.9	3.1	0.043	0.82	1.0	*		
	5.4	95	8.1	6.5	8.6	0.0039	0.69	1.0	**		$2.2 \cdot 10^4$
10^5; 10^3	12.7	43.1	50.4	2.8	2.9	0.048	0.81	1.0	*		
	17.2	>100	16.0	8.6	10.6	0.0023	0.75	1.0	**		$4.8 \cdot 10^3$
$5 \cdot 10^4$; $5 \cdot 10^2$	18.0	60.9	55.1	2.8	3.0	0.046	0.81	1.0	*		
	24.3	>100	18.1	9.0	11.0	0.0021	0.76	1.0	**		$3.4 \cdot 10^3$
10^4; 10^2	40.2	>100	60.5	2.9	3.0	0.044	0.82	1.0	*		$2.7 \cdot 10^2$
	54.3	>100	19.3	9.3	11.2	0.0020	0.76	1.0	**		$3.0 \cdot 10^3$
$5 \cdot 10^3$; $5 \cdot 10^1$	56.9	>100	60.5	2.9	3.0	0.044	0.82	1.0	*		$1.7 \cdot 10^2$
	79.6	>100	20.7	9.5	11.5	0.0019	0.77	1.0	**		$3.0 \cdot 10^3$
$2 \cdot 10^3$; $2 \cdot 10^1$	88.9	>100	63.4	2.9	3.1	0.043	0.82	1.0	*		$1.0 \cdot 10^2$
	>100	>100	20.7	9.5	11.5	0.0019	0.82	1.0	**	$5.1 \cdot 10^3$	$3.0 \cdot 10^3$

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Метод исследования ядерных реакций
в зарядово-несимметричных мюонных комплексах

Предложен метод экспериментального определения скоростей реакций ядерного синтеза в $d\mu$He-молекулах в состояниях с $J=1$ и $J=0$ (J — орбитальный момент системы), а также эффективной скорости вращательного перехода между этими состояниями. Показано, что информация об искомых параметрах может быть найдена путем анализа выходов и временных распределений продуктов реакций ядерного синтеза в $d\mu$He-молекулах (протонов; γ-квантов), а также мю-рентгеновского излучения ($E_x = 6{,}85$ кэВ), измеренных в экспериментах при трех (и более) различных значениях плотности D_2+He-смеси. Проведена оптимизация экспериментов, запланированных на мезонной фабрике PSI (Швейцария), с целью получения прецизионной информации об искомых параметрах в предложении реализации различных механизмов $1 \to 0$-перехода.

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Method of Investigation of Nuclear Reactions
in Charge-Nonsymmetrical Complexes

A method for experimental determination of the nuclear fusion rates in the $d\mu$He molecules in the states with $J=0$ and $J=1$ (J is the orbital momentum of the system) and of the effective rate of transition between these states (rotational transition 1–0) is proposed. It is shown that information on the desired characteristics can be found from joint analysis of the time distribution and yield of products of nuclear fusion reactions in deuterium-helium muonic molecules and muonic X-ray obtained in experiments with the D_2+He mixture at three (and more) appreciably different densities. The planned experiments with the D_2+He mixture at the meson facility PSI (Switzerland) are optimized to gain more accurate information about the desired parameters on the assumption that different mechanisms for the 1–0 transition of the $d\mu$He complex are realized.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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