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LONGITUDINAL INTERFERENCE OF THE DIFFRACTION FREE WAVE FIELDS. I. THEORY

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1. INTRODUCTION

It is well known that wave field of the wave length λ localized in the restricted volume of the lateral dimension r will diffract in such a way that at distances $z \gg r^2/\lambda$ the wave field will have the form of the divergent spherical wave with the beam width angle of the order of λ/r. Meanwhile it has been shown in [1] that there exists an axial-symmetric wave field in the free space that meets the wave equation

$$(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}) E(\vec{r}, t) = 0, \tag{1}$$

where $\vec{r} = \vec{r}(x, y, z)$ is the observation vector, and that does not change along the z-axis. At any $z > 0$ this wave field is of the same form as at $z = 0$. As was shown in [1] the wave field

$$E(\vec{r}, t) = \exp(i\beta z)\exp(-i\omega t)\, J_0(\alpha\rho), \tag{2}$$

does possess this property, where $\rho^2 = x^2 + y^2$, and $J_0(\bullet)$ is the Bessel function of zero order of the first kind. The special limited case of this class diffracted free wave fields is the plane wave into which the wave field (2) goes at $\alpha \to 0$.

The diffraction free wave fields postulated in [1] were named in [2] as "Bessel-form beams". It was noted that restricted Bessel- form beams can be produced in lasers with ring mirrors and with output beams in the form of the ring.

The production of the Bessel-form beams in the system with narrow ring slit was treated in [2,3]. In [4] it was proved that z-projection of the wave vector of the monochromatic diffraction wave field must be constant for all partial components, as otherwise the phase relations between these components will be changed along z-axis thus introducing the spatial variations of the whole wave field. It was pointed out in [4] that the axial nonsymmetruc wave fields of the form

$$E(\rho, \phi) = \cos m\phi \cdot J_m(\alpha\rho). \tag{3}$$

with Bessel function $J_m(\bullet)$ of the m-order form indeed a new class of the diffraction free wave fields. The wave field of the form (2) is a conical one, while the wave field of the form (3) has the form of the spiral (winding) stair with m-threads.

The properties of the diffraction free wave fields were treated in detail many years ago by J.Dyson[5], who proved that the intensity of the light diffracted by the circular diffraction grating is of the form

$$I(\rho) = \frac{4\pi^2 \ell_1 \ell_2 \lambda}{a^2(\ell_1 + \ell_2)^3} J_0^2(\alpha\rho), \tag{4}$$

where ℓ_1 is the distance from the observation point to the circular diffraction grating, ℓ_2 is the distance from the circular diffraction grating to the screen with observation point $P(\rho, \ell_2, \phi)$ and

$$\alpha = \frac{2\pi\ell_1}{a(\ell_1 + \ell_2)} \equiv \frac{1}{\rho_0^{CDG}}, \tag{5}$$

with "a" being the constant pitch of the circular diffraction grating.

An Archimedian spiral grating of constant pitch "a" was treated in[5] as well. The radial distribution of the light intensity in the diffraction pattern is described by the equation

$$I(\rho) = \frac{4\pi^2 \lambda \ell_1 \ell_2}{a^2(\ell_1 + \ell_2)^3} J_1(\alpha\rho), \tag{6}$$

with α given in (5).

The annular aperture of small width which creates diffraction free wave fields with J_0^2 - dependence along the radial coordinate has been treated by G.C. Steward[6] in 1928. The concise review of the phenomena with conical wave fronts was given in[7]. We see that diffraction free wave fields are indeed very old ones, and only J.Durbin in 1987 has introduced this very useful term. As diffracted free wave fields can be produced by conical and stair-like wave fronts we may consider this class of the wave fields as the mesooptical one[7].

In this paper, the new phenomenon, "the longitudinal interference of the diffraction free wave fields", is explained. The experimental arrangements proposed for observation of such phenomenon are presented: a) the system with two coaxial circular diffraction gratings, b) the system with one circular diffraction grating which produces at least two diffraction orders, c) the system with two narrow concentric transmitting rings. The evolution of the longitudinal interference of the light in the volume between the generator and the detector is analyzed. Finally the technique for suppression of the longitudinal modulation of the light intensity on the optical axis of the system is suggested.

2. CONICAL WAVE FRONTS

To produce the diffraction free wave field of the form $J_0(\alpha\rho)$ (4), it is sufficient to use the axial symmetrical conical wave front which in the vicinity of the optical axis produces the light intensity distribution

$$I(\rho) = |J\ (\alpha\rho)|^2. \tag{7}$$

For this aim we may use the circular diffraction grating or kinoform axicon[8].

The main difference between spherical and conical wavefields consists in the structure of the crossover on the optical axis of the system. In Fig.1 the crossover for the collimated beam (a) and for the spherical lense (b) are shown. The effective length of the crossover is equal to

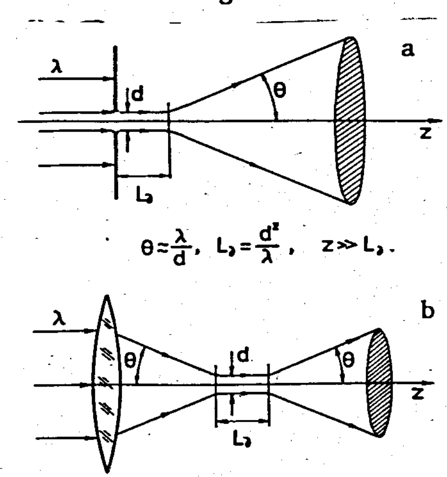

Fig.1. The structure of the crossover in the case of the spherical wave front.

$$L = \frac{d^2}{\lambda}, \tag{8}$$

where d is the diameter of the crossover and λ is the wave length. The distribution of the light intensity $L(\rho)$ has the form[9,10]

$$I(\rho) = \left| \frac{J_1(\alpha\rho)}{\alpha\rho} \right|^2. \tag{9}$$

In the case of the conical wavefront the distribution of the light intensity is described by the eq.(4). The length of the crossover L is equal to

$$L = \frac{D}{2} \sin\theta, \tag{10}$$

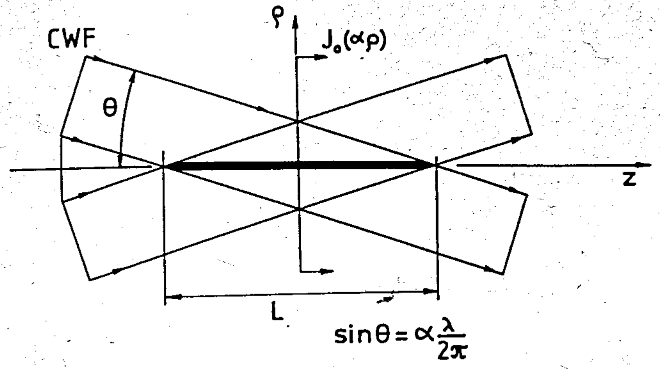

Fig.2. The structure of the crossover in the case of the conical wave front.

where D is the external diameter of the conical wavefront in the region of the crossover (Fig.2).

The diffraction free wave field with helix structure

$$E(\vec{r},t) = \exp(i\beta z)\exp(-i\omega t)\exp\left(i\frac{\phi}{2\pi}\right) J_1(a\rho) \qquad (11)$$

can be produced by Archimedian spiral grating with one thread (pass), the structure of which is shown in Fig.3. The light intensity on the optical axis is zero on the whole length L, defined by eq.(10), and the diameter of the first bright ring is equal to $1/a$ in eq.(5).

The radial distribution of the light intensity of the form

$$E(\vec{r},t) = \exp(i\beta z)\exp(-i\omega t)\exp\left(i\frac{\phi}{\pi}\right) J_2(a\rho) \qquad (12)$$

can be produced by the Archimedian spiral grating with 2 threads (passes).

In the experiments [8, 11, 12] the diffraction free wave field was observed over the length L = 30 m with diameter of the crossover of the order of 100 μm for the wave length $\lambda = 0.63$ μm. This "laser string" can be used for the metrological purposes with spatial resolution of 30 μm over the length L = = 50 m with kinoform axicon of the diameter 100 mm.

The longitudinal distribution of the light intensity over the focal segment produced by axicon was analysed in [13]. The profile of the conoid axicon in the form proposed in [14] gives rise to the constant light intensity along the crossover. The application of the axicon for generation of very long (~ 1 m)

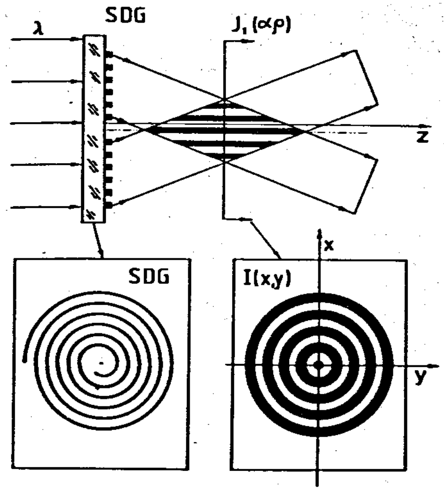

Fig.3. The properties of the spiral diffraction grating: top - the meridional cross section of the interference fringes, bottom to the left - the view of the spiral diffraction grating, bottom to the right - the picture of the interference fringes in one of the cross sections.

discharge in gas is described in [15, 16]. The conical wave fronts produced by axicon were used for radial and axial pumping of X-ray lasers [17]. The complex polarization effects in the reflective axicons are treated in [18, 19].

3. LONGITUDINAL INTERFERENCE OF TWO COAXIAL CONICAL WAVE FRONTS

Let us consider two coaxial conical wave fronts which can be described by two wave vectors $\vec{k}_1$ and $\vec{k}_2$ with $|\vec{k}_1| = |\vec{k}_2|$ (Fig.4). In the meridional cross section of these two wave fronts in the region of the mutual superposition of these two wave fronts in the region of the mutual superposition of these two wave fields we observe the interference fringes oriented along the bisectrix of the angle made by two wave vectors $\vec{k}_1$ and $\vec{k}_2$. The period of these fringes is equal to

$$a = \frac{\lambda}{\sin(\theta_2 - \theta_1)}. \qquad (13)$$

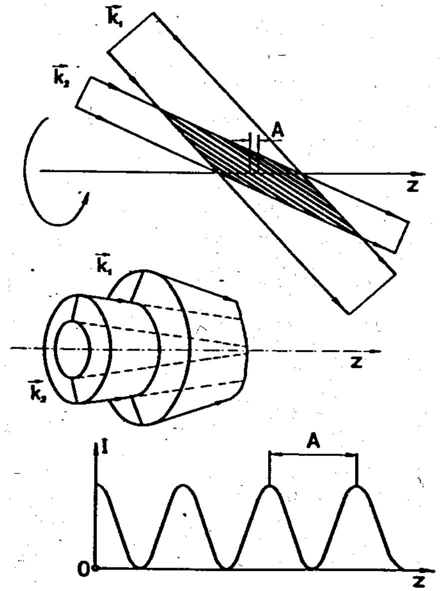

Fig.4. Two coaxial conical wavefronts, the structure of the interference fringes and the longitudinal distribution of the light intensity on the optical axis of the system: $\vec{k}_1$ and $\vec{k}_2$ are two wave vectors in the given meridional cross section, A - period of the light intensity modulation.

where θ_2 and θ_1 are the angles between the wave vectors $\vec{k}_2$ and $\vec{k}_1$ with optical axis of the system, respectively. The orientation angle θ_0 of the interference fringes with optical axis of the system z is equal to

$$\theta_0 = \frac{1}{2}(\theta_1 + \theta_2). \tag{14}$$

In the real system each point of this meridional cross section corresponds to a circle. Only the points lying on the optical axis of the system are fixed. Thus we can observe the longitudinal interference of two coaxial conical wavefronts. The period A of this light intensity modulation on the optical axis shown in the bottom of the Fig.4 is equal to

$$A = \frac{a}{\sin\left(\frac{\theta_2 + \theta_1}{2}\right)} = \frac{\lambda}{\sin(\theta_2 - \theta_1)\cdot\sin\left(\frac{\theta_2 + \theta_1}{2}\right)}. \tag{15}$$

The most straightforward experimental arrangement for observation of this longitudinal interference of two coaxial conical wavefronts is the system with two coaxial circular diffraction gratings G_1 and G_2 shown in Fig.5. The working parts of these two coaxial circular diffraction gratings are chosen such that the region of the mutual superposition of two fields is restricted near the optical axis of this system.

We can use only one circular diffraction grating G (Fig.6) if the later produces at least two diffraction orders. To match the intensities of the corresponding diffraction orders an attenuator in the form of a rotating polarizer P is inserted before the circular diffraction grating G. The external diameter of this rotating polarizer P must be smaller than the diameter of the external working part of the grating G. In such a configuration we have $\theta_2 = 2\theta_1$ and the period A_1 of the longitudinal interference is equal to

$$A_1 = \frac{\lambda}{\sin\theta_1 \cdot \sin\frac{3}{2}\theta_1}. \tag{16}$$

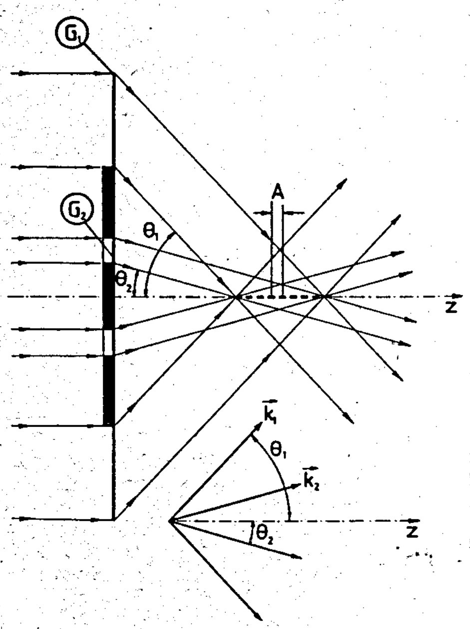

Fig.5. The experimental arrangement with two coaxial circular diffraction gratings and the wave vectors of the diffracted components: θ_1 and θ_2 are the orientation angles of the diffracted light into the plus first diffraction orders of two coaxial circular diffraction grating.

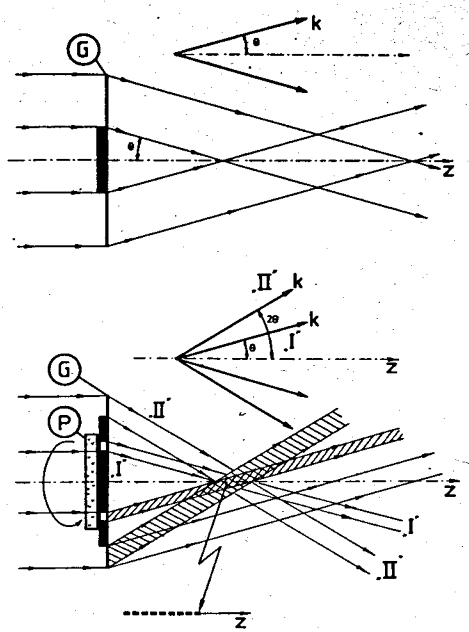

Fig.6. The experimental arrangement with one circular diffraction grating G in which the interference fringes are produced by different diffraction orders of the same circular diffraction grating: P - the rotating polarizer to match the intensities of light in two different orders.

The light intensity modulation is observed in the region of mutual superposition of two different diffraction orders.

The most simple experimental arrangement for observation of the longitudinal interference consists of two coaxial conical wave fronts produced by two coaxial narrow transmitting rings on the screen. The components of the transmitted light go initially separately and then these components cross the optical axis. In this region we see the longitudinal interference of the light.

4. HOW TO SUPPRESS THE LONGITUDINAL INTERFERENCE OF TWO CONICAL WAVE FRONTS

Now let us consider the technique of suppressing of the longitudinal interference of two coaxial conical wave fronts in the case when this phenomenon would be undesirable. For this purpose we may use the Hilbert phase jump in the form of the ring on the optical plate [20]. The construction of this filter is shown in Fig.7. It consists of two halves with 180° azimutal

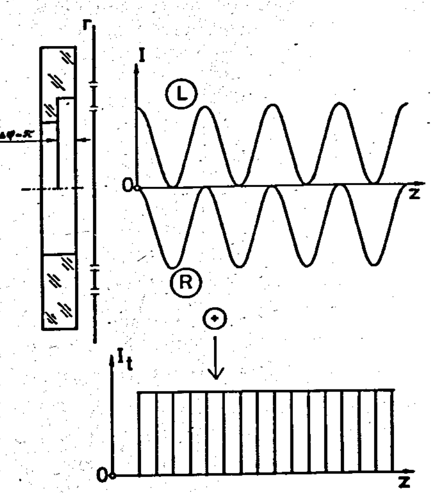

Fig.7. The formation of the longitudinal interferences in two halves of the filter in the system with two coaxial narrow transmitting rings on the screen and the sum of these pictures on the optical axis of the system.

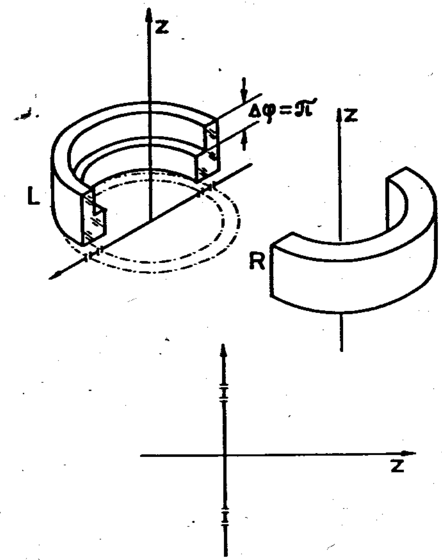

Fig.8. The view of two halves of the ring phase jump filter with $\Delta\phi = 180^\circ$ (see text).

angular interval. One half of this filter shown in Fig.7 is a glass plate. The second half of this filter shown in the top of Fig.7 is a ring phase jump with phase difference $\Delta\phi = 180^\circ$ for given wave length λ. The radius of this ring with phase jump is just intermediate value of the radii of two coaxial narrow rings. Under the influence of this filter the interference picture produced by the upper part (L) of this filter is in antiphase with respect to the interference picture produced by the bottom part (R) of this filter. The sum of these two mutually antiphase interference pictures is the constant function of z-coordinate. As this cancellation is taking place in each meridional cross section we got the complete smoothing of the light intensity modulation on the optical axis of the system. The constructions of two halves of the ring phase jump filter with $\Delta\phi = 180^\circ$ are shown in Fig.8.

5. LONGITUDINAL INTERFERENCE OF MANY COAXIAL CONICAL WAVE FRONTS

It is easy to prove that the function a(z) which describes the amplitude of the longitudinal interference of many coaxial conical wave fronts with light intensity

$$I(z) = |a(z)|^2 \qquad (17)$$

is equal to the Fourier Transform of the angular field $g(\theta)$ with many wave vectors $\vec{k}_i$ $(i = 1, 2, 3, \ldots)$ which on being superposed give rise to the interference picture in one meridional half-plane.

For the case of two wave vectors $\vec{k}_1$ and $\vec{k}_2$ we have

$$g(\theta) = \delta(\theta - \theta_1) + \delta(\theta - \theta_2) \qquad (18)$$

and

$$a(z) = \cos \gamma \frac{z}{\lambda}, \qquad (19)$$

where

$$\gamma = \sin(\theta_2 - \theta_1). \qquad (20)$$

For the case when the number of the conical components is high and the corresponding wave vectors represent the equidistant set along the axis $\sin\theta_i$ the periodic structure of the longitudinal interference on the optical axis will be periodic as well as the angular spectra of light diffracted on the periodic grating with noncosinusoidal form of groves and with small number of groves.

In the case when the nanber of the conical wavefronts is very high and the angular distribution of the wave vectors is irregular, we have

$$a(z) \to \frac{\sin \frac{\mu z}{\lambda}}{\left(\frac{\mu z}{\lambda}\right)}. \qquad (21)$$

where $\mu = \sin\theta_{AV}$, $\theta_{AV} = 1/2\,\theta_{max}$, and θ_{max} is the angle of the cone which is filled by the irregular oriented wave vectors of conical components. The eq.(21) is just equal to the distribution of the optical field along the longitudinal coordinate in the vicinity of the crossover produced by the traditional lens /21/.

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