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ON SOME PECULIARITIES OF VAVILOV-CERENKOV RADIATION

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1985

Fifty years have elapsed since the publication in 1934 in Doklady Akad. Nauk S.S.S.R, of two papers - a paper by P.A.Cerenkov and a paper by S.I.Vavilov * [1,2]. P.A.Cerenkov has summed up a comprehensive experimental material available then on the properties of glow in liquids induced by γ-rays, except for the directivity of radiation discovered later (in 1936). The paper by S.I.Vavilov contains the analysis of these results which brought him to the conclusion that the observed glow could not be the luminescence of a liquid and that the light appeared due to Compton electrons.These two papers should be considered as the two parts of one and the same work - experimental and theoretical.

The ideas advanced by S.I. Vavilov were of great importance for the development of both theory and experiment. They stimulated the experiments on the observation of glow induced in liquids in the magnetic field in order to seek for the connection between the direction of motion of electrons and polarization vector of light. The result was unexpected - it appeared that the direction of motion was connected with the angular distribution of radiation which was strongly anisotropic. This peculiarity was surprising and made the search for its interpretation especially urgent. The discussions of the problem which began in 1936 in collaboration with I.E.Tamm were a success. Recalling the remote past I would like to note that the mutual work with I.E.Tamm was very essential for me. It was the beginning of my career as a theoretician, and my further study of Vavilov-Cerenkov radiation and related topics has its origin there.

Since then many authors discussed the Vavilov-Cerenkov Radiation (VCR). In this paper I would like to linger over some peculiarities of the phenomenon that maybe have not always been paid attention to.

I shall begin with the role of the phase and group velocities of light in VCR. Everyone knows the construction based on the Huygens principle (Fig.1). It immediately gives the answer about

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* The title of Cerenkov's paper [1] was "The Visible Glow of Pure Liquids under the Action of γ-Rays"; and Vavilov's [2], "On the Possible Cause of the Blue γ-Glow in Liquids". Both papers came to the Editors of Doklady on May 27, 1934.

$$\cos\theta = \frac{c}{n(\omega)v}\ ; \quad \sin\varphi = \sin\left(\frac{\pi}{2} + \theta\right) = \frac{c}{n(\omega)v}\ ; \quad v = \frac{c}{n(\omega)}\ ,$$

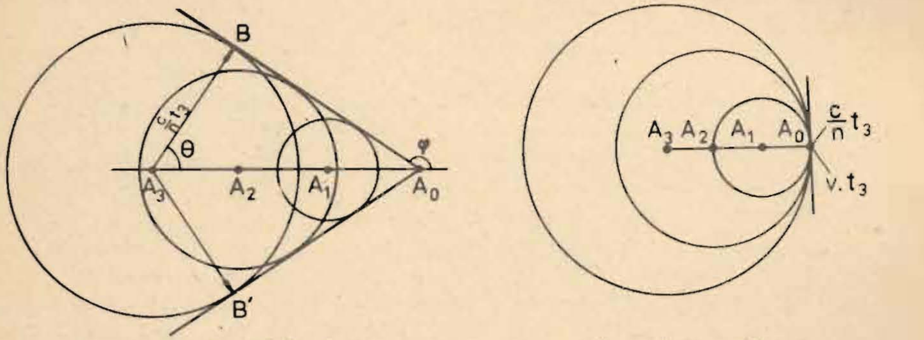


Fig. 1. A simplified Huygens construction that allows to find the characteristic angle θ_ω.

the direction of k -vector satisfying the following condition

$$\cos\theta_\omega = \frac{c}{vn(\omega)} = \frac{1}{\beta n(\omega)}\ . \qquad (1)$$

As is well known under $n(\omega)$ one understands the refractive index for the given frequency ω. In the optical isotropic medium for the light of given frequency the condition for the threshold is that the velocity of particles should be equal to the phase velocity of light

$$v = \frac{c}{n(\omega)} \qquad (2)$$

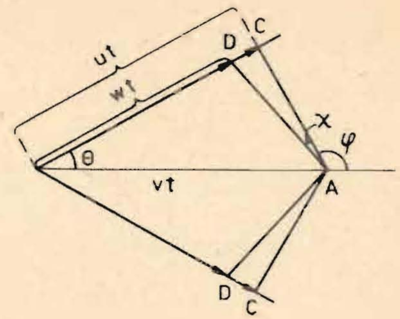
for which the angle θ_ω equals zero. The characteristic angle really depends only on phase velocity of light while the group velocity of light in that case is not essential.

From Fig.1 it follows that there is a cone enveloping spherical waves with generatrices forming the angle ϕ with the direction of motion

$$\sin\phi = \sin\left(\frac{\pi}{2} + \theta\right) = \frac{1}{\beta n(\omega)}\ . \qquad (3)$$

However, strictly speaking, it is not on the surface of this cone the VCR electromagnetic energy concentrates. The matter is that the Huygens construction applied here, though demonstrative, is oversimplified, and, therefore, not exact. In the construction to the light pulse arising in each point of the trajectory the constant velocity of propagation c/n is ascribed. Thus the light dispersion in the medium is not accounted for, while the theory does not allow this neglection. If one makes the same

Fig. 2. The wave cone (generatrix AC) and the group cone (generatrix AD).



plot for monochromatic waves, it should be assumed that the waves of frequency ω are radiated not in pulse, but continuously, and there is then not a single, but a vast number of cones satisfying eq.(3). These cones, of course, cannot be connected with the electromagnetic energy radiated. From here nevertheless follows that the direction of k -vector for the frequency ω is really determined by eq. (1). At the same time it is not always noticed that the light energy is transmitted not at a phase velocity, but at a group one, which in the medium with dispersion differs from the former and equals

$$W = \frac{c}{n(\omega) + \omega\dfrac{dn(\omega)}{d\omega}}\ . \qquad (4)$$

In order to find an instant position of the centre of a group of waves in the given narrow frequency range one should in the isotropic medium take W and not $u = c/n$ along the k -vector. The generatrices AD indicate the instant position of the group cone for the frequency ω (Fig.2). The angle between the two wave cones χ (first calculated by I.E.Tamm) may vary in a wide range, but as it seems to me it is not measured experimentally as yet. It is far from being an easy task in the practically interesting cases. For example, as is known the time difference in the detection of particles and signals due to VCR is measured in order to determine the height where the showers of cosmic particles form in atmosphere. As a rule it is assumed then that the VCR light propagates with phase velocity $u = c/n$. Strictly speaking it is not correct, but here as a simple calculation shows the interchange of u for W does not affect the result within the accuracy of the experiment. (The relative error does not exceed $\sim 10^{-5}$). The principle side of the matter is another question and we shall return to it in the further. Let us note that the fact that there is a cone of a group of waves is not at all in contradiction with the Huygens construction, if one applies it in sequence for at least two neighbour frequencies. Then as it was emphasized earlier we have to consider each point of the

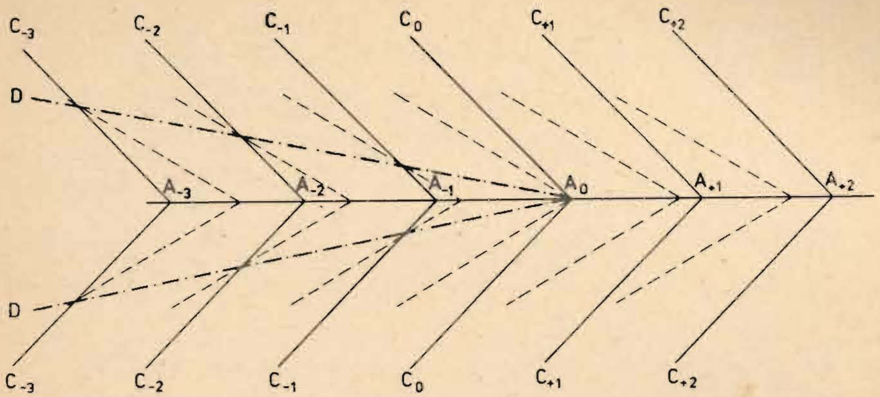


Fig.3. The Huygens plot for monochromatic waves. Solid lines - for the frequency ω, dashed lines - for the frequency $\omega + \Delta\omega$. The intersection of the corresponding generatrices gives the instant position of the group cone (dash-dot line).

trajectory as a source of the monochromatic waves of frequency ω generated continuously. The moment when the particle passes the given point determines only the phase of the wave. It is easy to see that as a result there is really a vast number of cones being the phase surface with generatrices forming the angle ϕ with the axis. Figure 3 presents the cross section in the plane of the drawing of an instant position of a series of such cones with a phase difference of 2π. Therefore, the distance between their apexes is the path the particle covers during a period of oscillation of waves with frequency ω (in the figure the generatrices of cones are $A_{-2}C_{-2}$, $A_{-1}C_{-1}$; A_0C_0, A_1C_1, etc.). The apex of the only one of them coincides with the instant position of the particle (let us call it the wave cone). In order to find this cone one should consider the same picture, but for another frequency, say, a little larger, e.g., $\omega' = \omega + \Delta\omega$. The phases of those two waves coincide in the point of the particle and consequently so do the apexes of the cones. In the case illustrated in Fig.3 it is the point A_0. Other cones do not coincide, since they have a little different ϕ angle in the presence of light dispersion and, moreover, the distance between cones is shorter due to a shorter oscillation period. The cross sections are shown by the dashed line. The intersection of the corresponding generatrices of both cones determines the surface of equal phases for the two frequencies ω and ω', i.e., the group cone. It can be shown that the discussed graphical method gives as it should be the same result as that shown in Fig.2 [4].

It is only natural to try to find out the role of the group velocity in VCR. For that one should ask oneself about the minimal velocity at which the radiation appears and not about the velocity at which the radiation with the given frequency is induced. The answer is simple - the minimal velocity corresponds to the maximal value of n

$$v_{min} = \frac{c}{n_{max}}. \qquad (5)$$

If the value of n is maximal then $dn/d\omega = 0$ and consequently the phase velocity coincides with the group velocity. Thus the condition for the threshold of VCR appearance corresponds to the case [5,6] when the velocity of particle first achieves the value of the group velocity

$$v = W. \qquad (6)$$

This relationship could have been considered as a mere coincidence if it were not true for all threshold phenomena of the radiator moving uniformly in the medium. It is the same for the appearance of the complex Doppler effect, of the anomalous Doppler effect [5,6] and with some reservations it is true for the appearance of the X-ray transition radiation [7]. So, the general nature has also the statement that the velocity of motion equal to the group velocity of light is the condition for the threshold of appearance of the complex effect, i.e., the radiated frequency splits into two components. Therefore, the VCR is always the complex effect, because, though at a given angle the radiation with only one frequency is observed, there is the other unseen frequency component in the anomalous dispersion area. It can be easily seen that the condition (6) for the threshold of appearance of VCR is also true for the optically anisotropic media which cannot be at all considered evident in advance. The condition (1) for the characteristic angle θ holds also in the anisotropic medium, if n is the refractive index for the k-vector with a given polarisation and direction [8].

At the same time, however, the direction of the ray along which, as is known, the group velocity is directed does not, generally speaking, coincide with the k-vector, but forms with it a certain angle α. Then in anisotropic medium, not only the phase velocity u appears essential, but also the velocity of phase propagation along the ray. Let us denote it u′ (see Fig.4).

$$u' \cos\alpha = \frac{c}{n} = u. \qquad (7)$$

The general condition for the appearance of the Vavilov-Cerenkov Radiation of frequency ω should be formulated in the following way: the threshold velocity of the source should equal

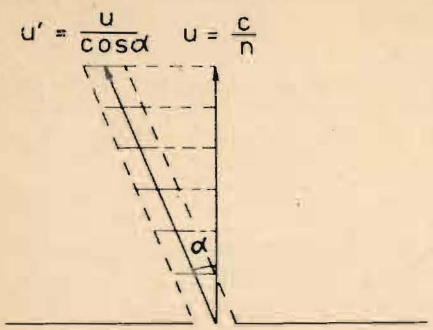


Fig.4. The normal to the wave (u - phase velocity) and the direction of the ray (u' - wave velocity along the ray) in the anisotropic medium.

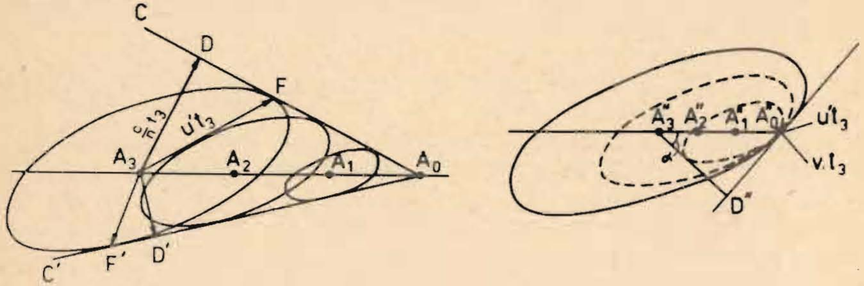


Fig.5. A simplified Huygens plot for the anisotropic medium. The direction of the ray is determined by the tangent points of elipse and generatrices of the wave cone (see A_3F and A_3F' in the figure to the left). Vector k *is directed along the normal to generatrices of the wave cone (see AD and AD'). Under threshold conditions the direction of the ray coincides with the direction of velocity* v *and* v *is being equal to* u'. *The vector* k *is directed along the normal to the tangent plane (see $A_3''D''$). It forms the angle* α *with the direction of the ray.*

the velocity of waves along the ray in the direction of motion. In other words

$$v = u'. \qquad (8)$$

The threshold velocity v coincides always with the direction of the ray and not with the k-vector, that, generally speaking, forms the angle α with v. (In optically isotropic medium the velocity $u = u'$ and it is the same in all directions). Hence, the condition (2) is a special case of the more general condition (8). The correctness of the condition (8) may be easily proved in the frame of the Huygens principle applied for the optically anisotropic medium [5,9] (Fig.5).

The condition for the appearance of VCR and not only of the given radiation frequency is

$$v_{min} = u'_{min}. \qquad (9)$$

For the frequency ω at which $u' = u'_{min}$ the derivative of the wave velocity along the ray equals zero and hence the velocity itself equals the group velocity. This again brings us to the conclusion that the threshold for the radiation to appear is defined by eq. (6), i.e., it is determined by the group velocity, or in other words the above connection proved to be general.

First we obtained the conditions for the threshold of appearance of the complex and anomalous Doppler effects [10]. L.I.Mandelshtam in his lecture has drawn attention to the essential role of the group velocity *.

Another question I wish to discuss here is the duration of VCR pulse arising when the charged particle goes through the radiator. Besides its theoretical value the question has the practical importance, since the Cerenkov counters are widely used in various coincidence schemes. P.A. Cerenkov [1] had shown even in his first experiments that the radiation he observed could not be reduced neither by introducing strong quenching agents of luminiscence nor by changing the temperature of the liquid. It was this that made S.I.Vavilov [2] think that the observed light was not the luminiscence at all, since the necessary characteristic of the latter according to S.I.Vavilov is a finite time of excitation of atoms and molecules which is of the order or more than 10^{-10} sec. However, the duration of the light pulse is not always due to the time of excitation only. If the radiator has some length, then it takes different times for the light from its points to arrive at the detector. For example, the light pulse from the radiator 3 cm long will be of duration not less than 10^{-10} sec under the condition that the light simultaneously leaves each point of the radiator. At the same time one knows that the Cerenkov counter may be more than 100 cm long. A question naturally arises: how will it affect the light pulse duration in VCR? The answer might have been the simplest if there were no dispersion of light in the medium. Really, if the detector summarizes the radiation coming at an angle θ, then signals from all the points of the radiator must arrive at the counter simultaneously. But the dispersion makes it different.

The picture to be observed is shown schematically in Fig.6. The dashed line is the trajectory of the particle, d is the width of the beam detected. Such limitations for the width always exist in each real counter. They are due to either the diaphragm, or the size of a focusing instrument, or the geometry of the light

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* In the published lecture [11] it is done in short and not distinct enough.

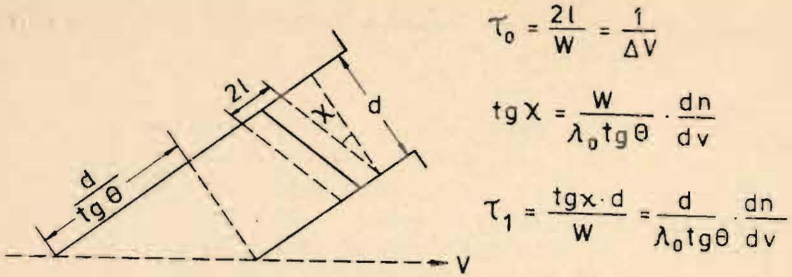

Fig.6. The distribution of the light signal from the particle moving in the counter. The time τ_0 is determined by the finite length of the group waves, and τ_1 by the fact that the group cone forms the angle χ with the wave cone.

source itself. For the sake of simplicity let d be the diameter of the diaphragm. The group surface forms the angle χ with the wave surface. The detected spectrum is always limited in frequency and, therefore, the group of waves has a finite length too. It is denoted by 2ℓ in the figure. Let us find the time which it takes the group of waves in the given frequency range $v \pm \Delta v$ to go through the diaphragm. The time period evidently goes into two parts. Since the group has a length 2ℓ it will go through each point for some time τ_0. Besides that, the group passed through different points not simultaneously, since the group surface forms the angle χ with the wave surface. Let us denote this additional time by τ_1. Now I shall consider τ_0 and τ_1 separately. It is easy to see that

$$\tau_0 = \frac{2\ell}{W} = \frac{1}{\Delta v}\,. \qquad (10)$$

Usually there is no large increase of pulse duration due to τ_0. It does not exceed 10^{-12} for visible light if the width of the group is $\Delta\lambda \sim 10$ Å. It is evident that, if the light dispersion in the radiator is low, the group width can be taken more than 10 Å and τ_0 may be neglected.

In order to find τ_1 one should know χ. It can be easily shown that [4]

$$tg\,\chi = \frac{W}{\lambda_0 tg\,\theta}\,\frac{dn}{dv}\,. \qquad (11)$$

From where it follows

$$\tau_1 = \frac{tg\,\chi \cdot d}{W} = \frac{d}{\lambda_0 tg\,\theta}\,\frac{dn}{dv}\,. \qquad (12)$$

So the coefficient in front of dn/dv is expressed in portions

$\lambda_0 = n\lambda$ — difference of paths from the farest and the nearest point of the radiator the light from which goes through the diaphragm, i.e., reaches the detector (see Fig.6). This difference is large when the radiator is long and θ is small. It may achieve 10^6 in modern Cerenkov counters. However, they are as a rule the gas counters in which dn/dv is very small, and thus τ_1 is not large. With solid radiators the time increases to $10^{-10} - 10^{-11}$ sec. In several cases it seemingly may even serve as a method for the measurement of θ [4]. So the difference between the phase velocity and group velocity is essential in some cases.

It should be noted that the time τ_1 may, in principle, be reduced. Really, it appears due to the dependence of θ on refractive coefficient. So, it is enough to transmit VCR as it is often done for precise measurements of θ through an achromatic system to reduce τ_1 to minimal possible value. Some of these achromatic systems were discussed by the author in his paper published in 1956 [4].

Here the angle θ is supposed to be practically constant. If the spectral width of the beam is so wide that the change in the refractive coefficient and consequently in the angle θ is considerable, then the pulse duration increases by an additional time τ_2 that may be large. All the above stated depends essentially of course on the type of the counter used and, first of all, on its geometry and the material of the radiator.

The fact that in any Cerenkov counter the radiation is always summed up over some finite path of the charged particle brings forward additional peculiarities of the phenomenon. They may be especially important at small θ, i.e., in the vicinity of the theoretically expected threshold.

A sharp directivity of VCR at a characteristic angle θ is often considered to be the integral part of the phenomenon. But a simple consideration shows that the radiation goes in some angle range $\Delta\theta$ near θ_ω that is the wider the shorter the radiator is. It is the so-called diffraction width of the peak directly connected with the coherent length ℓ', being equal in the simplest case of wave summation in the θ direction to [10]

$$\ell' = \frac{\pi v}{\omega\,|1 - \beta n \cos\theta|} = \frac{\beta\lambda_0}{2\,|1 - \beta n \cos\theta|}\,. \qquad (13)$$

According to the definition ℓ' is the length along which the phase of the waves from both its ends arriving at an infinitely far point differs by π. The value of ℓ' formally turns into infinity for the characteristic angle θ_ω though in reality it always has some finite length. The width of the diffraction peak $\Delta\theta$ is determined by the fact that the length $\mathcal{L}$ of the counter comprises two coherent lengths ℓ'.

From here under supposition of very small θ_ω one has

$$\Delta\theta_\omega = \frac{\lambda_0}{\mathfrak{L} n \sin\theta_\omega} \qquad (14)$$

So, if the angle θ_ω is not very small, the range $\Delta\theta_\omega$ is first of all determined by the ratio between the wavelength and the length of the radiator. It has a small but finite value and this probably may set limits for the accuracy of θ_ω measurements. The value of $\Delta\theta$ can be easily measured, if one takes the radiator of thickness comparable with λ. For the radiator from mica of thickness $\mathfrak{L} = 3\lambda_0$ the value of $\Delta\theta$ reaches 30° and the experiment demonstrates good agreement with theoretical predictions.

Other peculiarities are connected with the behaviour of VCR near the threshold. The diffraction width did not allow the threshold to be sharp. Really, for $\beta n = 1$, i.e., the characteristic angle $\theta = 0$, the intensity must not be equal to zero due to the finite width of the radiation peak. If it were possible to observe the radiation strictly in the direction of motion of the particle, then it would have appeared that the threshold for the appearance of radiation with frequency ω shifts a little to lower values, i.e.,[12]

$$\beta n = 1 - \frac{\beta\lambda}{\mathfrak{L}}. \qquad (15)$$

When the length of the radiator is large in comparison with the wavelength of light the displacement of the threshold is insignificantly small. However, if the radiator is thin, the shift is large as is shown in ref.[12].

Besides, near the very threshold the simple relationship (1) for the characteristic angle does not hold already. The matter is that at a finite length of the counter one of the coefficients of intensity proportionality is the squared sinus of radiation angle which makes the observed angle of radiation larger. Nearer to the threshold the intensity of VCR decreases, but the angle θ always has a finite value. Strictly speaking, the intensity never turns into zero even for βn less than the practical threshold (15), since there is always the radiation due to a limited trajectory. The latter radiation is identical with the bremsstrahlung or with the transition radiation, which cannot be separated from VCR above its threshold, though make a little contribution into it. They become essential near the threshold and below it. This really takes place in the gas Cerenkov counters for relativistic particles. One should take into account then the particular geometry of the counter.

I wished to show here that in the latter case as well as in other problems considered in the present paper the Vavilov-Cerenkov Radiation is a more complicate phenomenon, than the idealized picture conventionally used for it.

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Франк И.М. E14-85-168

Некоторые особенности излучения Вавилова-Черенкова

Приводится текст доклада, прочитанного в июле 1984 года на Всесоюзном семинаре "Черенковские детекторы и их применение в науке и технике", посвященном пятидесятилетию открытия Вавилова и Черенкова. В докладе рассмотрен ряд особенностей излучения Вавилова-Черенкова /ИВЧ/, на которые не всегда обращают внимание. Применение принципа Гюйгенса, с помощью которого обычно поясняют природу явления, приводит в случае анизотропных сред к обобщенному условию порога возникновения ИВЧ данной частоты. Последовательное применение принципа Гюйгенса для двух близких монохроматических частот позволяет определить мгновенное положение группы волн. Далее показано, что условием возникновения ИВЧ /а не данной частоты этого излучения/ является равенство скорости частицы минимальной величине групповой скорости света в веществе радиатора. Продолжительность сигнала, регистрируемого детектором черенковского счетчика, также во многом зависит от групповой скорости света в радиаторе и конечно от геометрии и конструкции счетчика. Поскольку любой радиатор счетчика имеет ограниченную длину, излучение всегда наблюдается в некотором интервале углов, прилегающем к характерному; это так называемая дифракционная ширина, которая тем больше, чем короче радиатор. Ее следует учитывать особенно в области, близкой к порогу излучения. Ряд особенностей наблюдается у ожидаемого по элементарной теории порога излучения. Вблизи порога ИВЧ неотделимо от переходного излучения, наблюдаются особенности в угловом распределении, порог не является резким и всегда имеется подпороговое излучение /переходное и тормозное/.

Работа выполнена в Лаборатории нейтронной физики ОИЯИ.

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Frank I.M. E14-85-168

On Some Peculiarities of Vavilov-Cerenkov Radiation

The paper is a reprint of the report made by the author in June 1984 at the All-Union seminar on "Cerenkov Detectors and Their Application in Science and Technology". There are considered some peculiarities of the Vavilov-Cerenkov Radiation (VCR) not always paid attention to. The Huygens principle conventionally used to explain the nature of the phenomenon leads in the case of anisotropic medium to the generalized threshold conditions for the appearance of VCR of a given frequancy. A consequent application of the Huygens principle to two neighbour monochromatic frequencies allows one to find the immediate position of the wave group. Further it is shown that the condition for the appearance of VCR (and not of a given frequency of radiation) is that the velocity of particle should be equal to the group velocity of light inside the radiator. The duration of the pulse registered in the detector is also strongly dependent on the group velocity of light inside the radiator and, of course, on the geometry and construction of the counter. Since any radiator of the counter has a finite length, the radiation is observed in some angle range around the characteristic angle, being called the diffraction width, which is the larger the shorter the radiator is. It should be accounted for, especially near the radiation threshold. Several peculiarities are observed near the threshold predicted within the elementary theory. Near the VCR threshold and inseparable from the transition radiation one observes a peculiar angular distribution, i.e., the threshold is not sharp since there always exists a subthreshold radiation (transition or bremsstrahlung).

The investigation has been performed at the Laboratory of Neutron Physics, JINR.

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