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NEW POSSIBILITIES FOR INVESTIGATION OF COLLECTIVE SPIN-SPIN INTERACTIONS USING DYNAMIC COOLING OF RADIOACTIVE NUCLEI

## 1. INTRODUCTION

To measure anisotropy of the angular distribution of $\beta$ - or $\gamma$-radiation is the most sensitive method for investigation of superfine spin interactions of oriented nuclei. Depending on the type of interaction, the radiation anisotropy allows determination of magnetic and quadrupole nuclear moments, internal local magnetic fields, etc. ${ }^{1 /}$.

In connection with the recent development of a new method of nuclear polarization, known as "dynamic cooling" $/ 2,3$, it is of interest to analyse new, still unused possibilities of investigating the collective spin-spin interactions in condensed media by anisotropy of $\beta^{-}$and $\gamma^{-r a d i a t i o n . ~ A c c o r d i n g ~ t o ~}$ the idea of dynamic cooling, any nuclei in a diamagnetic matrix with admixture of paramagnetic ions can be polarized by superhigh-frequency field irradiation near the EPR line centre, the lattice temperature being $\sim 0.5 \mathrm{~K}$ in the field $\mathrm{H}_{0} \simeq 20 \mathrm{kGs}$. As is known $/ 4$, it is dynamic cooling in diols $\mathrm{C}_{3} \mathrm{H}_{6}(\mathrm{OH})_{2}$, $\mathrm{C}_{2} \mathrm{D}_{4}(\mathrm{OH})$ and the like that allows now the record polarization of ${ }^{1} \mathrm{H},{ }^{2} \mathrm{D},{ }^{13} \mathrm{C}$. Un1ike the case in the solid-effect or in the Jeffries method ${ }^{/ 3 /}$, in the dynamic cooling method, if there is no spurious leakage, all nuclei in diamagnetic atoms get polarization corresponding to the common spin temperature $/ 5,6 /$. Hence one obtains an interesting opportunity of measuring polarization of non-radioactive nuclei (e.g. deuterons) by anisotropy of the angular distribution of the radio-decay products. It will be shown below that this method is about $10^{7}$ times sensitive than the NMR for the nuclear concentration needed for the experiment, requires no equipment calibration by the so-called non-amplified signal from spins that are in thermal equilibrium with the lattice ${ }^{77 /}$. Another consequence of the higher sensitivity is a possibility of finer, as compared with the known methods, investigations of electron, electron-nuclear and nuclear-nuclear collective spin-spin interactions and kinetics of nuclear spin relaxation.
2. MEASUREMENT OF SPIN TEMPERATURE

## BY RADIATION ANISOTROPY

The method to be discussed consists in the following. A paramagnetic admixture with the concentration of $\mathrm{C}_{\mathrm{S}} \ll \mathrm{C}_{1} \quad\left(\mathrm{C}_{\mathrm{S}}=\right.$ $=2 \cdot 10^{20} \mathrm{sp} / \mathrm{cm}^{3}$ ), which is necessary for nuclear polarization by dynamic cooling, is introduced in a diamagnetic matrix with nuclei of spin I to be investigated (e.g. protons, deuterons, etc). Besides, in order to measure the spin temperature (polarization of non-radioactive nuclei in the target), an admixture of radioactive long-lived nuclei of spin J , which are part of non-paramagnetic molecules, is introduced with the concentration of $\mathrm{C}_{\mathrm{J}} \ll \mathrm{C}_{\mathrm{S}}$. After cooling the sample down to low temperatures and switching on the magnetizing field $H_{0}$, the sample is irradiated by the SHF field at a frequency corresponding to the deep cooling of the reservoir of dipole-dipole interactions of electron spins ${ }^{/ 3 /}$. In this case all nuclear spin systems, including the radioactive admixture, contact with the cooled dipole-dipole reservoir. As a result, the radioactive nuclei are polarized, and the polarization can be detected by anisotropy of $\beta$ - or $\gamma$-radiation. If the spin temperatures of all nuclei are equal, the radiation anisotropy allows the spin temperature of non-radioactive nuclei to be measured. It should be noted in this connection that equality of spin temperatures of different nuclei is proved experimentally $/ 5 /$ and theoretically $/ 6 /$ only for sufficiently high concentrations $C_{I}$ and $C_{J}$. Below this is assumed to be also valid for very small concentrations $C_{J}$, when the spin diffusion can be neglected. It is essential that this method allows one both to check experimentally the validity of this assumption and to clear out the character of the process of establishment of the balanced spin state.

If a radioactive nucleus is of spin $J$ and its magnetic moment in nuclear magnetons is equal to $\mu$, populations of the states with different spin projections $m$ on the magnetic field direction in the field $H_{0}$ at the spin temperature $T$ are described by the Boltzmann formula ${ }^{/ 8 /}$

$$
\begin{equation*}
\rho_{m m}^{(J)}=-\frac{e^{m x / J}}{\sum_{m=-J}^{J} e^{m x / J}}=\frac{e^{m x / J}}{\operatorname{sh}\left[x\left(1+-\frac{1}{2 J}\right)\right]} \cdot \operatorname{sh}\left(\frac{x}{2 J}\right), \tag{1}
\end{equation*}
$$

where
$\mathrm{x}=\mu \frac{\mathrm{eh}}{2 \mathrm{~m}_{\mathrm{p}} \mathrm{c}} \cdot \frac{\mathrm{H}_{0}}{\mathrm{kT}}$,
$e$ is the proton charge, $m_{p}$ is the proton mass, $c$ is the speed of light in vacuum, $k$ is Boltzmanns constant. The degree of nuclear polarization is determined by the Brillouin function:
$P=\frac{1}{J} \cdot \sum_{m=-J}^{J} m \rho_{m m}=B(x, J)$.
Populations coincide with the elements of the spin density matrix of the nuclei ${ }^{/ 9 /}$ which is a diagonal one in our case. The radiation anisotropy under discussion is directly related to the fact that at low spin temperatures the values of $\rho_{\mathrm{mm}}$ significantly differ from $(2 \mathrm{~J}+1)^{-1}$ corresponding to non-polarized nuclei.

## 3. BETA DECAY OF A POLARIZED NUCLEUS

The method discussed can be used, for example, to measure the spin temperature of protons (or deuterons) in propanediol with the admixture of paramagnetic $\mathrm{Cr}^{+\mathrm{v}}$ ions. In a matrix like this the $\pm 97 \%$ polarization of protons and $\pm 39 \%$ polarization of deuterons is practically achieved, which corresponds to the common spin temperature $\pm 1.3 \mathrm{mK}$ in the field $\mathrm{H}=2.7 \mathrm{~T}$. When radioactive nuclei are introduced in the matrix, the anisotropy of $\beta$-radiation appears; it only depends on the spin temperature of these nuclei. If $\beta$-transitions are allowed, anisotropy will be due to non-conservation of parity in processes caused by weak interaction $10,11 /$. In this case the angular distribution of $\beta$-electrons has the structure
$\frac{d N}{d \Omega}-1+a P \cos \theta$,
where $\theta$ is the angle between the electron (positron) emission direction and the external magnetic field direction, $P$ is the degree of polarization of radioactive nuclei, it is determined by formula (3).

In the case of pure Gamov-Teller transitions it is easy to calculate theoretically the parameter a. Already in the classical paper by Lee and Yang ${ }^{10 /}$ it was shown that, if the spins of the initial $J$ and final $J^{\prime}$ states are related as $\mathrm{J}^{\prime}=\mathrm{J}-1$, then
$a=-\frac{\langle v\rangle}{c}$,
where $\langle v\rangle$ is the electron velocity averaged for the $\beta$-spectrum. If $J^{\prime}=J+1$, then
$a=\frac{\langle V\rangle}{c} \cdot \frac{J}{J+1}$.
At $\mathrm{J}^{\prime}=\mathrm{J} \neq 0$ both Gamov-Teller and Fermi $\boldsymbol{\beta}$-transitions are possible, and the asymmetry parameter a depends on their relative contribution. In this case a can be determined experimentally by studying the $\beta$-decay of a nucleus with the known polarization.

Consider an example where the radioactive admixture consists of ${ }^{32} \mathrm{P}_{15}$ isotopes in $\mathrm{PH}_{3}$ molecules. Microquantities of this substance can be dissolved in propanediol. The half-life of the ${ }^{32} \mathrm{P}_{15}$ nucleus is 14.5 days, its spin $\mathrm{J}=1$, spin of the ${ }^{32} \mathrm{~S}_{16}$ final nucleus is equal to zero, the energy of the allowed $\mathrm{Ga}-$ mov-Teller $\beta$-transition is $1.7 \mathrm{MeV} / 12 /$. In this case $\langle v\rangle / \mathrm{c}=$ $=0.835$ and, according to Eq.(5), $a=-0.835$. If $T=1.3 \mathrm{mK}$, $H_{0}=2.7 \mathrm{~T}$, then $\mathrm{x}=-0.19$ and, according to Eq. (3), the polarization of phosphorus nuclei $P=B(-0.19,1)=-0.125$. The sign "minus" is due to the negative magnetic moment of phosphorous nuclei $\left(\mu=-0.252^{12 /}\right)$. Thus,
$\frac{d N}{d \Omega}=1+0,1 \cdot \cos \theta$.
So, reversing the polarization sign will result in a $20 \%$ difference in counting. The admixture radioactivity being 12 microcurie, the target will emit $-4 \cdot 10^{5}$ particles per second. A silicon detector 3 cm in diameter, placed 15 cm off the sample, will register $\sim 10^{3}$ events per second. It is easy to calculate that $\sim 0.3 \cdot 10^{6}$ counts are necessary to measure the coefficient 0.1 in formula (7) with an accuracy of $2 \%$ and, consequently, the statistics will be gathered in 300 sec ; in this case it is enough to have $5 \cdot 10^{11}$ radioactive molecules in the target under investigation.
4. GAMMA DECAY OF A POLARIZED NUCLEUS

A similar approach is possible for $\gamma$-radiators (long-lived isomers) which are suitable for very thick samples. If, for example, an excited nucleus is of spin $J \neq 0$, and the final nucleus is of spin $J^{\prime}=0$, ther the normalized angular distribution of $\gamma$-quanta with regard to the magnetic field direction will have the form $/ 9,18 /$
$W(\theta)=\frac{2 \mathrm{~J}+1}{8 \pi} \sum_{\mathrm{m}=-\mathrm{J}}^{\mathrm{J}} \rho_{\mathrm{mm}}\left[\left(\mathrm{d}_{\mathrm{m}, 1}^{(\mathrm{J})}(\theta)\right)^{2}+\left(\mathrm{d}_{\mathrm{m},-1}^{(\mathrm{J})}(\theta)\right)^{2}\right\}$,
where $d_{m, \pm 1}^{(J)}(\theta)$ is the Wignerfunction (elements of the matrix of finite rotations), $\rho_{\mathrm{mm}}^{(\mathrm{J})}$ are the populations of spin states related to the spin temperature via Eq.(1). A similar formula is possible at $J^{\prime} \neq 0$ for pure electric or magnetic $\gamma$-transitions of a certain multipolarity L:
$\mathrm{W}(\theta)=\frac{2 \mathrm{~J}+1}{8 \pi} \sum_{\mathrm{m}=-\mathrm{J}}^{\mathrm{J}} \sum_{\mu=-\mathrm{L}}^{\mathrm{L}}\left(\mathrm{C}_{J^{\prime} \mathrm{m}-\mu \mathrm{L} \mu}^{\mathrm{J}}\right)^{2} \rho_{\mathrm{mm}}^{(\mathrm{J})}\left[\left(\mathrm{d}_{\mu, 1}^{(\mathrm{L})}(\theta)\right)^{2}+\left(\mathrm{d}_{\mu,-1}^{(\mathrm{L})}(\theta)\right)^{2}\right]$.
Here $C$ is the Clebsch-Gordan coefficient.

## 5. CASCADE $\beta-y$-TRANSITIONS

A third possible way to measure the spin temperature is to investigate the angular anisotropy of $y$-quanta in cascade transitions. Long-lived $\beta$-active nuclei with the non-zero magnetic moment ( $\operatorname{spin} J \neq 0$ ) should be taken as the radioactive admixture. These nuclei are polarized in accordance with the spin temperature of the sample. Let the $\beta$-decay of these nuclei result in production of intermediate daughter nuclei with the non-zero spin $\mathrm{J}^{\prime}$, which in their turn undergo a rapid $y^{-}$ decay. As is below, if a certain total angular momentum $L$ is transferred to the electron and the anti-neutrino and their momenta are not registered (i.e. the averaging takes place), the polarization of the daughter nucleus immediately after the $\beta$-decay is directly connected with the polarization of the initial long-lived nucleus and thus with the initial spin temperature. Intermediate nuclei must be short-lived, so that their spin state could not noticeably change during their lifetime. Actually, this means that the life-time of an intermediate nucleus must be very short as compared with the inverse frequency of the hyperfine splitting of atomic levels ( $t \ll$ $\ll 10^{-8}-10^{-9} \mathrm{sec}$ ).

In this case the angular distribution of $\gamma$-quanta with regard to the magnetic field direction carries information on the common spin temperature of the initial radioactive nuclei from the admixture and of the non-radioactive nuclei under investigation.

Consider an arbitrary decay
$a^{(J)} \rightarrow b^{\left(J^{\prime}\right)}+X^{(L)}$
provided that the set of partiles (X), whose momenta are not fixed, take off definite total angular momentum $L$. Since in the considered case there are no distinguished directions except those related to the polarization of the particle (a), the polarization parameters of the particle (b) produced in the decay are only determined by the vector addition of angular momenta. In other words, the set of particles (X) can be replaced by a fictitios "particle" of spin L, assuming that the orbital momentum of the system ( $b+\mathrm{X}$ ) is equal to zero. In this case the elements of the spin density matrix of the particle (b) are related to the elements of the spin density matrix of the particle (a) through a simple relation

$$
\begin{equation*}
\rho_{m \tilde{m}}^{\left(\mathrm{J}^{\prime}\right)}=\sum_{\mathrm{m}=-\mathrm{L}}^{\mathrm{L}} \mathrm{C}_{\mathrm{J}^{\prime}+\mathrm{mL} \mu}^{\mathrm{J}_{\mathrm{m}}} \cdot \mathrm{C}_{J^{\prime} \tilde{\mathrm{m} L} \mu}^{\mathrm{J} \tilde{m}} \cdot \rho_{\mathrm{m}+\mu \tilde{\mathrm{m}}+\mu}^{(\mathrm{J})}, \tag{10}
\end{equation*}
$$

where C are the Clebsch-Gordan coefficients.
It is easy to see that, if the spin density matrix of the initial particle (a) is a diagonal one, the spin density matrix of the final particle (b) will be also a diagonal one, and

$$
\begin{equation*}
\rho_{\mathrm{mm}}^{\left(\mathrm{J}^{\prime}\right)}=\sum_{\mu=-\mathrm{L}}^{\mathrm{L}}\left(\mathrm{C}_{\mathrm{J}^{\prime} \mathrm{mL} \mu}^{\mathrm{Jm}+\mu}\right)^{2} \cdot \rho_{\mathrm{m}+\mu \mathrm{m}+\mu}^{(\mathrm{J})} \tag{11}
\end{equation*}
$$

It follows from Eq. (11) and the equality $\sum_{\mu}\left(C_{J^{\prime} m L \mu}^{J m+\mu}\right)=\frac{2 \mathrm{~J}+1}{2 \mathrm{~J}^{\prime}+1}$,
known in the angular momentum addition theory, that, if the decaying particle is not polarized $\left(\rho_{\mathrm{m}+\mu, \mathrm{m}+\mu}^{(\mathrm{J})}=(2 \mathrm{~J}+1)^{-1}\right.$ at $\mathrm{m}+\mu: \leq \mathrm{J}$ ), the produced particle (b) is also non -polarized:

$$
\rho_{\mathrm{m} \tilde{\mathrm{~m}}}^{\left(\mathrm{J}^{\prime}\right)}=\left(2 J^{\prime}+1\right)^{-1} \delta_{\mathrm{m} \tilde{\mathrm{~m}}} .
$$

Formula (10) is well known in the theory of $y$-decay of oriented nuclei, it determines the averaged for all directions of $\gamma$-quantum emission polarization parameters of the final nucleus resulted from a transition of a certain multipolarity
 what was said above, relations (10) and (11) are also associated with the $\beta$-decay of a nucleus (and with the K -capture) on condition that leptons receive some fixed angular momentum L and the direction of their emission is not registered. For instance, if there is an allowed Gamov-Teller $\beta$-transition, the
angular momentum of the electron-antineutrino (positron-neutrino) system is equal to one $/ 15 \%^{\prime}$. When $J-J=1$, the allowed Ga-mov-Teller transition is the main one (others are heavily suppressed), and polarization of a nucleus resulted from the $\beta$ decay is quite accurately described by formulae (10), (11) with $\mathrm{L}=1$.

Let us return to the method of measuring the spin temperature by studying cascade $\beta^{-\gamma}$ transitions and consider a case when the spin of the intermediate short-lived nucleus, produced in the $\beta$-decay of polarized admixture nuclei of $\operatorname{spin} J$, takes the values $J^{\prime}=\mathrm{J}-1$ or $\mathrm{J}^{\prime}=\mathrm{J}+1$. Let us also assume that the intermediate nucleus turns into the zero spin state through a $\gamma$-decay. Then, with allowance for Eq. (8) and Eq.(11), the angular distribution of $\gamma$-quanta, normalized to one and taken with regard to the magnetic field direction (quantization axis), will have the form
$W(\theta)=\frac{2 J^{\prime}+1}{8 \pi}, \sum_{m=1}^{J^{\prime}}\left(\rho_{m m}^{\left(J^{\prime}\right)}+\rho_{-m-m}^{\left(J^{\prime}\right)}\right)\left[\left(d_{m 1}^{\left(J^{\prime}\right)}(\theta)\right)^{2}+\left(d_{m,-1}^{\left(J^{\prime}\right)}(\theta)\right)^{2}\right]+$
$\left.+2 \rho_{00}^{\left(J^{\prime}\right)}\left(\mathrm{d}_{01}^{\left(\mathrm{J}^{\prime}\right)}(\theta)\right)^{2}\right\}$,
where

$$
\rho_{\mathrm{mm}}^{\left(\mathrm{J}^{\prime}\right)}=\mathbf{\Sigma}_{\mu=0, \pm 1}\left(\mathrm{C}_{\mathrm{J}^{\prime} \mathrm{m} \mu \mu}^{\mathrm{Jm}+\mu}\right)^{2} \rho_{\mathrm{m}+\mu \mathrm{m}+\mu}^{(\mathrm{J})},
$$

and the quantities $\rho_{\mathrm{m}+\mu \mathrm{m}+\mu}^{(\mathrm{J})}$ are related to the spin temperature
through Eq. (1). If $\mathrm{J}=3$, $\mathrm{J}=2$, we have
$W(\theta)=\frac{5}{8 \pi}-\left\{\frac{1}{2}-\left(\rho_{22}^{(2)}+\rho_{-2,-2}^{(2)}\right) \sin ^{2} \theta\left(1+\cos ^{2} \theta\right)+\right.$
$\left.+\frac{1}{2}\left(\rho_{11}^{(2)}+\rho_{-1,-1}^{(2)}\right)\left(4 \cos ^{4} \theta-3 \cos ^{2} \theta+1\right)+3 \rho_{00}^{(2)} \cos ^{2} \theta \sin ^{2} \theta\right\}$,
and
$\rho_{22}^{(2)}=\rho_{33}^{(3)}+\frac{1}{3} \rho_{22}^{(3)}+\frac{1}{15} \rho_{11}^{(3)}$,
$\rho_{11}^{(2)}=\frac{2}{3} \rho_{22}^{(3)}+\frac{8}{15} \rho_{11}^{(3)}+\frac{1}{5} \rho_{00}^{(3)}$,
$\rho_{00}^{(2)}=\frac{2}{5} \rho_{11}^{(3)}+\frac{3}{5}-\rho_{00}^{(3)}+\frac{2}{5} \rho_{-1-1}^{(3)}$,
$\rho_{-1,-1}^{(2)}=\frac{1}{5} \rho_{00}^{(3)}+\frac{8}{15} \rho_{-1-1}^{(3)}+\frac{2}{3} \rho_{-2-2}^{(3)}$,
$\rho_{-2-2}^{(2)}=\frac{1}{15} \rho_{-1-1}^{(3)}+\frac{1}{3} \rho_{-2-2}^{(3)}+\rho_{-3-3}^{(3)}$,
where
$\rho_{\mathrm{mm}}^{(3)}=\frac{\exp \left(\frac{\mathrm{m}}{3} x\right)}{\operatorname{sh}\left(\frac{7}{6} x\right)} \cdot \operatorname{sh}\left(\frac{x}{6}\right), \mathrm{m}=0, \pm 1, \pm 2, \pm 3$.
If $\mathrm{J}=2, \mathrm{~J}^{\prime}=1$,
$W(\theta)=\frac{3}{8 \pi}\left\{\frac{1}{2}\left(\rho_{\|}^{(1)}+\rho_{-1-1}^{(1)}\right)\left(1+\cos ^{2} \theta\right)+\rho_{00}^{(1)} \cdot \sin ^{2} \theta\right\}$,
$\rho_{\|}^{(1)}=\rho_{22}^{(2)}+\frac{1}{2} \rho_{\|}^{(2)}+\frac{1}{6} \cdot \rho_{00}^{(2)}$,
$\rho_{00}^{(1)}=\frac{1}{2} \rho_{\|}^{(2)}+\frac{2}{3} \rho_{00}^{(2)}+\frac{1}{2} \rho_{-1-1}^{(2)}$,
$\rho_{-1-1}^{(1)}=\frac{1}{6} \rho_{00}^{(2)}+\frac{1}{2} \rho_{-1-1}^{(2)}+\rho_{-2-2}^{(2)}$,
where
$\rho_{\operatorname{mm}}^{(2)}=\frac{\exp \left(\frac{m}{2} x\right)}{\operatorname{sh}\left(\frac{5}{4} x\right)} \cdot \operatorname{sh}\left(\frac{x}{4}\right), \quad m=0, \pm 1, \pm 2$.
If $\mathrm{J}=1, \mathrm{~J}^{\prime}=2$, the angular distribution of $\gamma^{\text {-quanta }}$ is described by formula (13) with
$\rho_{22}^{(2)}=\frac{3}{5} \rho_{11}^{(1)}$,
$\rho_{11}^{(2)}=\frac{3}{10} \rho_{11}^{(1)}+\frac{3}{10} \rho_{00}^{(1)}$,

$$
\begin{aligned}
& \rho_{00}^{(2)}=\frac{1}{10} \rho_{11}^{(1)}+\frac{1}{10} \rho_{-1-1}^{(1)}+\frac{2}{5} \rho_{00}^{(1)} \\
& \rho_{-1-1}^{(2)}=\frac{3}{10} \rho_{-1-1}^{(1)}+\frac{3}{10} \rho_{00}^{(1)}, \\
& \rho_{-2-2}^{(2)}=\frac{3}{5} \rho_{-1-1}^{(1)}
\end{aligned}
$$

where

$$
\begin{equation*}
\rho_{\mathrm{mm}}^{(1)}=\frac{e^{\mathrm{mx}}}{\operatorname{sh}\left(\frac{3}{2} x\right)} \cdot \operatorname{sh}\left(\frac{x}{2}\right) \tag{20}
\end{equation*}
$$

6. CALCULATION FOR CASCADE TRANSITION

$$
{ }^{22} \mathrm{Na}\left(3^{+}\right) \rightarrow{ }^{22} \mathrm{Ne}\left(2^{+}\right) \rightarrow{ }^{22} \mathrm{Ne}\left(\mathrm{O}^{+}\right)
$$

The half-life of the radioactive ${ }^{22} \mathrm{Na}$ nucleus is 2.6 years, the spin-parity is $3^{+}$, and the magnetic moment in nuclear magnetons $\mu=+1.75^{12}$. The Gamov-Teller $B^{+}$-transition with the maximum positron energy of 550 keV (or the K -capture) results in production of an intermediate ${ }^{22} \mathrm{Ne}$ nucleus of $\operatorname{spin} \mathrm{J}^{\prime}=2$. Its half-life is $3.7 \cdot 10^{-12}$ s, i.e. its life-time is very short as compared with the typical time of the hyper-fine level. splitting. The ${ }^{22} \mathrm{Ne}\left(2^{+}\right)$nucleus emits $\gamma$-quantum with the energy 1280 keV and changes into a stable isotope ${ }^{22} \mathrm{Ne}$ with the zero $\operatorname{spin} / 12 /$. The transition scheme is shown in the Figure.


Fig. The transition scheme of ${ }^{22} \mathrm{Na}\left(3^{+}\right)$.

In the situation under consideration the angular distribution of $1280 \mathrm{keV} \gamma$-quanta with regard to the magnetic field direction can be calculated by formulas (13)-(15) irrespective of whether the transition ${ }^{22} \mathrm{Na}\left(3^{+}\right) \rightarrow{ }^{22} \mathrm{Ne}\left(2^{+}\right) \rightarrow$ $\rightarrow{ }^{22} \mathrm{Ne}\left(0^{+}\right)$corresponds to the $\beta^{+}$-decay into a positron and a neutrino or the $K$-capture. In this case, according to Eq. (2), $\mathrm{x}=0.64 \cdot \frac{\mathrm{H}_{0}}{\mathrm{~T}}$,
where the magnetic field strength is given in tesla and the spin temperature in millikelvin. Simple calculations yield $W(x, \theta)=\frac{1}{4 \pi}\left[f_{1}(x)-f_{2}(x) \cos ^{4} \theta-f_{3}(x) \cos ^{2} \theta\right]$,
where
$f_{1}(x)=\frac{5}{2} \frac{\operatorname{sh}\left(\frac{x}{6}\right)}{\operatorname{sh}\left(\frac{7}{6} x\right)}\left[\operatorname{ch} x+\operatorname{ch}\left(\frac{2}{3} x\right)+\frac{3}{5} \operatorname{ch}\left(\frac{x}{3}\right)+\frac{1}{5}\right]$,
$f_{2}(x)=\frac{5}{2} \frac{\operatorname{sh}\left(\frac{x}{6}\right)}{\operatorname{sh}\left(\frac{7}{6} x\right)}\left[\operatorname{ch} x+\frac{1}{3} \operatorname{ch}\left(\frac{x}{3}\right)-\frac{7}{3} \operatorname{ch}\left(\frac{2}{3} x\right)+1\right]$,
$f_{3}(x)=\frac{5}{2}-\frac{\operatorname{sh}\left(\frac{x}{6}\right)}{\operatorname{sh}\left(\frac{7}{6} x\right)}\left[2 \operatorname{ch}\left(\frac{2}{3} x\right)-\frac{4}{5} \operatorname{ch}\left(\frac{x}{3}\right)-\frac{6}{5}\right)$.
If $x \ll 1$ (weak magnetic field, high spin temperature), then $\mathrm{f}_{1}(\mathrm{x})=1, \mathrm{f}_{2}(\mathrm{x})=\mathrm{f}_{3}(\mathrm{x})=0$ and, as expected, the angular distribution of $\gamma$-quanta is isotropic. If $x \gg 1$, sharp anisotropy occurs:

$$
\begin{equation*}
W(\theta)=\frac{5}{16 \pi}\left(1-\cos ^{4} \theta\right) \tag{26}
\end{equation*}
$$

It should be stressed that angular distribution (22) with the coefficients determined according to Eqs. $(23 \div 25)$ automatically satisfies the normalization condition

$$
\begin{equation*}
4 \pi \int_{0}^{1} w(x, \theta) d(\cos \theta)=1 . \tag{27}
\end{equation*}
$$

Let us consider the quantity

$$
f_{4}(x)=W(x, 0) / W(x=0)=f_{1}(x)-f_{2}(x)-f_{3}(x)
$$

It is easy to see that

$$
\begin{equation*}
f_{4}(x)=5-\frac{\operatorname{sh}\left(\frac{x}{6}\right)}{\operatorname{sh}\left(\frac{7}{6} x\right)}\left\lceil\frac{2}{3} \operatorname{ch}\left(\frac{2}{3} x\right)+\frac{8}{15} \operatorname{ch}\left(\frac{x}{3}\right)+\frac{1}{5}\right] \tag{28}
\end{equation*}
$$

Table. The values of the function $f_{9}(x), f_{2}(x), f_{3}(x)$ and $f_{4}(x)$, where $\mathrm{x}_{\mathrm{Na}}$ is determined according to Eq. (21) ; in the last two colums there are polarization of protons and deuterons at the same values of the spin temperature and the magnetic field

| $\left.x{ }^{22} \mathrm{Na}_{n+1}\right)$ | $\mathrm{f}_{1}(x)-1$ | $\mathrm{f}_{2}(x)$ | $\mathrm{f}_{3}(x)$ | $\mathrm{f}_{4}(x)$ | $P_{p}$ | $P_{D}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $1,81 \cdot 10^{-3}$ | $5,85 \cdot 10^{-8}$ | $3,57 \cdot 10^{-12}$ | $4,72 \cdot 10^{-7}$ | 1,000 | $2,9 \cdot 10^{-3}$ | $4.27 \cdot 10^{-4}$ |
| $6,28 \cdot 10^{-2}$ | $1,88 \cdot 10^{-4}$ | $1,26 \cdot 10^{-7}$ | $5,64 \cdot 10^{-4}$ | 0,9996 | 0,1 | $2,05 \cdot 10^{-2}$ |
| 0,127 | $7,66 \cdot 10^{-4}$ | $2,09 \cdot 10^{-6}$ | $2,29 \cdot 10^{-3}$ | 0,9985 | 0,2 | $4,14 \cdot 10^{-2}$ |
| 0,194 | $1,78 \cdot 10^{-3}$ | $1,13 \cdot 10^{-5}$ | $5,33 \cdot 10^{-3}$ | 0,9964 | 0,3 | $6,32 \cdot 10^{-2}$ |
| 0,265 | $3,32 \cdot 10^{-3}$ | $3,96 \cdot 10^{-5}$ | $9,94 \cdot 10^{-3}$ | 0,993 | 0,4 | $8,64 \cdot 100^{-2}$ |
| 0,344 | $5,54 \cdot 10^{-3}$ | $1,11 \cdot 10^{-4}$ | $1,65 \cdot 10^{-2}$ | 0,989 | 0,5 | 0,112 |
| 0,434 | $8,74 \cdot 10^{-3}$ | $2,78 \cdot 10^{-4}$ | $2,60 \cdot 10^{-2}$ | 0,982 | 0,6 | 0,141 |
| 0,543 | $1,35 \cdot 10^{-2}$ | $6,70 \cdot 10^{-4}$ | $4,00 \cdot 10^{-2}$ | 0,972 | 0,7 | 0,175 |
| 0,688 | 2,11 | $1,67 \cdot 10^{-3}$ | $6,22 \cdot 10^{-2}$ | 0,957 | 0,8 | 0,221 |
| 0,922 | $3,59 \cdot 10^{-2}$ | $5,08 \cdot 10^{-3}$ | 0,105 | 0,926 | 0,9 | 0,291 |
| 1,15 | $5,24 \cdot 10^{-2}$ | $1,13 \cdot 10^{-2}$ | 0,150 | 0,891 | 0,95 | 0,356 |
| 1,31 | $6,50 \cdot 10^{-2}$ | $1,81 \cdot 10^{-2}$ | 0,184 | 0,862 | 0,97 | 0,401 |
| 1,66 | $9,25 \cdot 10^{-2}$ | $4,04 \cdot 10^{-2}$ | 0,253 | 0,799 | 0,99 | 0,489 |
| 1,88 | 0,109 | $5,99 \cdot 10^{-2}$ | 0,292 | 0,757 | 0,995 | 0,540 |
| 2,38 | 0,144 | 0,121 | 0,360 | 0,663 | 0,9990 | 0,641 |

According to Eq. (22), we have $W(x, \pi / 2) / W(x, 0)=f_{1}(x)$. The intensity ratio of the radiation under the angles $\theta=0$ and $\theta=\pi / 2$ takes the form

$$
\begin{equation*}
\frac{W(x, 0)}{W\left(x, \frac{\pi}{2}\right)}=\frac{f_{4}(x)}{f_{1}(x)}=\frac{2}{3} \frac{10 \operatorname{ch}\left(\frac{2}{3} x\right)+8 \operatorname{ch}\left(\frac{x}{3}\right)+3}{5 \operatorname{ch}(x)+5 \operatorname{ch}\left(\frac{2}{3} x\right)+3 \operatorname{ch}\left(\frac{x}{3}\right)+1} \tag{29}
\end{equation*}
$$

The functions $f_{1}(x), f_{2}(x), f_{3}(x)$ and $f_{4}(x)$ are tabulated. In the Table $x_{N a}$ is determined according to Eq. (21); in the last two columns there are polarizations of protons and deuterons at the same values of the spin temperature and the magnetic field, calculated by Eq. (3) with $\mathrm{x}_{\mathrm{P}}=\mu_{\mathrm{P}} \mathrm{x}_{\mathrm{Na}} / \mu_{\mathrm{Na}}, \mathrm{x}_{\mathrm{D}}=\mu_{\mathrm{D}} \mathrm{x}_{\mathrm{Na}} \mathrm{x}_{\mathrm{Na}}$

At $H=27 \mathrm{kGs}, \mathrm{T}=1.3 \mathrm{mK}(\mathrm{x}=1.33)$ the normalized angular distribution of $y$-quanta is of the form
$W=\frac{5}{8 \pi}\left\{0,426-0,0705 \cdot \cos ^{2} A-0,0125 \cdot \cos ^{4} A\right\}$.
Under the same conditions it follows from Eq. (28) that $f_{4}(1.33)=0.86$, i.e. the effect is about $14 \%$.

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