объединенный
институт
ядерных
исследований
дубна

E13-89-61

P.Kozma

APPLICATION OF THE MONTE CARLO METHOD TO DETERMINE THE THICKNESS OF A ΔE-DETECTOR

Submitted to "NIM"

1989

The thickness of silicon ΔE-detectors can be measured with a calibrated gamma-ray source using Compton conversion /1/ for which the cross section is known. The spectrum produced by a ^{137}Cs gamma-ray source /662 keV/ in the Si ΔE-detector with a radius of 16 mm and a thickness of about 500 μm is displayed in Fig.1. The dotted curve is the experimental Compton shoulder whose edge appears at 478 keV; the full line represents the Klein-Nishina theoretical shape of the Compton profile. It is seen that the simple Klein-Nishina distribution does not adequately fit the measured Compton continuum region in the gamma-ray spectrum for an actual experimental arrangement. It should be noted that the theoretical distribution does not contain contributions from multiple scattered photons in a finite detector. However, a correction for Compton electrons escaping through the edge of the detector must be introduced. Moreover, an attentive approximation of the missing part of the spectrum due to the threshold effects of electronics is also needed. For these reasons, we have tried to fit the experimental energy spectra of Compton scattered electrons by the function calculated using the Monte Carlo method. We have used the Monte Carlo model of scattering and absorption events of a gamma-ray in a finite homogeneous medium as described in Ref./2/.

The geometry of a real experimental arrangement considered in the calculation is illustrated in Fig.2. A gamma-ray history starts by selecting a random initial direction θ_0, φ_0 from an isotropic source N_S, and the flight distance ℓ_0. Then Cartesian coordinates of the entry point $N_0(x_0, y_0, 0)$ and the point of the first interaction are computed. The type of interaction is determined from the comparison of the ratio of photoelectric and total absorption coefficients for initial energy E_0 to a generated random number. If an event of the photoelectric origin is produced, a given gamma-ray history is terminated. For a Compton scattering event, the Klein-Nishina distribution is sampled to determine the angle of the scattered gamma-ray and the history is continued. After the evaluation of the scattering angle, the values of photon and electron energies, as well as the polar angle of the scattered electron and its path length, can be calculated /see below/. In general, the state of the gamma-ray after n-th scattering, $S_n=(E_n,\Theta_n,\varphi_n,x_n,y_n,z_n)$, is specified by a set of quantities: energy E, coordinates Θ,φ describing the direction of motion in a spherical coordinate system with the z-axis as a polar one /see Fig.3/, and Cartesian coordinates x,y,z; $S_0=(E_0,\Theta_0,\varphi_0,x_0,y_0,z_0=0)$ being the state of a photon entering the detector. The steps in the calculation of S_{n+1} quantities, given S_n, are:

1. Energy

$$E_{n+1} = \frac{E_n}{1 + E_n/m_oc^2(1-\cos\omega_{n+1})}, \qquad /1/$$

where m_oc^2 = 511 keV and the photon scattering angle ω_{n+1} is sampled from the theoretical distribution.

2. Cartesian coordinates

$$\begin{aligned} x_{n+1} &= x_n + \ell_n \sin\Theta_n \cos\varphi_n \\ y_{n+1} &= y_n + \ell_n \sin\Theta_n \sin\varphi_n \\ z_{n+1} &= z_n + \ell_n \cos\Theta_n \end{aligned} \qquad /2/$$

with flight distance ℓ_n determined from the inverse probability distribution

$$\ell_n = \frac{1}{\mu(E_n)} \ln \frac{1}{1-Q} \quad ; \qquad /3/$$

$\mu(E_n)$ and Q are the total attenuation coefficients /3/for a gamma-ray of energy E_n and a random number, respectively.

3. Angular coordinates

$$\Theta_{n+1} = \arccos\left[\cos\Theta_n \cos\omega_{n+1} + \sin\Theta_n \sin\chi_{n+1}\right] \qquad /4/$$

$$\varphi_{n+1} = \varphi_n + \arcsin\left[\sin\chi_{n+1} \sin\omega_{n+1}/\sin\Theta_{n+1}\right]$$

where χ_{n+1} is the randomly selected azimuthal scattering angle /see Fig.3/ distributed uniformly between 0 and 2π.

The state of a Compton electron after /n+1/-th scattering is specified by its energy E^{el}_{n+1} and a polar angle ω^{el}_{n+1} with respect to the direction of motion of the gamma-ray. These quantities are given as a function of the energy E^{γ}_{n+1} of the gamma-ray and the scattering angle ω^{γ}_{n+1} of the photon after /n+1/-th scattering:

$$E^{el}_{n+1} = \frac{aE^{\gamma}_{n+1}(1-\cos\omega^{\gamma}_{n+1})}{1+a(1-\cos\omega^{\gamma}_{n+1})} \qquad /5/$$

$$\omega^{el}_{n+1} = \arctan\left[\frac{1}{(1+a)\tan(\omega^{\gamma}_{n+1}/2)}\right], \qquad /6/$$

where $a = E^{\gamma}_{n+1}/m_oc^2$. The path length of a scattered electron is obtained from the range, R_{el}, of the electron with energy E_{el} /in MeV/ in silicon /4/:

$$R_{el} = 0.1768E_{el}^{1.265-0.0954E_{el}} \quad /cm/ . \qquad /7/$$

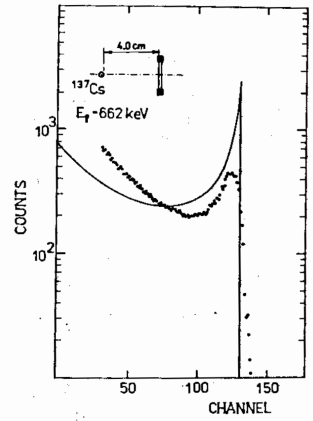

Fig.1. Spectrum produced by the ^{137}Cs gamma-ray source in the silicon ΔE-detector. The full line is the Klein-Nishina distribution.

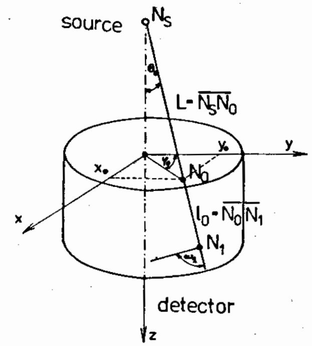

Fig.2. Geometry considered in the calculation.

In the case of the path length exceeding the dimension of a cylindrical detector in a given direction, the position of electron escape is calculated, and only an appropriate part of its energy is registered.

As a result of the procedure described here, the energy spectrum of Compton scattered electrons corrected for the escape through the edge of the detector is obtained. It should be noted that the results of the calculation are only of a random character. A limit on the accuracy of any set of Monte Carlo results is a statistical uncertainty which can be reduced by increasing the number of histories.

Fig.4 shows the experimental spectrum fitted with the Monte Carlo model calculations for the ^{137}Cs gamma-ray source and the real experimental arrangement. As can be seen, the calculated curve reproduces the shape of the measured Compton profile in a silicon ΔE-detector about 500 μm in thickness better than the Klein-Nishina theoretical distribution. It is evident that the corrected calculated curve can also produce a good approximation of the missing part of the experimental spectrum below the electronic threshold. One can conclude that the calculations, employing the simple Monte Carlo model of gamma-ray interaction in a silicon ΔE-detector, provide a very reliable spectrum of scattered electrons. The least-squares fit of this spectrum to an experimental Compton continuum, measured with a calibrated gamma-ray source, enables one to determine the thickness of a Si E-detector with a good accuracy.

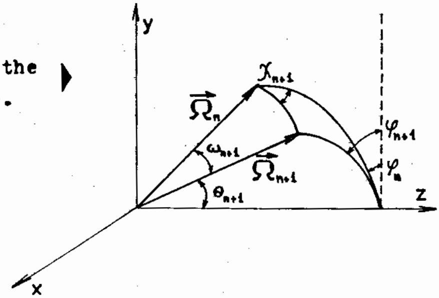

Fig.3. Angular representation in the spherical coordinate frame.

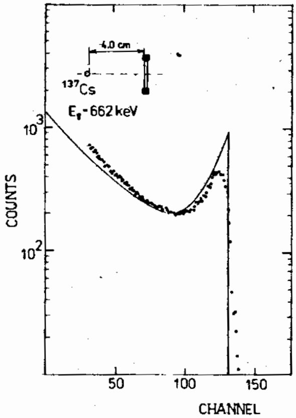

Fig.4. Resultant fit of the experimental spectrum by the Monte Carlo model curve.

The author is grateful to Prof.V.A.Nikitin for his interest in this work and a critical reading of the manuscript.

References

1. R.G.Miller and R.W.Kavanagh, Nucl.Instr.and Meth. 48 /1967/ 13.
2. P.Kozma, P.Bém and J.Vincour, Czech.J.Phys. B 30 /1980/ 503.
3. WM.J.Veigele, Atomic Data Tables 5 /1973/ 51.
4. R.P.Gardner, A.M.Yacout, J.Zhang and K.Verghese, Nucl.Instr.and Meth. A 242 /1986/ 399.

Received by Publishing Department
on January 31, 1989.