ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E15-97-329

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ON EXPERIMENTAL DETERMINATION OF CHARACTERISTICS OF NUCLEAR FUSION REACTIONS FROM MU-MOLECULAR RESONANCE STATES

Submitted to «Ядерная физика»

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1997

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К вопросу об экспериментальном определении характеристик реакций ядерного синтеза из мю-молекулярных резонансных состояний

Работа посвящена изучению зарядово-несимметричных дейтерий-гелиевых мюонных комплексов ($d\mu$He). Предлагается метод экспериментального определения скоростей реакции ядерного синтеза в $d\mu$He-молекулах в состояниях с $J=1$ и $J=0$ (J — орбитальный момент системы), а также парциальных скоростей радиационного распада данных комплексов в этих состояниях. Выполнение экспериментов предполагается на мезонных фабриках с использованием газовых и криогенных мишеней, заполняемых смесью дейтерия и гелия.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1997

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On Experimental Determination of Characteristics of Nuclear Fusion Reactions from Mu-Molecular Resonance States

Charge-nonsymmetrical deuterium-helium muon complexes ($d\mu$He) are studied. A method is proposed for experimentally determining the rates of nuclear fusion reactions in $d\mu$He molecules in the $J=1$ and $J=0$ states (J is the orbital moment of the system) and the partial rates for radiative decay of these complexes in these states. Experiments are supposed to be carried out at meson factories with gaseous and cryogenic targets filled with a mixture of deuterium and helium.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1997

1 Introduction

Investigation of the reactions among light nuclei at ultralow energies (eV – keV) is of great interest for many reasons. For example, to verify fundamental symmetries, one should experimentally compare the lengths of neutron and proton scattering from light nuclei (p, d, t, He, Li) in the indicated energy region. By now these scattering lengths for protons have only been deduced by extrapolation of experimental data from the high energy region (~ MeV) [1, 2]. Investigation of inelastic processes in this energy region, e.g. comparison of experimental and calculated yields and angular distributions of products from the radiative capture reactions $pd \to {}^3\mathrm{He} + \gamma$ and $n^3\mathrm{He} \to {}^4\mathrm{He} + \gamma$ may allow to extract information about the contribution of exchange meson currents [3–7].

Particular interest in low-energy nuclear reactions gives rise to some problems existing in astrophysics [8, 9]. An example is the observed deficiency in light elements (except He) in stars and Galaxy as compared with the predictions based on the theory of thermonuclear reactions and generally adopted star models. It turns out that deuterium burns up in stars at $T \geq 50$ eV, and Li at $T \geq 200$ eV [10].

To explain this phenomenon, star models are usually modified on the assumption that the cross sections for nuclear reactions in the astrophysical energy range (~ keV), deduced by extrapolation from the higher energy range (~ MeV), do not have any resonances or other anomalies in the ultralow-energy region. However, it is not impossible that nuclear cross sections in the low-energy region are of resonance character, which results, in turn, in intense burn-up of light elements [10].

Investigation of reactions among light nuclei in direct collisions in the energy region mentioned is practically impossible because of small cross sections for these processes $\sigma \sim 10^{-40} \div 10^{-45}$ cm^2 [11] (due to Coulomb repulsion between charged particles, which substantially reduces the probability of their approach as close as the nuclear force range) and very limited intensity of accelerated nuclear beams.

A method of investigating strong interactions among light nuclei in the ultralow-energy range with pulsed radially converged high-intensity ion beams, generated during liner plasma implosion (formation of a Z-pinch with an intensity of accelerated ion beam higher than 10^{20} particles per pulse) was proposed in [10, 12, 13] and a method of investigating nuclear reactions from the states of mu-molecular resonances HμHe, HμLi, HμBe (H $\equiv p, d, t$) was proposed in [10, 14] .

Reactions of nuclear fusion in these charge-nonsymmetrical muonic complexes[1] provide the same astrophysical energy range of nuclear collisions, namely ~ keV [15-17].

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[1]Charge-nonsymmetrical muonic complexes are mu-molecules in which one centre is the nucleus of a hydrogen isotope and the other is the nucleus of an element with $Z > 1$ (with introduction of this notion, mu-molecules of the hydrogen isotopes

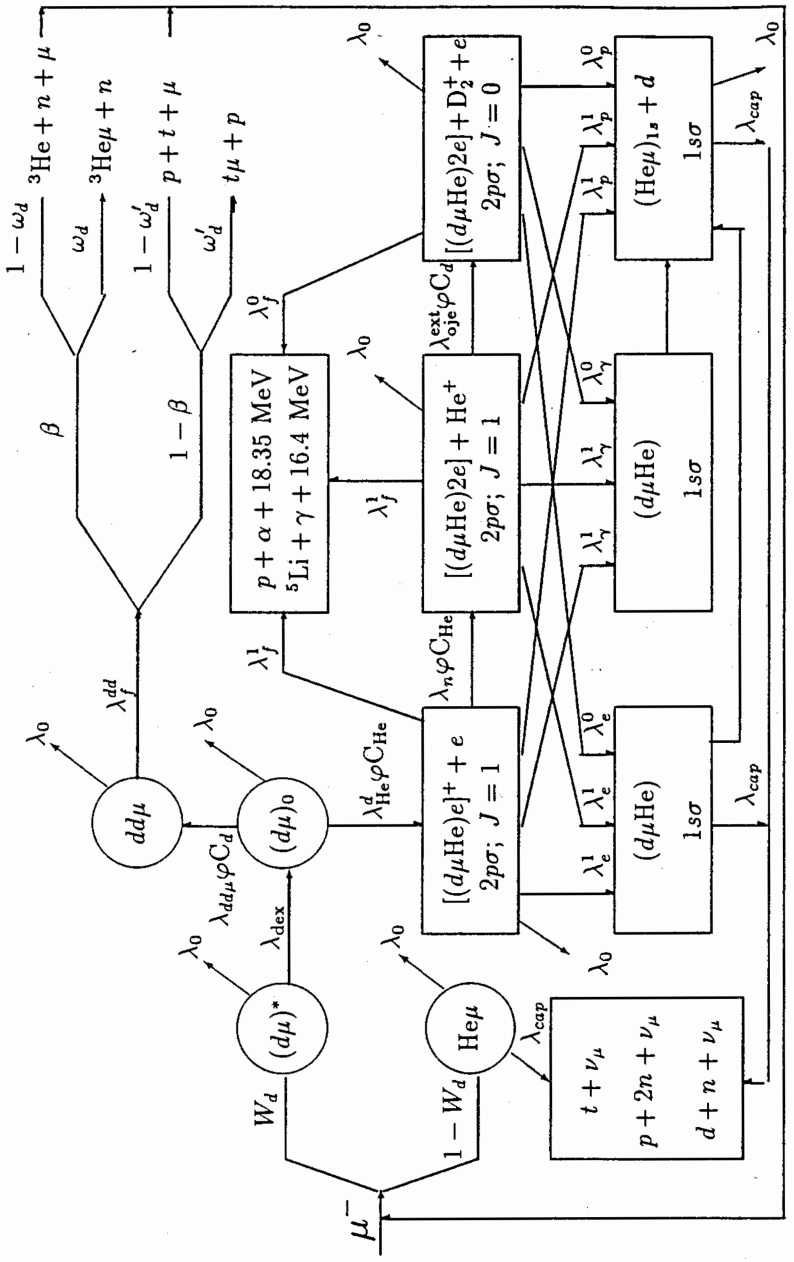

Figure 1: Mu-atomic, mu-molecular, and nuclear processes, occurring after muon stop in the $D_2 +^3$ He mixture. λ_{dex} is de-excitation rate of $d\mu$–atom; λ_f^{dd} is nuclear fusion rate in $dd\mu$–molecule. The notation is given in the text.

Analysis of the results obtained by both methods will make it possible, as we think, to gain unambiguous information on characteristics of nuclear reactions in the energy range of interest.

Investigation of nuclear fusion reactions in charge-nonsymmetrical muonic molecules like $d\mu^{3,4}$He , $t\mu^{3,4}$He is itself of interest. To know rate for formation and de-excitation of these complexes and partial rates for nuclear fusion reactions in them is necessary not only for correct description of the kinetics of mu-atomic and mu-molecular processes in mixtures of hydrogen isotopes but also for the optimum choice of experimental conditions for the effective muon-catalyzed dt reaction in a mixture of deuterium and tritium.

Besides, the study of nuclear reaction in charge-nonsymmetrical muonic molecules is a good test for the algorithm of the theoretical calculations of the rates for the processes in question.

In the present paper we propose a method for obtaining experimental information on the rates of nuclear fusion reactions in deuterium–helium muonic molecules. As the population of the states of the molecule with orbital moments 0 and 1 depends on the mixture density, it becomes possible to extract data on partial nuclear reaction rates from certain states.

2 Kinetics of processes in the D_2 + He mixture

As an example we consider a scheme of processes, occurring in the $D_2 +^3$ He mixture after negative muons stop in it (see Fig. 1). Muon transfer from muonic deuterium atoms ($d\mu$) to He nuclei takes place through the formation of an intermediate mu-molecular complex $d\mu$He in the excited state $2p\sigma$ (J = 1, J is the orbital moment of the system) [2] [30], which can decay to a He mu-atom and a deuteron with the rate λ_p (predissociation channel) or undergo a transition to the ground state $1s\sigma$ with emission of meso-X-rays of energy 6.85 keV or an Auger electron (the respective rates of the processes are λ_γ and λ_e), which is followed by the decay of the molecular complex into the nucleus of a hydrogen isotope and the muonic atom of He in the 1s ground state.[3] The scheme of molecular charge exchange of the $d\mu$ atom on the He nucleus is displayed in Fig. 2.

$pp\mu$, $pd\mu$, $dd\mu$, $dt\mu$, $tt\mu$, $pt\mu$ are charge-symmetrical).

[2] The $d\mu$He system is chosen for the experimental study because, unlike the case with other charge-nonsymmetrical complex, quite a large number of theoretical papers on investigations of this complex is now available [18-29].

[3] Muon transfer from mu-atoms of hydrogen isotopes to He nuclei $H\mu +$ He $\rightarrow$ He$\mu + H$ ($H \equiv p, d, t$; He $\equiv^3$ He,^{4} He) is strongly suppressed because of no intersections or pseudointersections of the term $2p\sigma$, corresponding to the system of the $H\mu+$ He type, and the term of the He$\mu + H$ system. In the early 1980s, the theoretically predicted [30] molecular mechanism for muon transfer from mu-atoms of hydrogen isotopes to He nuclei was found experimentally [31]. Late this discovery was experimentally confirmed in a few experiments [32-39] on the investigation of muon transfer from $p\mu$, $d\mu$ amd $t\mu$ atoms to He nuclei.

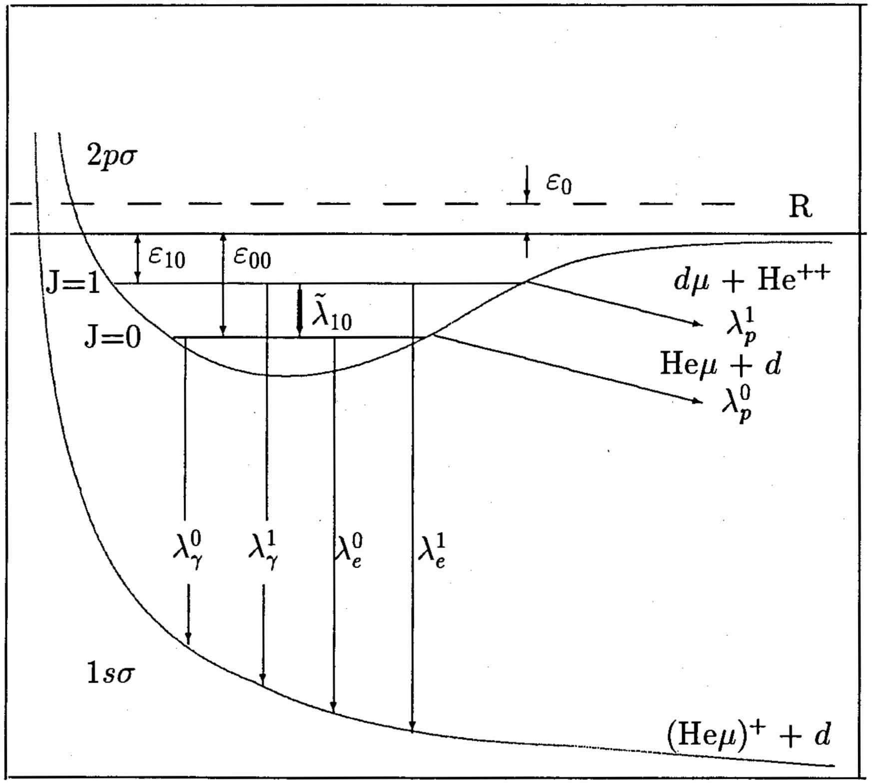

Figure 2: Scheme of molecular charge exchange of $d\mu$ atoms on He nuclei. ε_0—energy of collision of the $d\mu$ атома with the ^{3}He nucleus, ε_{10}, ε_{00}—binding energies of the $d\mu$He molecule in the states ($J = 1$; $\nu = 0$) and ($J = 0$; $\nu = 0$) (J is the orbital moment of the system, δ is the oscillation quantum number).

Since the $d\mu$He complex de-excitation rate is high as compared with its formation rate, by the rate for muon transfer from the $d\mu$ atom to the He nucleus is meant the rate for formation of these complexes.

Apart from the above-mentioned decay channels of the mu-molecular complex in the J = 1 state, competing processes are also possible, such as a nuclear fusion reaction in this complex or its rotational transition to the J = 0 state.

In the J = 0 state, as in the J = 1 state, a nuclear fusion reaction can proceed in the $d\mu$He complex with the rate λ_f^0 or this complex can decay with the rate λ_{dec}^0 ($\lambda_{dec}^0 = \lambda_p^0 + \lambda_\gamma^0 + \lambda_e^0$) via the above-mentioned three channels.

Now there are two mechanisms [40, 41] that describe the rotational transition of the $d\mu$He complex from the J = 1 state to the J = 0 state (as an example, the $1 \to 0$ transition of the $d\mu^3$He system by mechanism [40] is displayed in Fig.1).

According to [40], this transition is due to the external Auger effect in collisions of the $d\mu$He complex with D_2 molecules. This transition occurs in two steps:

(a)

$$\left[(d\mu\text{He})^{++}_{\substack{2p\sigma\\ J=1}}\, e\right]^+ + \text{He} \to \left[(d\mu\text{He})^{++}_{\substack{2p\sigma\\ J=1}}\, 2e\right] + \text{He}^+ \qquad (1)$$

(formation of a neutral helium quasi-atom; the rate of this process, normalized to the liquid hydrogen density $n_0 = 4.25 \cdot 10^{22}$ cm^{-3}, is $\lambda_n = 2 \cdot 10^{13}$ s^{-1});

(b)

$$\left[(d\mu\text{He})^{++}_{\substack{2p\sigma\\ J=1}}\, 2e\right] + \text{D}(\text{D}_2) \to \left[(d\mu\text{He})^{++}_{\substack{2p\sigma\\ J=0}}\, 2e\right] + \text{D}^+(\text{D}_2^+) + e \qquad (2)$$

(the external Auger effect; D and D_2 indicate the atom and molecule of deuterium; the rate for this process is $\lambda_{Oje}^{ext} = 8.5 \cdot 10^{11}$ s^{-1}).

In [41] another mechanism for the $1 \to 0$ transition of the $d\mu^3$He complex is considered. It also occurs in two steps:

(a)

$$\left[(d\mu^3\text{He})^{++}_{\substack{2p\sigma\\ J=1}}\, e\right] + \text{D}_2 \to \left[(d\mu^3\text{He})^{++}_{\substack{2p\sigma\\ J=1}}\, e\right]\text{D}_2 \qquad (3)$$

(cluster formation; the rate for this proces, normalized to the atomic density of liquid hydrogen, is $\lambda_{cl} = 7 \cdot 10^{13}$ s^{-1});

(b)

$$\left[(d\mu^3\text{He})^{++}_{\substack{2p\sigma\\ J=1}}\, e\right]\text{D}_2 \to \left[(d\mu^3\text{He})^{++}_{\substack{2p\sigma\\ J=0}}\, e\right]\text{D}_2^+ + e \qquad (4)$$

(internal Auger effect; the rate for this transition is $\lambda_{Oje}^{int} \simeq 10^{12}$ s^{-1}).

Since the nuclear reaction in $d\mu$He molecules

$$d\mu^3\text{He} \to p + \alpha + \mu + 18.35\ \text{MeV} \qquad (5a)$$

$$\to^5 Li\mu + \gamma(16.4\ \text{MeV}) \qquad (5b)$$

$$d\mu^4\text{He} \rightarrow {}^6Li\mu + \gamma(1.48\ \text{MeV}) \qquad (6)$$

may take place both in the J = 1 and J = 0 states, the yield of these reactions will depend on both the partial rates for nuclear fusion reactions in these states and on their population at the moment of the reaction occurrence.

The experimentally measured quantity is the effective reaction rate defined as $\lambda_f^{eff} = \alpha_1\lambda_f^1 + \alpha_0\lambda_f^0$, where α_1 and α_0 are the populations of the $d\mu$He complex states with J = 1 and J = 0 respectively (these values are determined by the ratio between the decay rates of the complexes in the states J = 0 and 1 and by the probability of the rotational $1 \rightarrow 0$ transition).

As to the yield of the 6.85-keV meso-X-rays (the transition of the $d\mu$He – complex from the $2p\sigma$ state to the $1s\sigma$ state), it is determined both by the partial rates for the radiative decay of the $d\mu$He complex in the states with J = 0, 1 $(\lambda_\gamma^0, \lambda_\gamma^1)$ and by the ratio between the rates for the decay of the mu-complex in these states $(\lambda_{dec}^0, \lambda_{dec}^1)$ and the effective rate for the rotational $1 \rightarrow 0$ $(\tilde{\lambda}_{10})$ transition.

It follows from what was said above that measurement of the yields of products from the fusion reactions in the muonic $d\mu$He complex and the yield of mu-X-ray radiation provides an opportunity to obtain experimental information on partial rates for fusion reactions and a variety of other characteristics of these complexes, on partial rates for the radiative transition of $d\mu$He molecules from the J = 0 and J = 1 states, and on the rate for the rotational transition between these states.

In [20-27] the partial probabilities of different decay channels are calculated for the $d\mu^3$He and $d\mu^4$He complexes and in [18, 19, 23, 28, 29] the nuclear fusion rates in them are estimated.

Table 1. Theoretical rates for predissociation (λ_p) and de-excitation of $d\mu^3$He and $d\mu^4$He complexes in the J = 1 state due to emission of 6.85-keV meso-X-ray radiation (λ_γ) or a conversion electron (λ_e).

Rate	$d\mu^3$He					$d\mu^4$He			
$10^{11}/s^{-1}$	[21,22]	[23]	[24]	[25,26]	[27]	[21,22]	[23]	[24]	[25,26]
λ_p^1	2.77	7.0	5.06	3.22	5.01	1.38	2.4	1.67	1.2
λ_γ^1	1.58			1.52		1.74			1.84
λ_e^1	0.41					0.43			

Partial rates for the decay of deuterium–helium complexes in the J = 0 states are calculated only in one paper [21], where they are

$$\lambda_p^0 = 3.58 \cdot 10^{11}\ s^{-1};\ \lambda_\gamma^0 = 1.8 \cdot 10^{11}\ s^{-1};\ \lambda_e^0 = 0.47 \cdot 10^{11}\ s^{-1},$$

for the dμ ^{3}He complex and

$$\lambda_p^0 = 1.85 \cdot 10^{11}\ s^{-1};\ \lambda_\gamma^0 = 1.94 \cdot 10^{11}\ s^{-1};\ \lambda_e = 0.49 \cdot 10^{11}\ s^{-1}$$

for the dμ ^{4}He complex.

Table 2. Estimates of rates for fusion reactions in $d\mu^3$He and $d\mu^4$He molecules in the J = 0 and J = 1 states.

Rate	Theory					Experiment	
(c^{-1})	[18]	[19]	[23]	[28]	[29]	[38]	[39]
$\lambda_f^0(d\mu^3\text{He})$	10^2	10^{11}	$3 \cdot 10^8$	$3.8 \cdot 10^6$	$\sim 10^6$	$\leq 4 \cdot 10^8$	$\leq 6 \cdot 10^5$
$\lambda_f^1(d\mu^3\text{He})$			10^6				
$\lambda_f^0(d\mu^4\text{He})$		10^{11}					

Table 2 lists the theoretical and experimental estimates of the rates for nuclear fusion reactions in $d\mu$He complexes available in the literature now. It follows from Table 2 that the calculated results for the $d\mu^3$He system are appreciably different. As to the experimental investigation of these systems, there are two estimates [38, 39] of the upper limit for the nuclear fusion rate in the $d\mu^3$He molecule, both obtained by the same measurement method.

Though the given estimates (see Tables 1 and 2) of the fusion reaction rates in charge-nonsymmetrical muonic molecules $d\mu^{3,4}$He are substantially smaller than their decay rate (practical use of muon-catalyzed nuclear fusion reactions in the deuterium–helium mixture for power production cannot be considered seriously), further careful experimental and theoretical investigation of these systems is nevertheless very important (see Introduction).

3 Measurement method

Experiments are supposed to be carried out at meson factories with gaseous and cryogenic targets filled with a mixture of deuterium and helium.

3.1 $D_2 + {}^3$He mixture

Experimental information on the partial rates for the fusion reaction in the $d\mu^3$He molecule in the J = 1 and J = 0 states $(\lambda_f^1,\ \lambda_f^0)$, on the partial rates for the radiative decay of this mu-complex in these states $(\lambda_\gamma^1,\ \lambda_\gamma^0)$, and on the effective rate for the rotational $1 \rightarrow 0$ transition can be obtained by measuring the yields and time distributions of 14.7 MeV protons (reaction (5a)), 16.4-MeV γ quanta (reaction (5(b)), and 6.86-keV meso-X-ray radiation. The yields and time distributions can be described by the following expressions:

$$\frac{dN^1_{p(\gamma)}}{dt} = N_\mu \lambda_{form} q_{1s} W_d \varepsilon_{p(\gamma)} \frac{\lambda^1_{f,p(\gamma)}}{\lambda_1 - \lambda_2} \times (e^{-\lambda_2 t} - e^{-\lambda_1 t}); \qquad (7)$$

$$\frac{dN^0_{p(\gamma)}}{dt} = N_\mu \lambda_{form} \frac{\tilde{\lambda}_{10}\lambda^0_{f,p(\gamma)}}{\lambda_2-\lambda_3}\varepsilon_{p(\gamma)} \times$$
$$\times\left[\frac{e^{-\lambda_3 t}}{\lambda_1-\lambda_3} - \frac{e^{-\lambda_2 t}}{\lambda_1-\lambda_2} + \frac{\lambda_2-\lambda_3}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}e^{-\lambda_1 t}\right]; \qquad (8)$$

$$\frac{dN^1_X}{dt} = N_\mu \lambda_{form} q_{1s} W_d \frac{\lambda^1_\gamma \varepsilon_X}{\lambda_1-\lambda_2} \times [e^{-\lambda_2 t} - e^{-\lambda_1 t}]; \qquad (9)$$

$$\frac{dN^0_X}{dt} = N_\mu \lambda_{form} \frac{\tilde{\lambda}_{10}\lambda^0_\gamma \varepsilon_X}{\lambda_2-\lambda_3} \times$$
$$\times\left[\frac{e^{-\lambda_3 t}}{\lambda_1-\lambda_3} - \frac{e^{-\lambda_2 t}}{\lambda_1-\lambda_2} + \frac{\lambda_2-\lambda_3}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}e^{-\lambda_1 t}\right]; \qquad (10)$$

$$\tilde{\lambda}_{10} = \frac{\lambda_n \lambda^{ext}_{Oje}\varphi^2 C_d C_{He}}{2(\lambda^1_{dec} + \lambda^{ext}_{Oje}\varphi C_d/2 + \lambda_n \varphi C_{He})} \quad \text{(according to [40]);} \qquad (11)$$

$$\tilde{\lambda}_{10} = \frac{\lambda_{cl} \lambda^{int}_{Oje}\varphi C_d}{2(\lambda^1_{dec} + \lambda^{int}_{Oje} + \lambda_{cl}\varphi C_d/2)} \quad \text{(according to [41]);} \qquad (12)$$

$$\lambda_1 = \lambda_{form} + \lambda_0 + \lambda_{dd\mu}\varphi\beta\omega_d C_d; \qquad (13)$$
$$\lambda_{form} = \lambda^d_{^3He}\varphi C_{He}; \qquad (14)$$
$$\lambda_2 = \lambda^1_{dec} + \tilde{\lambda}_{10} + \lambda_0 + \lambda^1_f; \qquad (15)$$
$$\lambda_3 = \lambda^0_{dec} + \lambda_0 + \lambda^0_f; \qquad (16)$$
$$\lambda^1_{dec} = \lambda^1_\gamma + \lambda^1_e + \lambda^1_p; \qquad (17)$$
$$\lambda^0_{dec} = \lambda^0_\gamma + \lambda^0_e + \lambda^0_p, \qquad (18)$$

where N_μ is the number of muon stops in the D_2 + He mixture; λ_0 is the free muon decay rate ($\lambda_0 = 0.455 \cdot 10^6\ s^{-1}$); $\lambda^d_{^3He}$ is the rate for the muon transfer from the $d\mu$ atom in the ground state to the ^{3}He nucleus; C_{He} and C_d are the atomic concentrations of helium and deuterium in the $D_2 +^3$ He mixture; $\varepsilon_{p(\gamma)}$ is the recording efficiency for protons (γ quanta) from fusion reaction (5); $\lambda^{1(0)}_\gamma$, $\lambda^{1(0)}_e$, $\lambda^{1(0)}_p$ are the partial rates for the decay of the $d\mu$He complex in the J = 1 and J = 0 states respectively; φ is $D_2 +^3$ He mixture density reduced to the liquid hydrogen density; $\tilde{\lambda}_{10}$ is the effective rate for the rotational transition $1 \to 0$ of the $d\mu^3$He system; $\lambda^{1(0)}_{f,p(\gamma)}$ are the partial rates for nuclear fusion in the $d\mu^3$He system in the J = 1 and J = 0 states with production of protons and γ quanta; q_{1s} is the probability for the $d\mu$ atom, formed in the excited state, to reach the ground state; W_d is the probability of the direct muon capture by the deuterium atom in the $D_2 +^3$ He mixture ($W_d = (1 + AC_{He}/(1 - C_{He}))^{-1}$, where $A = 1.7 \pm 0.2$ [42] is the ratio between

the probabilities of direct muon capture by the helium and deuterium atoms; ε_X is the recording efficiency for 6.85-keV mu-X-ray radiation; $\lambda_{dd\mu}$ is the rate for formation of $dd\mu$ molecules; β is the relative probability of the dd fusion channel with production of a neutron ($\beta = 0.58$ [43]); ω_d is the probability that muon "sticks" to the ^{3}He nucleus, resulting from the dd fusion reaction ($\omega_d = 0.122 \pm 0.003$ [43]).

Since $\lambda^{1(0)}_{dec}$, $\tilde{\lambda}_{10} \gg \lambda_{form}$, λ_0, $\lambda^{1(0)}_f$, $\lambda_{dd\mu}$, expressions (7)–(10) take the following form for times $t \gg 1/\lambda^{1(0)}_{dec}$:

$$\frac{dN^1_{p(\gamma)}}{dt} \simeq N_\mu \lambda_{form} q_{1s} W_d \frac{\lambda^1_{f,p(\gamma)}}{\lambda_4}\varepsilon_{p(\gamma)} e^{-\lambda_1 t}; \qquad (19)$$

$$\frac{dN^0_{p(\gamma)}}{dt} \simeq N_\mu \lambda_{form} q_{1s} W_d \tilde{\lambda}_{10} \frac{\lambda^0_{f,p(\gamma)}}{\lambda_4 \lambda^0_{dec}}\varepsilon_{p(\gamma)} e^{-\lambda_1 t}; \qquad (20)$$

$$\frac{dN_{p(\gamma)}}{dt} = \frac{dN^1_{p(\gamma)}}{dt} + \frac{dN^0_{p(\gamma)}}{dt} =$$
$$= \frac{N_\mu \lambda_{form} q_{1s} W_d \varepsilon_{p(\gamma)}}{\lambda_4}\left(\lambda^1_{f,p(\gamma)} + \frac{\tilde{\lambda}_{10}\lambda^0_{f,p(\gamma)}}{\lambda^0_{dec}}\right) \cdot e^{-\lambda_1 t}; \qquad (21)$$

$$\frac{dN^1_X}{dt} \simeq N_\mu \lambda_{form} q_{1s} W_d \varepsilon_X \frac{\lambda^1_\gamma}{\lambda_4} e^{-\lambda_1 t}; \qquad (22)$$

$$\frac{dN^0_X}{dt} \simeq N_\mu \lambda_{form} q_{1s} W_d \varepsilon_X \tilde{\lambda}_{10} \frac{\lambda^0_\gamma}{\lambda_4 \lambda^0_{dec}} \cdot e^{-\lambda_1 t}; \qquad (23)$$

$$\frac{dN_X}{dt} = \frac{dN^1_X}{dt} + \frac{dN^0_X}{dt} =$$
$$= N_\mu q_{1s} W_d \varepsilon_X \frac{\lambda_{form}}{\lambda_4}\left(\lambda^1_\gamma + \frac{\tilde{\lambda}_{10}\lambda^0_\gamma}{\lambda^0_{dec}}\right) \cdot e^{-\lambda_1 t}, \qquad (24)$$

where

$$\lambda_4 = \lambda^1_{dec} + \tilde{\lambda}_{10}. \qquad (25)$$

The total yields of recorded products from the fusion reactions in the $d\mu^3$He molecule and 6.85-keV mu-X-ray radiation per muon stopped in D_2 + ^{3}He mixture are defined as

$$N_{p(\gamma)} = N^1_{p(\gamma)} + N^0_{p(\gamma)} = N_\mu q_{1s} W_d \varepsilon_{p(\gamma)} \frac{\lambda_{form}}{\lambda_1(\lambda^1_{dec} + \tilde{\lambda}_{10})} \times$$
$$\times \left(\lambda^1_{f,p(\gamma)} + \frac{\tilde{\lambda}_{10}\lambda^0_{f,p(\gamma)}}{\lambda^0_{dec}}\right); \qquad (26)$$

$$N_X = N_\mu q_{1s} W_d \varepsilon_X \frac{\lambda_{form}}{\lambda_1(\lambda^1_{dec} + \tilde{\lambda}_{10})} \times$$

$$\times\left(\lambda_\gamma^1 + \frac{\tilde{\lambda}_{10}\lambda_\gamma^0}{\lambda_{dec}^0}\right). \qquad (27)$$

The expressions for yields of $d\mu^3$He fusion protons and mu-X-ray radiation per mu-molecular complex $d\mu^3$He have the form:

$$N_{p(\gamma)} = \varepsilon_{p(\gamma)} \frac{1}{\lambda_{dec}^1 + \tilde{\lambda}_{10}} \times \times\left(\lambda_{f,p(\gamma)}^1 + \frac{\tilde{\lambda}_{10}\lambda_{f,p(\gamma)}^0}{\lambda_{dec}^0}\right); \qquad (28)$$

$$N_X = \varepsilon_X \frac{1}{\lambda_{dec}^1 + \tilde{\lambda}_{10}} \times \times\left(\lambda_\gamma^1 + \frac{\tilde{\lambda}_{10}\lambda_\gamma^0}{\lambda_{dec}^0}\right). \qquad (29)$$

Approximating the time distributions of the 6.85-keV mu-X-ray radiation by (24), one can find the slope of the exponential λ_1 and thus, from (13), the rate of muon transfer from the $d\mu$ atom to the ^{3}He nucleus

$$\lambda_{^3\mathrm{He}}^d = \frac{\lambda_{form}}{\varphi C_{\mathrm{He}}} = \frac{\lambda_1 - \lambda_0 - \lambda_{dd\mu}\varphi\beta C_d\omega_d}{\varphi C_{\mathrm{He}}} \qquad (30)$$

($\lambda_{dd\mu}$, β, ω_d are supposed to be known quantities, their values being averages of the data of a variety of experimental investigations [43, 44]).

To find the desired parameters $\lambda_{f,p(\gamma)}^1$, $\lambda_{f,p(\gamma)}^0$, λ_γ^1, λ_γ^0 and $\tilde{\lambda}_{10}(\varphi)$ by joint analysis of the time distributions and yields of products of the fusion reaction in the $d\mu^3$He molecule and 6.85-keV mu-X-ray radiation (see (21), (24), (26), (27)), it is necessary to carry out a set of experiments with the $D_2 +^3$ He mixture for three and more appreciably different densities φ.

In the analysis of the experimental data the functions $\tilde{\lambda}_{10}(\varphi)$ are used in the parametric representations in terms of the rates of each of the two steps of the 1 → 0 transition of the $d\mu$He complex: in terms of λ_n, λ_{Oje}^{ext} for the transition [40] and in terms of λ_{cl}, λ_{Oje}^{int} for the transition [41] (see (11), (12).

The relative atomic concentration of helium may remain constant in these experiments. As follows from (11), (12), (21), (24), (26), (27), this analysis allows one not only to check the validity of one of the two proposed mechanisms for the rotational transition 1 → 0 of the $d\mu^3$He complex, but also to find the absolute values of the rates for each of the steps of this transition (λ_n, λ_{Oje}^{ext}) [40]; (λ_{cl}, λ_{Oje}^{int}) [41].

Note that the information on the desired parameters is gained on the assumption that the values of $\lambda_{dec}^{1(0)}$, q_{1s}, N_μ, W_d, $\varepsilon_{p(\gamma)}$ and ε_X are known (the values of λ_{dec}^1 and λ_{dec}^0 are obtained by averaging the data from [20-26]).

A distinctive feature of setting up the experiment in this way is that the yields and time distributions of protons and meso-X-ray radiation are approximated by (21), (24), (26), (27) with the real experimental rather than calculated values of $\lambda_{^3\mathrm{He}}^d$, i.e. with the values corresponding to the particular experimental conditions [45].

3.2 $D_2 +^4$ He mixture

The scheme of mu-atomic and mu-molecular processes in the $D_2 +^4$ He mixture is similar to that displayed in Fig. 1. The only difference from the $D_2 +^3$ He mixture is that muon transfer from $d\mu$ atoms to ^{4}He nuclei occurs with the formation of an intermediate mu-molecular complex $[[d\mu^4\mathrm{He}]^{++}e]^+$ in which nuclear reaction (6) is possible.

All reasoning that concerns variation in the rate for the muon transfer from $d\mu$ atoms to ^{4}He nuclei ($\lambda_{^4\mathrm{He}}^d$), the effective rate for the transition of the $d\mu^4$He complex from the excited $2p\sigma$ state with J = 1 to the state with J = 0, the partial rates for nuclear fusion reactions $\lambda_f^{1(0)}$ in the muonic molecule $d\mu^4$He, and the partial rates for the radiative decay of this molecule in the states with J = 1 and J = 0 ($\lambda_\gamma^{1(0)}$) is the same as for the experiments with the $D_2 +^3$ He mixture.

Below we give some recommendation on optimization, making of experiments with the deuterium–helium mixture and point to mistakes in determining the desired parameters for reasonable collection of statistics with a muon beam.

4 Discussion of the results

Figures 3 and 4 illustrate the relationships between the 14.7-MeV protons and 6.85-keV meso-X-ray radiation yields and the $D_2 +^3$ He density, per mu-molecular complex of $d\mu$He, obtained by (28) and (29) for two mechanisms [40, 41] of the rotational transition 1 → 0 of the $d\mu^3$He complex. The following values were used in the calculations:

$$\lambda_{dec}^1 = 6 \cdot 10^{11}\ \mathrm{s}^{-1}, \lambda_\gamma^1 = 1.55 \cdot 10^{11}\ \mathrm{s}^{-1}$$

—results of averaging the data from [20, 21, 22, 23, 24, 25, 26] ;

$$\lambda_{dec}^0 = 6 \cdot 10^{11}\ \mathrm{s}^{-1}\ [21];\ \lambda_\gamma^0 = 1.8 \cdot 10^{11}\ \mathrm{s}^{-1}\ [21];$$

$$\lambda_n = 2 \cdot 10^{13}\ \mathrm{s}^{-1}\ [40];\ \lambda_{Oje}^{ext} - 8.5 \cdot 10^{11}\ \mathrm{s}^{-1}\ [40];$$

$$\lambda_{cl} = 7 \cdot 10^{13}\ \mathrm{s}^{-1}\ [41];\ \lambda_{Oje}^{int} = 10^{12}\ \mathrm{s}^{-1}\ [41];$$

$$\lambda_{^3\mathrm{He}}^d = 2.2 \cdot 10^8\ \mathrm{s}^{-1}\ [46];\ \lambda_{dd\mu} = 0.04 \cdot 10^6\ \mathrm{s}^{-1}\ [44]$$

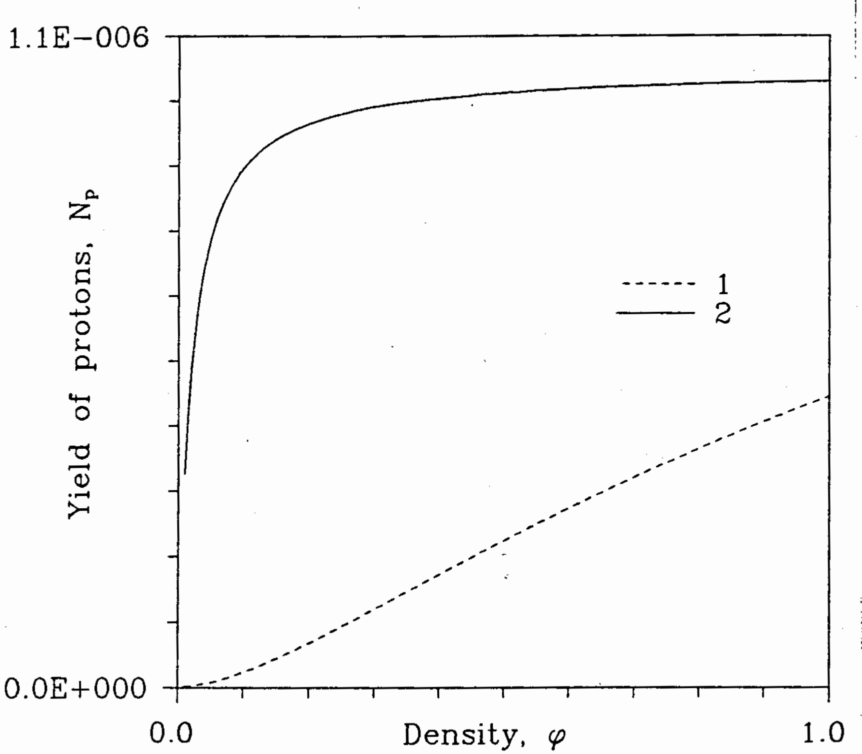

Figure 3: Proton yield N_p from reaction (5a) as a function of the $D_2 +^3$ He mixture density φ ($C_{He} = 0.1$) for two possible mechanisms of the rotational $1 \to 0$ transition of the $d\mu^3$He complex: 1—mechanism [40]; 2—mechanism [41] The probabilities are normalized to one $d\mu^3$He complex formed. The given relationships are obtained for the partial rates of the fusion reaction in the $d\mu^3$He molecule $\lambda^0_{f,p} = 10^6\ s_{-1}$; $\lambda^1_{f,p} = 10^4\ s^{-1}$.

(the values of $\lambda^d_{^3\mathrm{He}}$ and $\lambda_{dd\mu}$ for the range of the density φ from 0 to 1 were assumed to be constant and equal to their corresponding values at the temperature 30 K);

$$\omega_d = 0.122\ [43];\ \beta = 0.58\ [43];\ C_{He} = 0.1;\ C_d = 0.9;$$

$$\varepsilon_p = 0.3;\ \varepsilon_X = 5 \cdot 10^{-3}.$$

As is evident from Fig. 3 and 4, the given relationships $N_p(\varphi)$ and $N_X(\varphi)$ for the mechanisms [40, 41] of the $1 \to 0$ transition of the $d\mu^3$He complex greatly differ not only in shape but also in absolute value for the same densities of the $D_2 +^3$ He mixture.

This great different in the yields of products of the fusion reaction in the $d\mu^3$He complex undoubtedly allows not only the mechanism for the transition

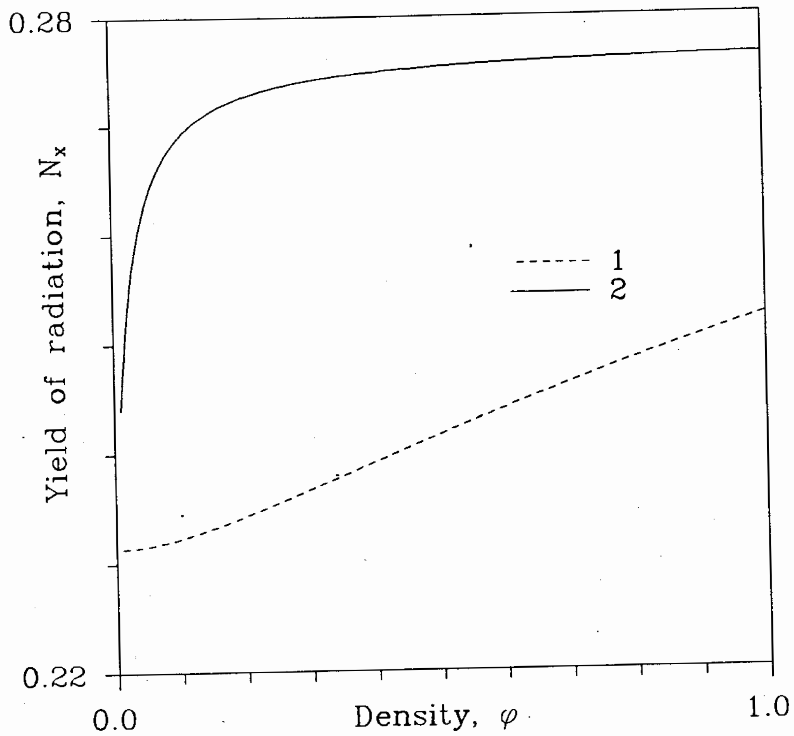

Figure 4: Yield of the 6.85-keV mu-X-ray radiation as a function of the $D_2 +^3$ He mixture density for different mechanisms of the $1 \to 0$ transition of the $d\mu^3$He system: 1—mechanism [40]; 2—mechanism [41]. The meso-X-ray radiation yields are normalized to one $d\mu^3$He complex formed. The given relationships are obtained for $\lambda^1_\gamma = 1.55 \cdot 10^{11}$ s^{-1}; $\lambda^0_\gamma = 1.8 \cdot 10^{11}$ s^{-1}.

of the mu-molecular compels from the J = 1 state to the J = 0 state to be unambiguously established but also the partial rates for the radiative decay of the complex from this states and the rates for each to the two steps of the mechanism [40, 41] for the $1 \to 0$ transition to be found.

To minimize the error in measurement of the desired parameters $\lambda^{1(0)}_f$, $\lambda^{1(0)}_\gamma$, $(\lambda_n, \lambda^{ext}_{Oje})$ [40] or $(\lambda_{cl}, \lambda^{int}_{Oje})$ [41] it is necessary:

(a) to simulate, by the Monte Carlo method, the experimental data (using formulas (11), (12), (21), (24), (26), (27) and the above values of q_{1s}, W_d, $\lambda^{1(0)}_{dec}$, $\lambda_{dd\mu}$, $\lambda^d_{^3\mathrm{He}}$) corresponding to different D_2 + ^{3}He mixture densities and values of the desired parameters;

(b) to carry out joint χ^2-analysis of the resulting data to choose the optimum experimental conditions (to choose at least three $D_2 +^3$ He mixture densities in the range from 0 to 1) corresponding to three exposures to the muon beam.

By now we have carried out a tentative statistical analysis of the yields of recorded 14.7-MeV protons and 6.85-keV meso-X-ray radiation, calculated by (26) and (27) (the yield of γ quanta from reaction (5b) was not included in the analysis because it was very small as compared with the yield of protons from reaction (5a): $N_\gamma/N_p \sim 10^{-4}$ for the energy of collisions of deuterons and ^{3}He nuclei 10 keV [47]). The following values of N_μ, q_{1s}, W_d were used in calculations:

$$N_\mu = 2 \cdot 10^4 \ \mu s^{-1}; \ q_{1s} = 0.75 \ [48]; \ W_d = 0.84 \ [42].$$

This analysis included variation in the $D_2 + ^3$He mixture density, partial rates for the nuclear fusion reaction in the $d\mu^3$He molecule $(\lambda^1_{f,p(\gamma)}, \lambda^0_{f,p(\gamma)})$, and rates for radiative decay of this molecule from the J = 1 and J = 0 states $(\lambda^1_\gamma, \lambda^0_\gamma)$ providing that the total data taking time in three exposures is 600 hours. The values of the rates $(\lambda_n, \lambda^{ext}_{Oje})$ and $(\lambda_{cl}, \lambda^{int}_{Oje})$ were fixed and equal to their values given above.

In this analysis, minimization of the errors of the desired parameters would mean a search for experimental conditions under which the sum of relative errors squared of each of the desired parameters would be the smallest.

The results of the analysis show that the situation with the determination of the partial rates for fusion in the $d\mu^3$He molecule and of their errors is different for two hypotheses of the $1 \to 0$ transition of this complex. For example, if the $1 \to 0$ transition mechanism [40] takes place, three exposures with the $D_2 + ^3$He mixture ($\varphi = 0.01;\ 0.5;\ 1.0$) provide a possibility of finding the rates for nuclear fusion in the $d\mu^3$He molecule in the J = 1 and J = 0 states with indication of their errors for the absolute values of the quantities in question differing by two–three orders of magnitude (in this case $\lambda^0_{f,p}$ is 10^5–10^6 s^{-1}) [4] :

$$\delta_{\lambda^0_{f,p}} \simeq 4\%, \ \delta_{\lambda^1_{f,p}} \simeq 15\% \text{ at } \lambda^0_{f,p} = 10^6 \text{ s}^{-1} \text{ and } \lambda^1_{f,p} = 10^4 \text{ s}^{-1};$$

$$\delta_{\lambda^0_{f,p}} = 12\%, \ \delta_{\lambda^1_{f,p}} \simeq 35\% \text{ at } \lambda^0_{f,p} = 10^5 \text{ s}^{-1} \text{ and } \lambda^1_{f,p} = 10^3 \text{ s}^{-1};$$

$$\delta_{\lambda^0_{f,p}} \simeq 5\%, \ \delta_{\lambda^1_{f,p}} \simeq 45\% \text{ at } \lambda^0_{f,p} = 10^6 \text{ s}^{-1} \text{ and } \lambda^1_{f,p} = 10^3 \text{ s}^{-1}.$$

If the $1 \to 0$ transition takes place through the mechanism [41], the possibility of determining the rates for fusion reactions in the $d\mu^3$He molecule with indication of errors is considerably reduced as compared with the case [40]:

$$\delta_{\lambda^0_{f,p}} \simeq 5\%, \ \delta_{\lambda^1_{f,p}} \simeq 70\% \text{ at } \lambda^0_{f,p} = 10^6 \text{ s}^{-1} \text{ and } \lambda^1_{f,p} = 10^4 \text{ s}^{-1};$$

$$\delta_{\lambda^0_{f,p}} \simeq 10\%, \ \delta_{\lambda^1_{f,p}} > 100\% \text{ at } \lambda^0_{f,p} = 10^5 \text{ s}^{-1} \text{ and } \lambda^1_{f,p} = 10^3 \text{ s}^{-1};$$

[4]This range of $\lambda^0_{f,p}$ values corresponds to the results of recent calculations [28, 29].

$$\delta_{\lambda^0_{f,p}} \simeq 6\%, \ \delta_{\lambda^1_{f,p}} > 100\% \text{ at } \lambda^0_{f,p} = 10^6 \text{ s}^{-1} \text{ and } \lambda^1_{f,p} = 10^3 \text{ s}^{-1}.$$

In this case the densities of the $D_2 + ^3$He mixture are $\varphi = 0.001,\ 0.5,\ 1.0$.

As to the errors in determination of the partial rates for the radiative decay of $d\mu$He complexes in the J = 1 and J = 0 states, they are as large as 10–20% for the two hypotheses of the $1 \to 0$ transition [40, 41].

This reasoning is valid right for our chosen setting-up of the experiment: the total data taking time is 600 h, the variation range of the deuterium–helium density φ is from 0 to 1.

Under these conditions the lower bounds for the measured rates of fusion reactions in $d\mu$He molecules are $\sim 10^3$ s^{-1}.

Note that if the difference between the fusion rates in the J = 0 and J = 1 states exceeds three orders of magnitude (for $\lambda^0_{f,p} \sim 10^5$ s^{-1}), the information on value of $\lambda^1_{f,p}$ can be obtained in an additional experiment with the mixture density $\varphi \approx 2 \cdot 10^{-4}$ (if the $1 \to 0$ transition occurs through the mechanism [40]) and $\varphi \approx 5 \cdot 10^{-6}$ (if the rotational transition occurs through the mechanism [41]).

At present an experiment with this low density of the $D_2 + ^3$He density ($\varphi \approx 5 \cdot 10^{-6}$) is technically impossible.

Finally, we point out that the planned experiment on the investigation of charge-nonsymmetrical complexes like $d\mu$He must involve at least three muon beam exposures with the $D_2 + ^3$He mixture densities $\varphi = 0.005,\ 0.5,\ 1.0$. Only after analyzing the results of this experiment one will be able to reveal the actual mechanism for the $1 \to 0$ transition of the $d\mu^3$He complex and to choose, if necessary, the proper conditions for additional experiments on gaining unambiguous information on the desired parameters.

This information is essential not only for correct description of the kinetics of muonic processes in the deuterium–helium mixture but also for correct comparison of the experimental and calculated data.

With the above kinetics of processes in the deuterium–helium mixture, the experimental upper bound for the fusion reaction rate in the $d\mu^3$He molecule $\lambda_{f,p}(d\mu^3\text{He}) < 6 \cdot 10^5$ c^{-1} [39], is the limit estimation of the effective nuclear fusion rate in a deuterium–helium molecule. Therefore, according to expressions (11), (12), (28), (29), the upper limits of the $\lambda^0_{f,p}$ for two feasible $1 \to 0$ transition mechanisms are: $\lambda^0_{f,p} < 7 \cdot 10^7$ s^{-1} (mechanism [40]) and $\lambda^0_f < 1.2 \cdot 10^6$ s^{-1} (mechanism [41]).

The authors are sincerely thankful to N.P.Popov for helpful discussions.

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Received by Publishing Department
on October 31, 1997.