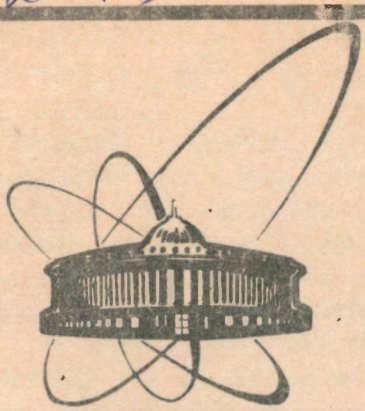

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E15-92-119

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CALCULATION OF THE BREMSSTRAHLUNG SPECTRA FROM THICK TUNGSTEN RADIATOR AS A FUNCTION OF PHOTON ENERGY AND ANGLE

1992

I. Introduction

The electron accelerators are powerful sources of gamma bremsstrahlung. They are widely used in the physics of photonuclear reactions and applied nuclear physics.

To obtain the dependence of the cross-sections of photonuclear reactions on the gamma-quanta energy, and to optimize the technique of activation analysis and isotope production, one must know the bremsstrahlung spectrum as a function of the incident electron energy, the observation angle, the target material and thickness. As there is not enough experimental data available, one has to calculate the bremsstrahlung spectra.

In principle there are several semianalytical methods for calculation of the bremsstrahlung spectra [1-6]. However, only in ref. [4-6] spectra for non-zero observation angles have been presented. This type of calculations are rather difficult, so one has to employ different approximations [7,8] still keeping in mind that they are not good for describing the spectrum behaviour near the upper limit.

In this paper we give a technique for calculation of bremsstrahlung spectra from a thick tungsten target in the energy range of 5-30 MeV at angles 0°-20°. The results obtained are compared with the available experimental data and Monte Carlo calculations.

II. Calculation technique

Let us consider a beam of electrons with kinetic energy T_e which arrive at a stopping target of thickness D (Fig.1). We divide this target into n layers of thickness $\Delta d=10^{-3}X_o$ [1], where X_o is the radiative length of the target [9]. Let the i-th layer be at a distance $d_i=(i-0.5)\Delta d$, and the mean kinetic energy of an electron in the i-th layer be $(T_e)_i$.

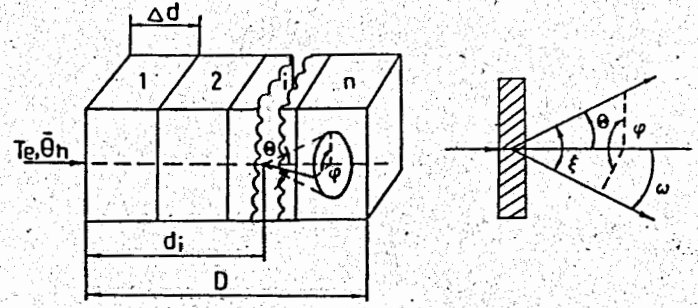

Fig.1. Schematic diagram of calculation procedure

Assuming that the "elementary" radiation spectrum, i.e. the radiation spectrum of an electron after one act of scattering is known, one adds together bremsstrahlung spectra from the whole target with allowance for electron scattering, absorption of radiation in the target material, probability of photon emission at a given angle, electron energy losses, and absorption of electrons.

In ref. [6] a formula was obtained for calculation of the bremsstrahlung spectrum from a thick target at different angles:

$$\frac{d^2Y}{dkd\Omega}\left(T_e,k,\omega\right)=\sum_{i=1}^{n}\eta_i(k,d_i,\omega)\;\tau_i(T_e,d_i)\;N_i\;\frac{d\sigma}{dk}\left[(T_e)_i,k\right]\;B_i(\omega). \qquad (1)$$

Here n is the total number of target layers, i - the number of the layer, $\eta_i(T_e,d_i,\omega)$ - the coefficient of photon absorption in the target material, $\tau_i(T_e,d_i)$ - the electron transmission coefficient, N_i - the number of atoms per unit of cross area seen by the electron in the i-th layer, $\frac{d\sigma}{dk}\left[(T_e)_i,k\right]$ - the "elementary" integral bremsstrahlung spectrum in the i-th layer, k - the photon energy, ω - the observation angle with respect to the beam axis. The quantity $B_i(\omega)$ is the number of photons emitted in a solid angle $d\Omega$ in the direction ω from the i-th layer. It is determined by the formula:

$$B_i(\omega)=\int_0^{2\pi}\int_0^{\pi}\frac{dF}{d\theta}\left[(T_e)_i,d_i,\bar{\theta}_n^2,\theta\right]\frac{d\sigma}{d\Omega}\left[(T_e)_i,\xi\right]\sin\theta\;d\theta\;d\varphi \qquad (2)$$

where $\frac{dF}{d\theta}\left[(T_e)_i,d_i,\bar{\theta}_n^2,\theta\right]$ is the angular distribution of electrons in the i-th layer, $\frac{d\sigma}{d\Omega}\left[(T_e)_i,\xi\right]$ is the angular distribution of the "elementary" spectrum, $\xi=\arccos\left(\cos\theta\cos\omega+\sin\theta\sin\omega\cos\varphi\right)$, (3) $\bar{\theta}_n$ is the half-width of the Gaussian distribution for the electrons incident on the target. The greatest difficulty in the described calculational method is to include all these factors as correctly as possible. For this purpose, we have used approximations deduced on the basis of different experimental data.

II.1. Mean energy of an electron in the i-th layer

It is well known that moving through matter electrons lose energy and change their direction of motion. On one hand, the energy losses can be due to ionization and radiation effects. On the other hand the changes in the direction of the electron motion are the result of collisions with the target atoms which in turn leads to further energy losses because of the longer trajectories.

Finally the mean kinetic energy of an electron in the i-th layer is determined by the formula:

$$(T_e)_1 = T_e - \Delta d_o^{eff.}\left(\frac{dT}{dS}\right)_{T_e} \qquad (4)$$

$$(T_e)_i = (T_e)_{i-1} - \Delta d_{i-1}^{eff.}\left(\frac{dT}{dS}\right)_{(T_e)_{i-1}} \quad (i=2,3,\ldots n) \qquad (5)$$

where $\left(\frac{dT}{dS}\right)_{(T_e)_i}$ are the mean energy losses of electrons per unit length, including the ionizing and radiative losses, viz.:

$$\left(\frac{dT}{dS}\right)_{(T_e)_i} = \left(\frac{dT}{dS}\right)^{rad.}_{(T_e)_i} + \left(\frac{dT}{dS}\right)^{ion.}_{(T_e)_i} \qquad (6)$$

$\Delta d_i^{eff.}$ is the effective electron path in the i-th layer.

The ionizing losses of energy for tungsten were determined by the formula:

$$\left(\frac{dT}{dS}\right)^{ion.}_{(T_e)_i} = \frac{0.153536}{\beta^2}(\rho Z/A)G\left[(T_e)_i\right] \quad \text{MeV/cm} \qquad (7)$$

where $\beta^2 = 1-1/(1+\tau_i)^2$, $\tau_i = (T_e)_i/mc^2$, $mc^2 = 0.5110034$ MeV.

The coefficient $G\left[(T_e)_i\right]$ for tungsten was determined in ref. [10].

The radiative losses for thin tungsten layers can be described by the formula:

$$\left(\frac{dT}{dS}\right)^{rad.}_{(T_e)_i} = 3.4910^{-3}(\rho Z^2/A)\left[(T_e)_i + mc^2\right]\psi\left[(T_e)_i, Z\right] \quad \text{MeV/cm} \qquad (8)$$

where the dimensionless function $\psi\left[(T_e)_i, Z\right]$ expresses the residual dependence of the losses on $(T_e)_i$ and Z. For Z from 1 to 100 and electron energy from 3 KeV to 10 GeV this function is approximated (with an error of less than 1%) by the expression:

$$\psi\left[(T_e)_i, Z\right] = d_1\left[1 + \sum_{k=1}^{4} f_k\left(\ln((T_e)_i)\right)^k\right] \Big/ \left[1 + \sum_{k=1}^{4} h_k\left(\ln((T_e)_i)\right)^k\right] \qquad (9)$$

with the coefficients d_1, f_k and h_k (k=1,2,3,4) calculated in ref. [11].

The number of atoms per unit area seen by the electron in the i-th layer in units atoms/cm^2, is given by:

$$N_i = (N_a/A)\rho\Delta d_i^{eff.} \qquad (10)$$

where N_a=6.022045 10^{23} moll^{-1} is Avogadro's number, A is the atomic weight of the atoms of the medium in units g/moll and ρ is the density of the medium in units g/cm^3.

The effective path of an electron in each layer was determined as follows:

$$\Delta d_o^{eff.} = \frac{\Delta d}{4}\left[1 + \frac{1}{(1-\bar{\theta}_n^2)}\right], \quad \Delta d_i^{eff.} = \Delta d\langle 1/\cos\theta\rangle_i \qquad (11)$$

where

$$\langle 1/\cos\theta\rangle_i = \int_o^{\pi}\left(1/\cos\theta\right)\frac{dF}{d\theta}\left[(T_e)_i, d_i, \bar{\theta}_n^2, \theta\right]\theta\, d\theta \,. \qquad (12)$$

The integral (12) is solved by the method described in ref. [12].

II.2. Electron transmission coefficient

The authors of ref. [1,6] have described the electron transmission coefficient by a function which had been earlier introduced by Ebert et.al. in ref. [13] for the experimental data in the energy range 4-12 MeV. In our case we prefer to use the formula given in ref. [14]. It is more suitable for describing the experimental results for electron energies 8 KeV-30 MeV and Z=4-82:

$$\tau_i(T_e, d_i) = \left[1 + \exp(-S_o)\right]\Big/\left\{1 + \exp\left[(S_o+2)(d_i/R_{ex}) - S_o\right]\right\} \qquad (13)$$

where $S_o = a_1\exp\left[-a_2/(1 + a_3\tau_o^{a_4})\right]$, $\tau_o = T_e/mc^2$, $a_1=b_1/Z^{b_2}$, $a_2=b_3Z^{b_4}$, R_{ex} is the extrapolated path of the electrons, a_i, b_i (i=1,2,3,4) are constants.

II.3. Gamma quantum absorption coefficient

Since the path of the emitted bremsstrahlung gamma quantum depends on the angle ω, we took account of photon energy absorption as in [6]:

$$\eta_i(k, d_i, \omega) = \exp\left[-\mu(k)(D-d_i)/\cos\omega\right] \qquad (14)$$

where $\mu(k)$ is the photon absorption coefficient. The relation $\mu(k)$ is given as a table in [15,16]. In practical calculations the data from these tables were approximated by polynomials with an accuracy better than 1%.

II.4. Energy and angular distribution of the "elementary" spectrum

Unfortunately, the analytical formula for the "elementary" bremsstrahlung spectrum can be derived only with some assumptions which are not suitable however for the energy and angular range of interest. Shiff [17] obtained an integral cross section with respect to photon emission angles:

$$\frac{d\sigma}{dk}\left[(T_e)_i, k\right] = \left(2\alpha Z(Z+1)r_o^2\mu/k\right)\left\{\left[1 + \left((T_e')_i/(T_e)_i\right)^2 - (2/3)\left((T_e')_i/(T_e)_i\right)\right]\left[\ln M(0) + 1 - (2/b)\operatorname{arctg}(b)\right] + \left((T_e')_i/(T_e)_i\right) \times \left[(2/b^2)\ln(1+b^2) + \left((4/3)(2-b^2)/b^3\right)\operatorname{arctg}(b) - (8/3)/b^2 + 2/9\right]\right\} \qquad (15)$$

where $\frac{1}{M(0)} = \left[\mu k / \left(2 (T_e')_i (T_e)_i \right) \right]^2 + \left(Z^{1/3}/C \right)^2$ (16)

$$b = \left[\left(2Z^{1/3} (T_e')_i (T_e)_i \right) / Ck\mu \right] \quad (17)$$

α is the fine structure constant, Z is the atomic number of the target material, r_o is the classical electron radius, $\mu = mc^2$, $(T_e')_i = (T_e)_i - k$, C is a dimensionless constant.

Generally speaking, the approximations used for derivation of the Shiff formula are valid only for $(T_e)_i \geq 50$ MeV, as pointed out in ref. [9,18]. However, the experiments on checking the bremsstrahlung spectrum showed good agreement with Shiff's results for lower electron energies as well [5]. Moreover, further calculations show that various approximations applied to the "elementary" spectrum slightly change the shape of the bremsstrahlung spectrum for thick targets, and the best agreement with the experiment is obtained with the Shiff spectrum as an "elementary" one.

To approximate the angular distribution of the "elementary" spectrum we used the results of ref. [3] where superposition of three Gaussian distributions was employed, providing better description of the experimental data:

$$\frac{d\sigma}{d\Omega}\left[(T_e)_i, k \right] = \sum_{k=1}^{3} \frac{a_k}{\pi\, b_k\, \bar{\theta}_b^2} \exp\left(- \frac{\xi^2}{b_k\, \bar{\theta}_b^2} \right), \quad (18)$$

where $\bar{\theta}_b^2 = mc^2 / \left[(T_e)_i + mc^2 \right]$ and the constants a_k, b_k (k=1,2,3) are calculated in [3].

II.5. Angular distribution of electrons

Passing through the target, electrons do not only lose energy but also undergo multiple scattering. The angular distribution of electrons $\frac{dF}{d\theta}\left[(T_e)_i, d_i, \bar{\theta}_n^2, \theta \right]$ broadens and leads to a smaller electron penetration depth as compared to the true path of the particle.

Measurements of the angular distributions of the electrons passing through thin and thick targets [19-25] show that in small penetration depths the mean square scattering angle increases almost linearly with the absorber thickness. In this region the angular distribution of the electrons is described by a Gaussian. The dependence of the squared half-width $\bar{\theta}_e^2$ on the target thickness is given by formula:

$\bar{\theta}_e^2 = \bar{\theta}_{e1/2}^2 / \ln 2$, $\bar{\theta}_{e1/2}^2 = (2.31 \log_{10}\Omega - 3.45) X_\gamma^2\, \Omega$ (19)

which describes the experimental data [23-25] much better. The

quantities Ω and X_γ are defined in ref. [26].

In depths larger than 1/4-1/3 of their real path, the electrons reach the region of full diffusion $d_{f.d.} = f(T_e, Z)$ [24] where the angular distribution width practically does not increase because electrons quickly leave the beam after large angle scattering. Unlike [1,6], where the angular distribution in this region was described by $\cos^2\theta$, the present paper (taking into account the experimental data [19-25]) represents the angular distribution as a Gaussian with a half-width:

$$\bar{\theta}_{f.d.} = 0.87 \pm 0.02 \text{ rad}. \quad (20)$$

II.6. Resultant formula

Using the Gaussian angular distribution for electrons and gamma-quanta, one can analytically estimate the fraction of photons emitted per solid angle unit in the direction ω with respect to the incident beam axis (2):

$$B_i(\omega) = \sum_{k=1}^{3} A_k \int_0^{2\pi}\!\!\int_0^{\infty} \exp\left(- \frac{\xi^2}{b_k \bar{\theta}_b^2} \right) \exp\left(- \frac{\theta^2}{\bar{\theta}_i^2} \right) \theta\, d\theta\, d\varphi, \quad (21)$$

where $A_k = \frac{a_k}{\pi^2 b_k \bar{\theta}_b^2 \bar{\theta}_i^2}$ and $\bar{\theta}_i^2 = \begin{cases} \bar{\theta}_e^2 + \bar{\theta}_n^2, & d_i < d_{f.d.} \\ \bar{\theta}_{f.d.}^2 + \bar{\theta}_n^2, & d_i \geq d_{f.d.} \end{cases}$

If one assumes that all angles (3) are much smaller than a radian (and in most cases it is so), then:

$$\xi^2 = \theta^2 + \omega^2 - 2\theta\omega\cos\varphi, \quad (22)$$

and $B_i(\omega) = \sum_{k=1}^{3} 2\pi A_k \exp\left(- \frac{\omega^2}{b_k \bar{\theta}_b^2} \right) \int_0^{\pi} \exp(-c_i\theta^2) I_o(f_i\theta)\, \theta\, d\theta$, (23)

where $c_i = \left[\left(\bar{\theta}_i^2 + b_k \bar{\theta}_b^2 \right) / b_k \bar{\theta}_b^2 \bar{\theta}_i^2 \right]$; $f_i = 2\omega / b_k \bar{\theta}_b^2$; I_o is the modified Bessel function [27]. For convenience we extend the limits of integration with respect to θ to infinity because integral (23) quickly tends to zero at $\theta > 15°$, and obtain the formula:

$$B_i(\omega) = (1/\pi) \sum_{k=1}^{3} \frac{a_k}{(b_k \bar{\theta}_b^2 + \bar{\theta}_i^2)} \exp\left\{ - \frac{\omega^2}{b_k \bar{\theta}_b^2} \left[1 - \frac{1}{\left(1 + b_k (\bar{\theta}_b^2 / \bar{\theta}_i^2) \right)} \right] \right\}. \quad (24)$$

Taking into account (1) and (24), we obtain the following expression for the bremsstrahlung spectrum:

$$\frac{d^2y}{dkd\Omega}\left[T_e,k,\omega\right] = (1/\pi)\sum_{i=1}^{n}\eta_i\,\tau_i\,N_i\left(\frac{d\sigma}{dk}\right)_i\sum_{k=1}^{3}\frac{a_k}{(b_k\bar{\theta}_b^2+\bar{\theta}_i^2)}\cdot$$

$$\cdot\exp\left\{-\frac{\omega^2}{b_k\bar{\theta}_b^2}\left[1-\frac{1}{\left(1+b_k(\bar{\theta}_b^2/\bar{\theta}_i^2)\right)}\right]\right\}. \quad (25)$$

III. Discussion of results

Unfortunately, there is much less data on the spectra of bremsstrahlung emitted at non-zero angles than for forward spectra. We know only one work [28] in the energy region of interest (5-30 MeV), with which we can compare our results. As can by seen in Fig.2, the calculation provides quite a good description of the spectrum shape for angles 0° and 12° for a tungsten target 5.8 g/cm^2 thick at an electron energy of 9.66 MeV.

Fig.3 shows our results compared with the ETRAN calculations [29] for a tungsten target 6.219 g/cm^2 thick at an electron energy of 10 MeV. The difference between the calculations is not more than 15-20% for angles other than 0°, which does not exceed the calculation accuracy varying within 10-15%.

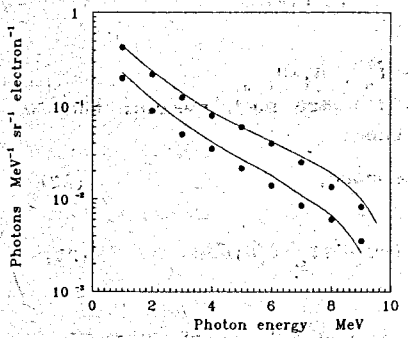

Fig.2.
Spectra of bremsstrahlung from 9.66 MeV electrons on a 5.8 g/cm^2 tungsten radiator for angles $\omega=0^o$ and 12^o from the present calculations (solid lines) and from data of ref. [28].

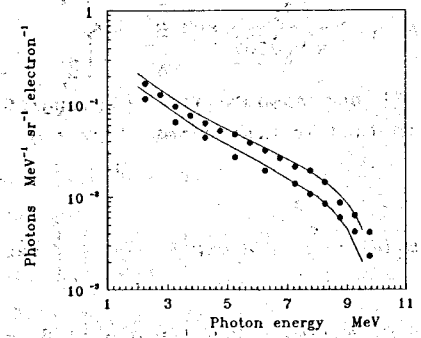

Fig.3.
Spectra of bremsstrahlung from 10 MeV electrons on a 6.219 g/cm^2 W radiator for angles $\omega=2.5^o$ and 7.5^o from the present calculations (solid lines) and from data of the computer code ETRAN [29].

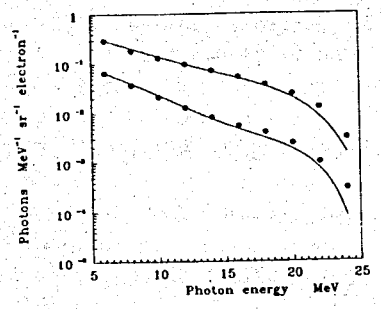

Fig.4.
Spectra of bremsstrahlung from 25.0 MeV electrons on a 6.8 g/cm^2 tantalum radiator for angles $\omega=1.7°$ and $5.1°$ from the present calculations (solid lines) and from data of ref. [30].

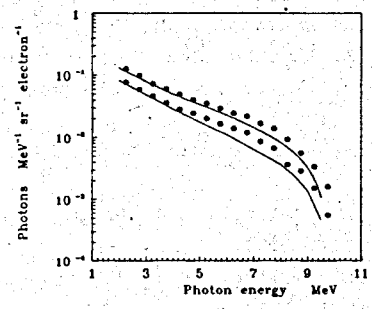

Fig.5.
Spectra of bremsstrahlung from 10 MeV electrons on a 5.899 g/cm^2 Al radiator for angles $\omega=2.5^o$ and 7.5^o from the present calculations (solid lines) and from data of the computer code ETRAN [29].

We have to point out that the proposed method is applicable not only for tungsten but also for radiators made of other materials. The comparison of our results with experimental data for a 6.8 g/cm^2 thick Ta target for electron energy 25 MeV and Monte-Carlo calculations for Al target 5.899 g/cm^2 thick for energy of the electrons 10 MeV is shown in Fig.4 and 5. The comparisons clearly demonstrate the usefulness of our method.

The authors are very much indebted to Prof. Yu.P. Gangrsky from JINR and Prof. N.P. Balabanov from Plovdiv University, Bulgaria for the encouragement and permanent interest in the work. They also wish to thank S.Tanaka, D.J.S. Findlay, A.Brahme and T.Tabata for sending their works.

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Received by Publishing Department
on March 20, 1992