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NEUTRON SCATTERING BY A FILM CONTAINING A ROTATING MAGNETIC FIELD

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Рассеяние нейтронов на тонкой пленке, содержащей вращающееся магнитное поле

Рассмотрено рассеяние нейтронов на тонкой пленке, содержащей вращающееся магнитное поле. Продемонстрировано, что существуют точки полной прозрачности пленки и полного отражения нейтронов от пленки. Поглощение нейтронов веществом пленки аномально велико в точке полного отражения. Коэффициенты отражения и прохождения имеют корневую особенность в точке открытия каналов, отвечающих рассеянию с переворотом спина.

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Перевод автора

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Neutron Scattering by a Film Containing a Rotating Magnetic Field

Neutron scattering by a thin film containing a rotating magnetic field is considered. It is shown that there exist points of total transparency and of total reflection from the film. Neutron absorption by the film is anomalous in the latter point. Transition and reflection coefficients are non-analytical in the point where the channels, corresponding to the spin-flip scattering, open up.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

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I. Introduction

Studies of particle scattering by the time-periodic potential of a simple shape are of great theoretical and experimental interest.

Of special note are such experimental results as ultracold neutron scattering by a vibrating film [1] and the interference of thermal neutrons passing through an oscillating magnetic field [2].

Important theoretical results correspond, first of all, to the analysis, carried out using rather simple models, of a rich circle of phenomena taking place in the scattering by the time-periodic potential. In [3, 4] one-dimentional scattering of a spinless particle by the $\left(u_0 + u_\tau \cos \Omega t\right)\delta(x)$ potential is considered, where t is the time, Ω is the frequency, x is the coordinate and $\delta(x)$ is the Dirac delta-function. In [5] the similar problem is considered for the rectangular potential oscillating in time as $\left(U_0 + U_\tau \cos \Omega t\right)$. But even these rather simple problems cannot be solved exactly. Each of them equivalent to an infinite set of equations. The authors use different approximations to solve this set of equations and consequently their results differ. It is stated in [3] that if $u_0 < 0$ the total reflection of neutrons with the fixed energy E takes place when the frequency is $\Omega = \Omega_0(E)$. It is asserted in [4] that in the indicated point a resonant reflection occurs but it is not total. In [5] resonant phenomena are not considered at all. As we shall see below there are various interesting phenomena existing in the scattering by the time-periodic potential. The authors of [3, 4, 5] neglected these phenomena as their results are presented in a rather complicated form.

So it is important to consider exactly soluble models and analyse in detail the arising phenomena. For example, in [6] neutron scattering by moving transversely diffraction grating is considered and some new interesting phenomena are discovered. In the present paper we will also consider the exactly soluble model: one-dimentional neutron scattering by a thin film contaning a rotating magnetic field.

2. Formulation of the Problem

Let us consider neutron beam scattering by a thin film. The film consists of two layers. The first layer is $a \sim 10^{-6}$ cm thick. It is composed of substance with the negative pseudo-potential $-U$, $U > 0$, $U \sim 10^{-7}$ ev. The imaginary part of the pseudo-potential is always negative. We denote it as $-V$, $V > 0$, $V \sim 10^{-4}U \sim 10^{-11}$ ev. The second layer is $b \sim 10^{-6}$ cm thick. It is of a ferromagnetic kind, so its pseudo-potential is $U_1 > 0$, $U_1 \sim 10^{-7}$ ev. Hence, $Ua \sim U_1 b \sim 10^{-13}$ $ev \cdot cm$. We consider that $W = Ua - U_1 b > 0$, $W \sim 10^{-13}$ $ev \cdot cm$. Let $-V_1$ be an imaginary part of the ferromagnetic pseudo-potential, $V_1 > 0$, $V_1 \sim 10^{-11}$ ev. Let

$W_* = Va + V_1 b$, $W_* > 0$, $W_* \sim 10^{-4} W \sim 10^{-17}\ ev \cdot cm$. The ferromagnetic layer is magnetized. The magnetic field H is about $10^3 - 10^4\ Gs$. The magnetic field vector is rotating in the plane of the film with the frequency $\Omega \sim 10^7\ sec^{-1}$.

The component k of the neutron wave vector, which is orthogonal to the film, satisfies the relation: $k << (a+b)^{-1}$, so $E = \hbar^2 k^2/2m << 10^{-6}\ ev$, where $\hbar$ is the Planck constant, m is the neutron mass. The neutron motion parallel to the film does not matter. So the described situation can be realized using ultracold neutrons or thermal neutrons moving of a small angle to the film. Under specified conditions the action of the film on neutrons can be described by the delta-function potential.

Let us introduce coordinates x_1, x_2, $x_3 = x$; x_1 and x_2 are directed along the film. Let us choose the x_3 axis as a magnetic field quantization axis. The neutron is described by a two-component wave function $\Psi(x,t) = \Big\{\Psi_+(x,t),\ \Psi_-(x,t)\Big\}$ (we have separated a neutron motion along the film). Under mentioned conditions the Schrödinger equation for the neutron has the form:

$$i\hbar\frac{\partial\Psi_\pm}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi_\pm}{\partial x^2} - \delta(x)\Big\{\Big(W + iW_*\Big)\Psi_\pm + \mu H b e^{\mp it\Omega}\Psi_\mp\Big\}, \qquad (1)$$

where μ is the neutron magnetic moment.

Let us note that the set of equations (1) is also correct if $W < 0$. The latter situation can be realized using the film consisting of one ferromagnetic layer.

The time dependence in (1) can be separated:

$$\Psi_+(x,t) = \psi_+(x)\exp\Big\{-itE/\hbar\Big\},\ \Psi_-(x,t) = \psi_-(x)\exp\Big\{-it(E-\hbar\Omega)/\hbar\Big\}. \qquad (2)$$

So (1) can be rewritten in the form:

$$\begin{aligned} E\psi_+ &= -\frac{\hbar^2}{2m}\frac{d^2\psi_+}{dx^2} - \delta(x)\Big\{\Big(W + iW_*\Big)\psi_+ + \mu H b\psi_-\Big\}, \\ \Big(E - \hbar\Omega\Big)\psi_- &= -\frac{\hbar^2}{2m}\frac{d^2\psi_-}{dx^2} - \delta(x)\Big\{\Big(W + iW_*\Big)\psi_- + \mu H b\psi_+\Big\}. \end{aligned} \qquad (3)$$

3. Bound States

Though our main aim is to describe scattering, in the present section we will investigate the problem of neutron bound states in the film potential. It will be important for the analysis of rezults of the solution of the scattering problem.

As the film potential includes an imaginary part, any normalizable wave function is damped with time. So we defined bound states as normalizable solutions of the eigenvalue problem (3) with the complex E, $\Im E < 0$. Since we wouldlike to get normalizable solutions, then $\Re E < 0$, and so $E = -\varepsilon - i\hbar\Gamma$, $\varepsilon > 0$, $\Gamma > 0$, $\tau = \Gamma^{-1}$

denotes the bound state lifetime. In this case the solution of (3) has the form: $\psi_\pm = A_\pm \exp\Big\{-\kappa_\pm |x|\Big\}$, where

$$\kappa_+ = \sqrt{\Big(2m/\hbar^2\Big)\Big(\varepsilon + i\hbar\Gamma\Big)},\ \kappa_- = \sqrt{\Big(2m/\hbar^2\Big)\Big(\varepsilon + \hbar\Omega + i\hbar\Gamma\Big)},\ \Re\kappa_\pm > 0. \qquad (4)$$

Inserting the described solution into (3) gives the equation for ε and Γ:

$$\Big\{\sqrt{\frac{2\hbar^2}{m}\Big(\varepsilon + i\hbar\Gamma\Big)} - W - iW_*\Big\}\Big\{\sqrt{\frac{2\hbar^2}{m}\Big(\varepsilon + \hbar\Omega + i\hbar\Gamma\Big)} - W - iW_*\Big\} - \Big(\mu H b\Big)^2 = 0. \qquad (5)$$

The equation (5) can be represented as an algebraic equation of the forth degree for $\Big(\varepsilon + i\hbar\Gamma\Big)$. So its solution has a very complicated form. But this equation has a simple solution for two important cases.

a) $W = W_* = 0$, i. e. scattering takes place only in a rotating magnetic field. As damping is lack in this case, $\Gamma = 0$. As $\varepsilon > 0$, we have the only equation in this case:

$$\varepsilon = \sqrt{\Big(\hbar\Omega/2\Big)^2 + \Big\{m\Big(\mu H b\Big)^2/\Big(2\hbar^2\Big)\Big\}^2} - \hbar\Omega/2. \qquad (6)$$

b) $H = 0$. In this case, if $W > 0$ $\Big(W >> W_*\Big)$, the following solution exists:

$$\varepsilon = \varepsilon_0 = \Big(mW^2\Big)/\Big(2\hbar^2\Big),\ \Gamma = \Gamma_0 = \Big(mWW_*\Big)/\Big(2\hbar^3\Big),\ \psi_+ \neq 0,\ \psi_- = 0. \qquad (7)$$

If $W > 0$, $\Big(mW^2\Big)/\Big(2\hbar^2\Big) > \hbar\Omega$, there is also the second solution:

$$\varepsilon = \varepsilon_1 = \varepsilon_0 - \hbar\Omega,\ \Gamma = \Gamma_0,\ \psi_+ = 0,\ \psi_- \neq 0. \qquad (8)$$

These two solutions correspond to two neutron polarizations which are independ if $H = 0$.

4. Solution of the Scattering Problem

Now let us consider the scattering problem, i.e. the set of equations (3) with the corresponding boundary condition and fixed $E > 0$. Let us introduce dimentionless values:

$$\begin{aligned} u + iv &= \Big(W + iW_*\Big)\sqrt{\Big(2m\Big)/\Big(E\hbar^2\Big)},\ f = \mu H b\sqrt{\Big(2m\Big)/\Big(E\hbar^2\Big)}, \\ \omega &= \Big(\hbar\Omega\Big)/E,\ z = x\sqrt{\Big(2mE\Big)/\hbar^2}. \end{aligned} \qquad (9)$$

Now we can rewrite (3) in the form:

$$\begin{aligned} \psi_+ &= -\frac{d^2\psi_+}{dz^2} - \delta(z)\Big\{\Big(u + iv\Big)\psi_+ + f\psi_-\Big\}, \\ \Big(1 - \omega\Big)\psi_- &= -\frac{d^2\psi_-}{dz^2} - \delta(z)\Big\{\Big(u + iv\Big)\psi_- + f\psi_+\Big\}. \end{aligned} \qquad (10)$$

In our case $u \sim 1$, $u > 0$; $v \sim 10^{-4}$, $v > 0$; $\omega \sim 1$. We consider $f < 1$ and $f \sim 10^{-1}$ or $f^2 \sim 10^{-1}$.

Two different cases exist for (10): $\omega > 1$ and $\omega < 1$ (may be $\omega < 0$).

a) $\omega > 1$, i.e. $\hbar\Omega > E$. In this case the neutron kinetic energy is so small, that the neutron spin cannot flip by scattering. The solution of (10) is given by:

$$\psi_+(z) = e^{iz} + (\alpha - 1)e^{-iz}\ , \text{ if } z < 0\ ;\ = \alpha e^{iz}\ , \text{ if } z > 0\ ;$$
$$\psi_-(z) = \beta \exp\{-\sqrt{\omega - 1}|z|\}\ , \tag{11}$$

where

$$\alpha = \frac{2i\left(u + iv - 2\sqrt{\omega - 1}\right)}{\left(2i + u + iv\right)\left(u + iv - 2\sqrt{\omega - 1}\right) - f^2}\ ,$$
$$\beta = \frac{-2if}{\left(2i + u + iv\right)\left(u + iv - 2\sqrt{\omega - 1}\right) - f^2}\ . \tag{12}$$

Let us introduce transition T_+ and reflection R_+ coefficients for the scattering without spin-flip. Let us also introduce transition T_- and reflection R_- coefficients for the scattering with spin-flip. As $\omega > 1$, then $T_+ = |\alpha|^2$, $R_+ = |\alpha - 1|^2$, $T_- = R_- = 0$.

Let us also introduce the damping coefficient $\Delta = 1 - T_+ - R_+ - T_- - R_-$, $0 \leq \Delta \leq 1$. If $v = 0$ (i.e. $W_* = 0$), then $\Delta = 0$. In our case, when $\omega > 1$, Δ is given by:

$$\Delta = \frac{4v\left\{\left(u - 2\sqrt{\omega - 1}\right)^2 + f^2\right\}}{\left\{u\left(u - 2\sqrt{\omega - 1}\right) - f^2\right\}^2 + 4\left(u - 2\sqrt{\omega - 1}\right)^2} \tag{13}$$

(in (13) we have neglected the terms $\sim v^2$).

b) $\omega < 1$. In this case we should substitute the expression $\psi_-(z) = \beta \exp\{i\sqrt{1 - \omega}|z|\}$ for $\psi_-(z)$ in (11). We also should replace the expression (12) for α, β by the substitution: $\sqrt{\omega - 1} \to -i\sqrt{1 - \omega}$. The transition and reflection coefficients are given by: $R_+ = |\alpha|^2$, $T_+ = |1 - \alpha|^2$, $R_- = T_- = \sqrt{1 - \omega}|\beta|^2$.

5. Analysis of the Solution

Now we will discuss the most interesting phenomena taking place in the described scattering.

a) Let us neglect damping. Then it follows from (12) that when $u = 2\sqrt{\omega - 1}$, then $\alpha = 0$, $T_+ = 0$, $R_+ = 1$, i.e. the total reflection of neutrons from the film occurs in this point. This situation can be realized if $u > 0$, i.e. if $W > 0$: the film potential must be attractive. The total reflection condition may be rewritten in the form:

$$\hbar\Omega = E + \left(mW^2\right)/\left(2\hbar^2\right) = E - (-\varepsilon_0)\ , \tag{14}$$

where $-\varepsilon_0$ was introduced in (7); it is the value for the bound state energy in the film without an external rotating magnetic field.

We can put forward the following interpretation of the described phenomena: the resonant reflection takes place when the energy of one quantum of the rotating magnetic field is equal to the difference of the neutron energy and the energy of the bound state (see (14)). In this case the rotating field brings the neutron into the bound state, and the neutron occupies this state for a rather long time. The resonance has to be strongly pronounced when the magnetic field is small (i.e. when f is small) since in this case the localized neutron occupies the bound state longer.

We can advaice the following argument in favour of this interpretation. As follows from (11), the square of the norm of $\psi_-(z)$ is equal to $|\beta|^2/\sqrt{\omega - 1}$. This value describes the probability of the bound state population. It follows from (12) that usually (if $u \sim 1$, $\omega \sim 1$): $|\beta|^2 \sim f^2$. But when the total reflection condition is true, then $|\beta|^2 \sim f^{-2}$, i.e. $|\beta|^2$ increases in the resonant region as $f^{-4} \sim 10^2 - 10^4$.

When $u \sim 1$, $\omega \sim 1$, the resonant width $\delta\omega \sim f^2$, i.e. $\delta\Omega/\Omega \sim f^2$, $\delta E/E \sim f^2$. If $\delta E/E \sim 10^{-1}$ in the neutron beam, the resonance is observable if $f^2 \sim 10^{-1}$.

b) Now let us take into account the small damping v and consider the damping coefficient Δ. As follows from (13), usually (when $u \sim 1$, $\omega \sim 1$): $\Delta \sim v$. But when the total reflection condition is true, then $\Delta \sim vf^{-2}$, i.e. Δ increases in the resonant region as f^{-2}. So the resonant reflection is followed by the resonant neutron damping in the film. The damping is connected with a long stay of neutrons in a bound state, i.e. in the region of the film damping potential.

c) The presence of damping leads to a weak percolation of neutrons through the film even in the resonant point. As follows from (12), $T_+ \approx \left(v/f^2\right)^2$ in the resonant point. If $f^2 \sim 10^{-1}$, then $T_+ \sim 10^{-6}$. If $f \sim 10^{-1}$, then $T_+ \sim 10^{-4}$.

d) Now let us consider the following fact which looks like a paradox in the frame of our interpretation of the resonant reflection. The condition (14) describes the resonance between the neutron energy and the bound state energy in the film without an external magnetic field. But the magnetic field changes the value of the bound state energy, as follows, for example, from (6). So the condition (14) describes the exact resonance with the level, which does not really exist in the system: the film in a magnetic field behaves as if it remembered what properties it had without a magnetic field!

Comparing (6), (12) and (13), one can see the following. While the shift of the bound level by a magnetic field is small, the resonant point is fixed, the resonant width is small and the resonant damping is large. When the bound level shift by a magnetic field becomes large, the resonant width becomes large and the resonant damping becomes small, but the resonant point remains fixed. The latter fact is extraordinary unusual. It demonstrates new type of the resonant behaviour of the quantum system.

e) Now let us consider another interesting point. We neglect damping again. It follows from (12), that if $v = 0$ and $u\left(u - 2\sqrt{\omega - 1}\right) = f^2$, then $\alpha = 1$, $T_+ = 1$, $R_+ = 0$, i.e. the film is absolutely transparent in this point. This total transparency condition can be rewritten in the form:

$$\hbar\Omega = E + \frac{mW^2}{2\hbar^2}\left\{1 - \left(\frac{\mu H b}{W}\right)^2\right\}^2 , \quad W \geq \mu H b \, . \tag{15}$$

The points of total reflection and total transparency are close. If one use dimentionless units (9), the distance between these two points is $\sim f^2$ (when $u \sim 1$, $\omega \sim 1$).

f) The presence of damping leads to a weak reflection of neutrons from the film even in the point (15). As follows from (12), in this point $R_+ \approx v^2(u^2 + f^2)/f^4$. If $f^2 \sim 10^{-1}$, then $R_+ \sim 10^{-6}$. If $f \sim 10^{-1}$, then $R_+ \sim 10^{-4}$.

g) Now let us consider scattering when $E \to +0$. As follows from (9), (12), in the vicinity of this point:

$$\alpha \approx Q\sqrt{E} \, , \; Q = \frac{i\hbar\sqrt{2/m}\left(W + iW_* - \sqrt{2\hbar^3\Omega/m}\right)}{\left(W + iW_*\right)\left(W + iW_* - \sqrt{2\hbar^3\Omega/m}\right) - \left(\mu H b\right)^2} , \tag{16}$$

i.e. the film is opaque when $E \to +0$.

If we neglect damping (i.e. put $W_* = 0$ in (16)), the point exists where the denomination of Q is equal to zero. It comes when:

$$\hbar\Omega = \frac{mW^2}{2\hbar^2}\left\{1 - \left(\frac{\mu H b}{W}\right)^2\right\}^2 , \quad W \geq \mu H b \, , \tag{17}$$

i.e. when the total transparency condition (15) is true for $E = 0$. In this case the film is transparent at the zero energy, or (if we take into account small damping) up to very small energy.

h) Now let us consider the vicinity of the point $\omega = 1$. In this region:

$$T_+ \approx \frac{4u^2}{(u^2 - f^2)^2 + 4u^2} + \frac{16uf^2(u^2 - f^2)}{\left[(u^2 - f^2)^2 + 4u^2\right]^2}\sqrt{\omega - 1} \, , \; \omega > 1 \, ,$$

$$T_+ \approx \frac{4u^2}{(u^2 - f^2)^2 + 4u^2} - \frac{32u^2 f^2}{\left[(u^2 - f^2)^2 + 4u^2\right]^2}\sqrt{1 - \omega} \, , \; \omega < 1 \, ;$$

$$R_+ \approx \frac{(u^2 - f^2)^2}{(u^2 - f^2)^2 + 4u^2} - \frac{16uf^2(u^2 - f^2)}{\left[(u^2 - f^2)^2 + 4u^2\right]^2}\sqrt{\omega - 1} \, , \; \omega > 1 \, ,$$

$$R_+ \approx \frac{(u^2 - f^2)^2}{(u^2 - f^2)^2 + 4u^2} - \frac{8f^2(u^2 - f^2)^2}{\left[(u^2 - f^2)^2 + 4u^2\right]^2}\sqrt{1 - \omega} \, , \; \omega < 1 \, ;$$

$$R_- = T_- = 0 \, , \; \omega > 1 \, ; \quad R_- = T_- \approx \frac{4f^2}{(u^2 - f^2)^2 + 4u^2}\sqrt{1 - \omega} \, , \; \omega < 1 \, . \tag{18}$$

We have neglected damping in (18), so $R_+ + T_+ + R_- + T_- = 1$. When $\omega < 1$ (i.e. when $E > \hbar\Omega$), a new scattering channel which corresponds to spin-flip scattering, opens up. It is known [7] that in the standard stationary scattering theory the opening of new scattering channels leads to a non-analytical behaviour of cross-sections in old channels. This non-analytical behaviour takes place in the point where these new scattering channels open up. It is connected with the fact that the scattering matrix is unitary.

A scattering matrix for the general case of systems with the time-periodic Hamiltonian is constructed in [8]. It is shown there that the scattering matrix in the time-periodic case is also unitary. According to this fact, the coefficients R_+ and T_+ are non-analytical in the point $\omega = 1$, as one can see from (18).

So the non-analytical behaviour of cross-sections in old channels in the point of opening of new channels occurs both in time-periodic scattering and in time-independent scattering.

6. Conclusion

We see that the exact solution of the problem for neutron scattering by a thin film which contains a rotating magnetic field, has led us to the discovery of some new phenomena. The most interesting of these phenomena are: the existence of points of total reflection and of total transparency of the film; an anomalous damping in the total reflection point; an unusual behaviour of the total reflection point as a function of a bound state energy; a non-analytical behaviour of transition and reflection coefficients in the point of opening of spin-flip scattering channels.

It seems that the exact solution of problems for neutron scattering by films with finite thickness and compound films containing a rotating magnetic field holds much promise. New qualitative effects may be found out by these solutions. In addition, these solutions will make it possible to raise a problem of quantitative calculation for real experiments.

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