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THE LONDON PENETRATION DEPTH MEASUREMENT USING SQUID MICROSCOPE

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1. Introduction

The London penetration depth is one of the most important parameters of superconductor [1]. Its value is determined by the concentration of the superconducting carrier density $\lambda_L \propto \sqrt{\frac{m}{ne^2}}$, where m is the mass, e is the charge, n is the concentration of the superconducting carriers, and usually constitutes 500 - 5000 angstrom. Such a small value determines a difficulties of its measurement. But the temperature dependence of λ_L gives the valuable information for understanding of the mechanism of superconductivity [2], and thus its investigation is very important

Let us consider the most precise for such measurements "inductive" method [3-5]. In this method, the changing of the inductance of some system of coils with the temperature changing of London penetration depth of the sample under investigation is measured. The usual size of pick-up coil in such a system is 1 cm. Thus, in this method it is necessary to measure the changes of the signal which are 10^5 times smaller than the signal itself. In spite of the method of this changes measurement (resonance technique, SQUID susceptometry or otherwise), it is clearly seen that the changing of the sizes of pick-up coils with the temperature or temperature dependence of the susceptibility of the constructive materials of an installation give a considerable error in such measurements. Thus, the miniaturisation of the pick-up coils is a direct way to increase the accuracy of this method. Due to decrease of the geometrical size of the pick-up coils, the level of the signal also decreases. Therefore, for such measurements the most sensitive technique, like SQUID microscopy, must be used [6-12].

In this paper, we consider the possibilities of the SQUID microscope in such measurements. In the beginning, we consider a linear response of superconductor in an external magnetic field of arbitrary configuration. Then we solve this problem for a thin superconducting film and propose the scheme of a device for the measurement of the local London penetration depth for the thin film and for the bulk samples.

2. Superconductor in an external magnetic field

Let us consider a superconductor in an external magnetic field. The London equation in a form of the measured values is

$$\lambda^2 \mathrm{rot}\mathbf{J} + \mathbf{H} = 0, \quad (1)$$

and

$$\Delta \mathbf{J} - \frac{1}{\lambda^2}\mathbf{J} = 0. \tag{2}$$

The magnetic field generated by some external source and screening currents of the superconductor is

$$\mathrm{rot}(\mathbf{H_0} + \mathbf{H_1}) = \mathbf{J_0} + \theta \mathbf{J_s}, \tag{3}$$

where $\mathbf{H_0}$ is the field generated by the external current $\mathbf{J_0}$, $\mathbf{H_1}$ is the field generated by the screening current of the superconductor $\mathbf{J_s}$, θ is the Heaviside function ($\theta = 1$ inside the superconductor, $\theta = 0$ outside). From (3) we obtain:

$$-\Delta \mathbf{H_1} = \theta \mathrm{rot}\mathbf{J_s} + \delta_s(\mathbf{n})\begin{pmatrix} -J_{sy}(0) \\ J_{sx}(0) \\ 0 \end{pmatrix}, \tag{4}$$

where $\mathbf{n}$ is a vector normal to the surface of the superconductor and coincident with the z axis of the local coordinate system xyz, J_{sx} and J_{sy} are the surface components of the screening current, $\delta_s(\mathbf{n})$ is the Dirac delta-function.

Now, we convolute the equation (4) with the fundamental solution of equation (2) A^0 (the sign * means convolution in the three-dimensional space). Keeping in mind the equation (1) and the existence of function $A^0 * \mathbf{H}$, we obtain:

$$-\Delta A^0 * \mathbf{H}_1 = -\frac{1}{\lambda^2} A^0 * \theta\,(\mathbf{H}_1 + \mathbf{H}_0) + A^0 * \delta_s(\mathbf{n})\begin{pmatrix} -J_{sy}(0) \\ J_{sx}(0) \\ 0 \end{pmatrix} \tag{5}$$

or

$$\mathbf{H}_1 = -\frac{1}{\lambda^2} A^0 * (1-\theta)\mathbf{H}_1 + \frac{1}{\lambda^2} A^0 * \theta \mathbf{H}_0 + A^0 * \delta_s(\mathbf{n})\begin{pmatrix} -J_{sy}(0) \\ J_{sx}(0) \\ 0 \end{pmatrix}. \tag{6}$$

In any case, the field outside the superconductor may be written as a field of double layer, positioned on the surface of the superconductor:

$$_{\mathrm{out}}\big| \mathbf{H}_1 = \mathbf{H^1} * \delta_s'(\mathbf{n})\, \mathbf{m}_s , \tag{7}$$

where $\mathbf{H^1}$ is a magnetic field of a monopole and $\mathbf{m}_s$ is a magnetic moment perpendicular to the surface. Composing (6) and (7), we could write the equation for the surface of the superconductor and for the component $\mathbf{H}$ normal to the surface:

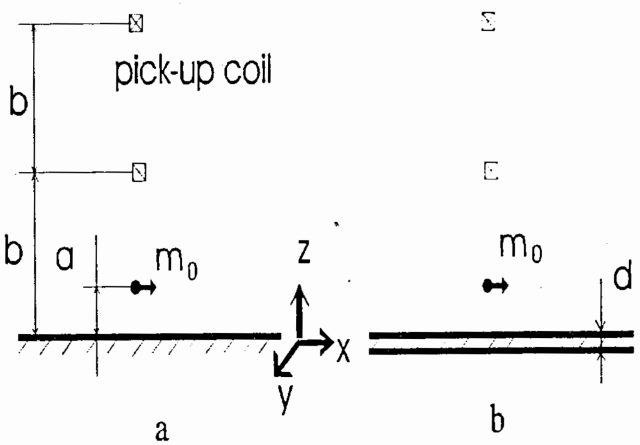

Figure 1. The scheme of the location of the excitation loop m_0, the pick-up loop, and the coordinate system for a) half-space, b) thin film

$$_{s,\mathbf{n}}\big| \left(\mathbf{H^1} + \frac{1}{\lambda^2} A^0 * (1-\theta)\mathbf{H^1} \right) * \delta_s'(\mathbf{n})\, \mathbf{m}_s = \frac{1}{\lambda^2} A^0 * \theta \mathbf{H}_0 . \tag{8}$$

Thus, we transform the three-dimensional vector problem to the surface problem. The $\mathbf{m}_s$ is perpendicular to the surface of the superconductor and absolutely determines the field generated by the superconductor outside its volume. To determine surface current of the superconductor, it is necessary to solve equation (6) for the surface of the superconductor and then the magnetic field inside the superconductor is determined. Usually, in practical problems, it is necessary to determine only the response of the superconductor in an external excitation field, i.e. the 'outside' field.

3. The scheme of the London penetration depth measurement

Let us consider the following scheme for the measurement of the London penetration depth. Near the infinite superconducting film (or half-space for the bulk superconductor) the excitation coil generating the external magnetic field (see Figure 1) is positioned. It is located at distance a from the film and may be considered as a point dipole m_0. For the measurement of the magnetic flux, generated by the screening currents of the film, a pick-up coil located at a distance b, is used. It has the sizes $b \times b$. Using equation (8), let us calculate the magnetic flux through the pick-up coil and its dependence on λ.

3.1. The case of a half-space

Let the surface of a superconductor be determined as z = 0, so that superconductor is positioned at z < 0 (see Figure 1a). The external field generated by the excitation coil is equal

$$\mathbf{H}_0 = \mathbf{H}^1 * \mathbf{m}_0\delta(z-a)\,\delta'(x)\delta(y),$$

where $\mathbf{m}_0$ has only the x-component.

In this case, we could make the Fourier transformation of the equation (8) by two coordinates x and y ($f(k)=\int_{-\infty}^{\infty} f(x)e^{-ikx}dx$ and $f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(k)e^{ikx}dk$) and obtain

$$_{z=0}\big|\ (H^1_z(k_1,k_2,z) + \frac{1}{\lambda^2}A^0(k_1,k_2,z) * \theta(z)H^1_z(k_1,k_2,z)) * m_1(k_1,k_2)\,\delta'(z) =$$

$$\frac{1}{\lambda^2}A^0(k_1,k_2,z) * (1-\theta(z))H^1_z(k_1,k_2,z) * ik_1 m_0\,\delta(z-a). \qquad (9)$$

The convolution in this equation is made only by z coordinate and

$$H_z^{\,1}(k_1,k_2,z) = \frac{1}{2}sign(z)e^{-\sqrt{k_1^2+k_2^2}\,|z|},$$

$$A^0(k_1,k_2,z) = -\frac{1}{2\sqrt{k_1^2+k_2^2+\frac{1}{\lambda^2}}}e^{-\sqrt{k_1^2+k_2^2+\frac{1}{\lambda^2}}\,|z|}.$$

Performing integration in (9) by z and setting z=0 we obtain for m_1

$$m_1 = -m_0 ik_1\frac{e^{-\sqrt{k_1^2+k_2^2}\cdot a}}{\lambda^2\sqrt{k_1^2+k_2^2}\cdot\left(\sqrt{k_1^2+k_2^2+\frac{1}{\lambda^2}}+\sqrt{k_1^2+k_2^2}\right)^2} \qquad (10)$$

The x component of the field, generated by m_1 is

$$H_x(k_1,k_2,z) = \frac{1}{2}sign(z)ik_1e^{-\sqrt{k_1^2+k_2^2}\,|z|}\cdot m_1. \qquad (11)$$

The flux trough the pick-up coil is

$$\Phi = \int_b^{2b} dz \int_{-b/2}^{b/2} dy H_x(x=0,y,z),$$

To simplify, we make the integration by y from -∞ to +∞ and use the feature of the Fourier transform $f(x=0)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(k)dk$. In this case, we obtain

$$\Phi_1 = \frac{m_0}{4\pi}\int_{-\infty}^{\infty}(e^{-2b|k_1|}-e^{-b|k_1|})\frac{e^{-|k_1|a}}{\left(\sqrt{(\lambda k_1)^2+1}+|\lambda k_1|\right)^2}dk_1. \qquad (12)$$

Assuming that $\lambda \ll a \ll b$ and adding to Φ_1 the flux Φ_0 from the external field, we obtain Φ_{tot}

$$\Phi_{tot} = -\frac{m_0}{2\pi}\cdot\frac{1}{b}\left(1-\frac{3}{2}\frac{\lambda}{b}\right). \qquad (13)$$

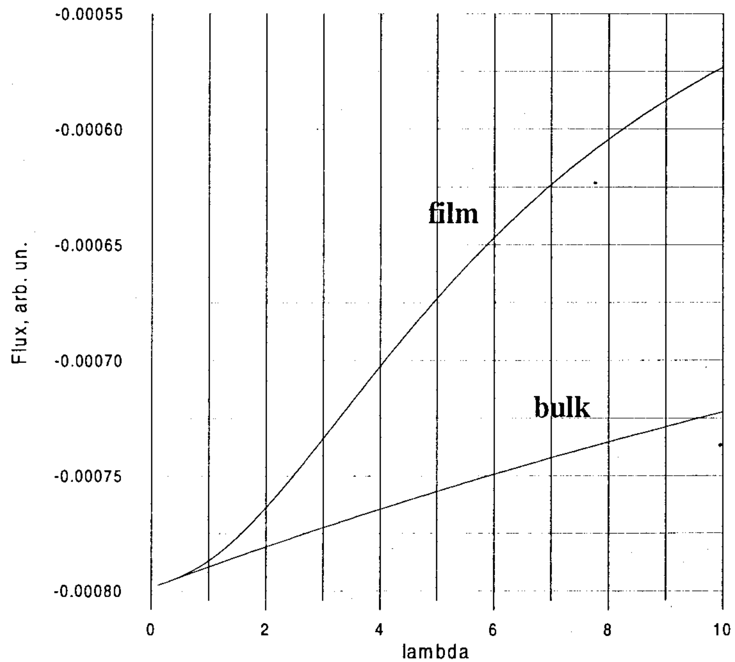

Figure 2. The dependence of the magnetic flux Φ_{tot} through the pick-up loop for the bulk sample and the thin film. The calculation was performed using formula (12) and analogues formula for the thing film. Lambda is shown in units λ/d, a/d = 10, and b/d = 100.

Therefore, the flux measured by the pick-up coil has a linear dependence on λ. The standard sizes for coils of a SQUID microscope are $a = 1\mu m, b = 10\mu m$. These sizes give a considerable change of the microscope signal with the change of the λ on the level of few hundreds of angstroms. The exact dependence of Φ_{tot} from the λ is shown in Figure 2.

3.2. The case of thin film

Let us consider the case of a thin superconducting film with boarders z=0 and z=-d, where d is the thickness of the film (see Figure 1b). We use the same logic as in equation (9), but it is necessary to write this equation for two surfaces with moment distributions m_1 (z = 0) and m_2 (z = - d). Moreover we chose m_1 and m_2 in such a way that m_1 completely determines the field above the film a m_2 determines the field below.

For m_1 we obtain

$$m_1 = -\frac{m_0 i k_1}{\sqrt{k_1^2+k_2^2}} e^{-\sqrt{k_1^2+k_2^2}a} \frac{\left[\left(1-\frac{1}{B}\right)\left(e^{\sqrt{k_1^2+k_2^2+\frac{1}{\lambda^2}}d} - e^{-\sqrt{k_1^2+k_2^2}d}\right) + \frac{1}{B'}\left(e^{-\sqrt{k_1^2+k_2^2+\frac{1}{\lambda^2}}d} - e^{-\sqrt{k_1^2+k_2^2}d}\right)\right]}{\left[\left(1-\frac{1}{B}\right)(B-1)e^{\sqrt{k_1^2+k_2^2+\frac{1}{\lambda^2}}d} - \frac{1}{B}e^{-\sqrt{k_1^1+k_2^2+\frac{1}{\lambda^2}}d}\right]} \qquad (14)$$

where

$$B = 2\lambda^2\sqrt{k_1^2+k_2^2+\frac{1}{\lambda^2}}\left(\sqrt{k_1^2+k_2^2}+\sqrt{k_1^2+k_2^2+\frac{1}{\lambda^2}}\right)$$

$$B' = 2\lambda^2\sqrt{k_1^2+k_2^2+\frac{1}{\lambda^2}}\left(\sqrt{k_1^2+k_2^2}-\sqrt{k_1^2+k_2^2+\frac{1}{\lambda^2}}\right)$$

Performing the same procedures as above we obtain Φ_{tot} for the thing film.

$$\Phi_{tot} = -\frac{m_0}{2\pi}\cdot\frac{1}{b}\left(1-\frac{3}{2}\cdot\frac{\lambda}{b}cth\frac{d}{\lambda}\right), \qquad (15)$$

if $cth(d/\lambda) << b/\lambda$. The exact dependence of Φ_{tot} on λ, when λ/d changes from 0.1 to 10, is shown in Figure 2. It is clearly seen that the dependence $\Phi_{tot}(\lambda)$ for the film is even stronger than for the bulk sample.

Let us evaluate the limit of the λ resolution for such a SQUID microscope using (13).

$$\lambda_{min} = \frac{4\pi b^2}{3m_0}\Phi_{min}.$$

Assuming $b = 10\mu m, m_0 = 10^{-4}cm\cdot 10^{-4}cm\cdot 10^{-3}A, \Phi_{min} = 10^{-6}\varphi_0 \approx 2\cdot 10^{-13}G\cdot cm^2$, we obtain λ_{min} ~ 1 nm. Hence, for SQUID microscope this problem is absolutely real. Moreover, the device proposed gives a possibility to measure a local penetration depth, because the main screening currents are induced on a surface of a superconductor in a spot of a size a.

4. Conclusions

In this paper, we developed the method for solution of the problem of a superconductor in an external magnetic field. Using this method, we considered the behaviour of the thin film and bulk superconductors in the external magnetic field. The scheme for measurement of the local London penetration depth using a SQUID microscope of the appropriate construction was proposed. It was shown that the signal of such a microscope is linear with the change of λ and the resolution of 1 nm in λ measurement may be attained. The device proposed may be extremely powerful in the investigations of the parameters of superconductors and the mechanism of superconductivity.

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Измерение лондоновской локальной глубины проникновения при помощи СКВИД-микроскопа

Предложен метод решения задачи о сверхпроводнике во внешнем магнитном поле. При помощи этого метода рассмотрено поведение тонкой сверхпроводящей пленки и массивного образца во внешнем магнитном поле. Предложена схема измерения лондоновской локальной глубины проникновения с использованием СКВИД-микроскопа специальной конструкции. Показано, что сигнал микроскопа линейно зависит от лондоновской глубины проникновения λ, и при ее измерении может быть достигнуто разрешение в 1 нанометр.

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Obukhov Yu.V. E13-98-208

The London Penetration Depth Measurement Using SQUID Microscope

A method for solution of the problem of superconductor in an external magnetic field is developed. Using this method, the behaviour of the thin film and bulk superconductors in an external magnetic field is considered. A scheme for measurement of the local London penetration depth using a SQUID microscope of the appropriate construction is proposed. It is shown that the signal of such microscope is linear with the change of the London depth λ and it is possible to attain a resolution of 1 nm at λ measurement.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

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