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DIRAC DELTA-PLUS (OR MINUS) FUNCTION IN OPTICS AND MESOOPTICS

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1. Delta function as a new mathematical concept has been introduced by the physicist P.A.Dirac for treatment of the quantum mechanical quantities involving infinities of different kinds. Subsequently, Dirac delta function $\delta(x)$ was used any time when the final result is determined by the convolution of the impulse function with some continuous function.

The Dirac delta-plus (or minus) function $\delta_+(x)$ intended for investigation of the causal functions and of the dispersion relations enters naturally into Hilbert optics and mesooptics. In this branch of the imaging optics the experimentator makes use of the Dirac delta-plus function $\delta_+(x)$ in his day-to-day work.

The review article begins with the introduction of the backgrounds and of the underlying relations. Then the properties of the coherent imaging system containing various spatial frequency filters are treated. They include the knife-edge filter, the phase-edge filter and the diffraction grating with phase jump. The analysis of the mesooptical imaging system used for observation of the straight line particle tracks in nuclear research emulsion is presented. It is shown that in a general case the convolution kernel of the mesooptical system is a superposition of the form $a_1\delta_+(x) + a_2(d/dx)\delta_+(x)$.

2. The Dirac delta function $\delta(x)$ is a generalized function or a distribution, and we must at first consider the properties of the distributions. For this purpose we introduce two key concepts: the scalar product of two functions $f(x)$ and $g(x)$ /2, 3/

$$\langle f(x), g(x) \rangle \equiv \int f(x) g(x) dx, \tag{1}$$

and the convolution of two functions $f_1(x)$ and $f_2(x)$:

$$f(x) = f_1(x) \otimes f_2(x) \equiv \int f_1(\xi) f_2(x - \xi) d\xi . \tag{2}$$

The scalar product, Eq. (1), is a functional, that is a process of assigning some number $\langle f, g \rangle$ to the function $f(x)$, the function $g(x)$ being given. For example, the delta function $\delta(x)$ is fully defined by the relation:

$$\langle \delta(x), g(x) \rangle = \int \delta(x) g(x) dx = g(0) . \tag{3}$$

The convolution of two functions $f_1(x)$ and $f_2(x)$ is a new function $f(x)$, defined by Eq. (2). If $g(x)$ is a "good" function which vanishes at infinity, then we can give the rule for finding the derivative of the generalized function $f(x)$ /3/ :

$$< \frac{d f(x)}{d x}, g(x) > = < f(x), - \frac{d g(x)}{d x} >, \quad (4)$$

and

$$< \frac{d^2 f(x)}{d x^2}, g(x) > = < f(x), + \frac{d^2 g(x)}{d x^2} >. \quad (5)$$

Let $Y(x)$ be the unit step function or the Heaviside function defined as

$$Y(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0. \end{cases} \quad (6)$$

With Eq. (4) we may find the derivative of the unit step function $\frac{dY(x)}{dx}$. We have /3/

$$< \frac{dY(x)}{dx}, g(x) > = < Y(x), - \frac{dg(x)}{dx} > =$$

$$= - \int_0^\infty \frac{dg(x)}{dx} dx = - g(x) \Big|_0^\infty = g(0) = <\delta(x), g(x) >, \quad (7)$$

or

$$\frac{dY(x)}{dx} = \delta(x). \quad (8)$$

To calculate the Fourier transform of the generalized function $f(x)$ we must use a Parseval equation which can be written in the form of scalar products:

$$< f(x), g^*(x) > = \frac{1}{2\pi} < \hat{\mathfrak{F}}[f(x)], \hat{\mathfrak{F}}^*[g(x)] >, \quad (9)$$

where $\hat{\mathfrak{F}}$ denotes both the Fourier transformation and the Fourier transform of the function presented in brackets.

Let us find the Fourier transform of the generalized function $\delta(x)$ defined in Eq. (3). According to Eq. (9), we have

$$<\hat{\mathfrak{F}}[\delta(x)], G(\omega)> = 2\pi < \delta(x), g(x) > = <\delta(x), \int_{-\infty}^{\infty} e^{i\omega x} G(\omega) d\omega > =$$

$$= \int_{-\infty}^{\infty} G(\omega) d\omega = < 1(\omega), G(\omega) >. \quad (10)$$

Thus

$$\hat{\mathfrak{F}}[\delta(x)] = 1(\omega), \quad (11)$$

where $1(\omega)$ is the unit constant function of ω. Similarly, we get

$$< \hat{\mathfrak{F}}[\delta(x - x_0)], G(\omega)> = 2\pi < \delta(x - x_0), g(x) > = < \delta(x - x_0),$$

$$\int_{-\infty}^{\infty} e^{i\omega x} G(\omega) d\omega > = \int_{-\infty}^{\infty} e^{i\omega x_0} G(\omega) d\omega = < e^{i\omega x_0}, G(\omega) >, \quad (12)$$

or

$$\hat{\mathfrak{F}}[\delta(x - x_0)] = e^{i\omega x_0}. \quad (13)$$

In a similar way we may prove that

$$\frac{dF(\omega)}{d\omega} = \hat{\mathfrak{F}}[(ix) \cdot f(x)], \quad (14)$$

$$(-i\omega) \cdot F(\omega) = \hat{\mathfrak{F}}[\frac{df(x)}{dx}]. \quad (15)$$

Two theorems concerning the convolution of two functions /1,3/ will be used in this article:

$$\text{I. } f_1(x) \otimes f_2(x) \xrightarrow{\hat{\mathfrak{F}}} \hat{\mathfrak{F}}[f_1(x)] \cdot \hat{\mathfrak{F}}[f_2(x)], \quad (16)$$

$$\text{II. } f_1(x) \cdot f_2(x) \xrightarrow{\hat{\mathfrak{F}}} \frac{1}{2\pi} \hat{\mathfrak{F}}[f_1(x)] \otimes \hat{\mathfrak{F}}[f_2(x)]. \quad (17)$$

3. Let the real-valued symmetrical function $f_o(x)$ be given in the interval $-\infty < x < \infty$. To transform this function into the causal one /1/, it is sufficient to multiply it by the unit step function, Eq. (6). To prove that the new function $f(x) = f_o(x) \cdot Y(x)$ is indeed a causal one, we use the theorem (17) which states that multication of two functions in the x-space is transformed into convolution of their Fourier transtorms in the ω-space. As $Y(x)$ is a distribution /2/, we cannot find its Fourier transform in a usual way. We must follow an indirect way instead. We know that the derivative of the unit step function $Y(x)$ is the Dirac delta function, Eq. (8), and that differentiation in the x-space turns into multiplication of the Fourier transform of the initial function by $(i\omega)$ in the ω-space, Eq. (15). Thus we have

$$\hat{\mathfrak{F}}[\frac{d}{dx} Y(x)] = i\omega \cdot \hat{\mathfrak{F}}[Y(x)]. \tag{18}$$

On the other hand

$$\hat{\mathfrak{F}}[\frac{d}{dx} Y(x)] = \hat{\mathfrak{F}}[\delta(x)] = 1(\omega). \tag{19}$$

We come to an equation

$$i\omega \cdot \hat{\mathfrak{F}}[Y(x)] = 1(\omega). \tag{20}$$

According to /2,3/

$$\omega \cdot \delta(\omega) = 0 \tag{21}$$

and the solution of Eq. (20) takes the form

$$\hat{\mathfrak{F}}[Y(x)] = \frac{1}{i\omega} + C_1 \delta(\omega), \tag{22}$$

where C_1 is the unknown constant. To find them, it is sufficient to take into account that

$$Y(x) + Y(-x) = 1(x), \tag{23}$$

and that the Fourier transform of the left-hand part of Eq. (23) is equal to

$$\hat{\mathfrak{F}}[1(x)] = 2\pi \cdot \delta(\omega). \tag{24}$$

Finally we get

$$\hat{\mathfrak{F}}[Y(x)] = 2\pi[\frac{1}{2}\delta(\omega) + \frac{1}{2\pi i\omega}] = 2\pi\delta_+(\omega), \tag{25}$$

where the function

$$\delta_+(\omega) = \frac{1}{2}\delta(\omega) + \frac{1}{2\pi i\omega}, \tag{26}$$

is the Dirac delta-plus function of ω. We have proved a well-known theorem: the Fourier transform of the unit step function is equal to the Dirac delta-plus function up to a constant factor /1/. It remains to prove that the function $f(x) = f_o(x) \cdot Y(x)$ is indeed a causal one. Let us rewrite the right-hand side of the equation

$$f_o(x) \cdot Y(x) \xrightarrow{\hat{\mathfrak{F}}} F_o(\omega) \otimes \hat{\mathfrak{F}}_\omega[Y(x)], \tag{27}$$

where

$$F_o(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f_o(x) e^{-i\omega x} dx, \tag{28}$$

is the Fourier transform of the function $f_o(x)$.

We have

$$F_o(\omega) \otimes \hat{\mathfrak{F}}_\omega[Y(x)] = F_o(\omega) \otimes 2\pi[\frac{1}{2}\delta(\omega) + \frac{1}{2\pi i\omega}] =$$

$$= \pi[F_o(\omega) + \frac{1}{\pi i} F_o(\omega) \otimes \frac{1}{\omega}] = \pi[F_o(\omega) + \frac{1}{\pi i}\oint \frac{F_o(\omega')}{\omega - \omega'} d\omega'], \tag{29}$$

where the integral taken in the sense of the Cauchy principal value /1,2/ is equal to the Hilbert transform of the function $F_o(\omega)$:

$$\frac{1}{\pi i}\oint \frac{F_o(\omega')}{\omega - \omega'} d\omega' = \hat{\mathfrak{H}}[F_o(\omega)]. \tag{30}$$

Eq. (30) is known as a Kramers-Kronig relation which expresses the dispersion relation: the real and the imaginary parts of the Fourier transform of the causal function are Hilbert transforms of each other up a sign.

The delta-plus function, Eq. (26), is commonly used in quantum electrodynamics [4,5], in spectroscopy and in radioengineering. In this article it will be shown how the properties of the Dirac delta-plus function presented here manifest themselves in imaging optics and mesooptics.

4. Let us consider a simple imaging system which enables one to accomplish the spatial frequency filtering of the image's field [6]. For this purpose it is sufficient to expose the object-transparency by the convergent light beam and to set an imaging lens in the spatial frequency plane (Fig. 1). The two-dimensional (2D) Fourier transform of the transparency function $f_0(x, y)$ is mapped at the spatial frequency plane (ω_x, ω_y):

$$F_0(\omega_x, \omega_y) = \frac{1}{(2\pi)^2} \int dx \int dy\, f_0(x, y)\, e^{-i\omega_x x} e^{-i\omega_y y}. \tag{31}$$

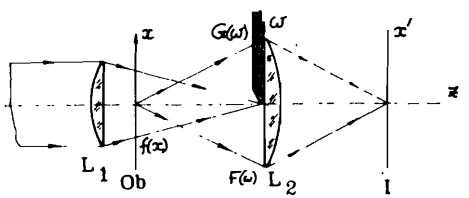


Fig. 1. Coherent optical imaging system containing the spatial frequency filter: L_1 *– lens which produces the convergent light beam,* Ob *– object-transparant,* G *– knife-edge filter,* L_2 *– imaging lens,* I *– image plane.*

Here the integral is extended over the field of view of the imaging system. To elucidate the essence of the filtering process, we may restrict ourselved to the one-dimensional (1D) transparency $f_0(x)$. Its 1D Fourier transform is equal to

$$F_0(\omega_x) = \frac{1}{2\pi} \int f_0(x)\, e^{-i\omega_x x} dx. \tag{32}$$

Let us insert a 1D knife-edge filter with the amplitude transparency [7]

$$G(\omega_x) = Y(\omega_x), \tag{33}$$

into the spatial frequency plane (ω_x, ω_y). Now the lens is only exposed by the positive spatial frequencies of the far field diffraction picture, $F(\omega_x)$. Behind the knife-edge filter with transparency (33) we have a causal function of the spatial frequency

$$F(\omega_x) = F_0(\omega_x) \cdot Y(\omega_x), \tag{34}$$

which after the inverse Fourier transformation [7] accomplished by the lens is going into the filtered image with the field amplitude

$$f_\Phi(x) = \frac{1}{2}[f_0(x) + i\chi(x)], \tag{35}$$

where $\chi(x)$ is the Hilbert transform of the function $f_0(x)$ [7]. As functions $f_0(x)$ and $i\chi(x)$ are in quadrature with each other, that is they are shifted in phase by 90°, the light intensity in the filtered image is the sum of its quadrates [7]

$$I_\Phi(x) = \frac{1}{4}[|f_0(x)|^2 + |\chi(x)|^2]. \tag{36}$$

When we need to observe only one function $\chi(x)$ without admixture of the input image field $f_0(x)$, the phase-edge filter with transparency [8]

$$G(\omega) = i\,\mathrm{sgn}\,\omega \equiv i \begin{cases} 1, & \omega > 0, \\ -1, & \omega < 0, \end{cases} \tag{37}$$

must be inserted into the spatial frequency plane. Here the sign function $\mathrm{sgn}\,\omega$ is related to the unit step function $Y(\omega)$ by the equation

$$\mathrm{sgn}\,\omega \equiv 2Y(\omega) - 1(\omega). \tag{38}$$

The inverse Fourier transform of the sign function $\mathrm{sgn}\,\omega$ only consists of an imaginary part of the Dirac delta-plus function multiplied by two:

$$\hat{\mathfrak{F}}[\mathrm{sgn}\,\omega] = \hat{\mathfrak{F}}[2Y(\omega) - 1(\omega)] = \{2\delta_+(x) - \delta(x)\} = \frac{1}{\pi i x}. \tag{39}$$

Unlike the quantum mechanics and the physical optics the coherent Hilbert optics enables one to construct a convolution kernel which differs in principle from $\delta(x)$ function as well as from $\delta_\pm(x)$ functions. To prove this, let us use the diffraction grating with phase jump [9] as a spatial frequency filter. The relevant diffraction grating has a distance "a′" between two neighbouring central grooves which is equal to $a' = a/2$, where "a" is the grating period, the phase jump line being on the line, defined by the equation $\omega_x = 0$.

The amplitude of the field appeared in the first diffraction order is described by the Hilbert transform $\chi(x)$ of the input field $f_0(x)$ [7]. However, if the distance between two neighbouring central grooves is equal to $a' \neq a/2$, the intensity of the signal in the first diffraction order is equal to

$$I(x) = |\cos\alpha \cdot \chi(x) + \sin\alpha \cdot f_0(x)|^2 , \tag{40}$$

where α is the angle which differs from the nominal value $\alpha_{nom} = 0$. In this case the convolution kernel is equal to

$$\delta_{HIL\,B}(x) = \sin\alpha \cdot \delta(x) + \cos\alpha \cdot \frac{1}{\pi x} \tag{41}$$

instead of a classical convolution kernel

$$\delta_+(x) = \frac{1}{2}\,\delta(x) + \frac{1}{2\pi i x} . \tag{42}$$

The inherent property of the convolution kernel, Eq. (41), is that its real and imaginary components are in phase instead of being in a quadrature. Due to this we may observe the direct interference between the field $f_0(x)$ and $\chi(x)$ in accordance with Eq. (40). This interference phenomenon was observed in the experiment [7].

5. Now let us treat the mesooptical counterpart of the imaging system shown in Fig. 1. We recall here that the imaging system is called a mesooptical one when the object point is mapped into a straight-line segment (the longitudinal mesooptics) or into the circle (the transversal mesooptics) [10]. To construct a mesooptical system with the transversal mesoopticity, it is sufficient to add a negative conical lens to the circular lens in Fig. 1 [11,12] (Fig. 2). Now the object point goes into a circle in the plane of the mesooptical images. As before, the object is exposed by the convergent light beam. Despite the exotic construction and seeming uselessness the mesooptics found application in high energy physics for observation of the straight line particle tracks in nuclear research emulsion [10,12-16]. This is due to the fact that the straight line segment is mapped by the mesooptical imaging system into itself and twice multiplexed. Two mesooptical images of the same straight line particle track are displayed in the plane perpendicular to the optical axis of the system. Positions of the mesooptical images are defined by the orientation angle of the particle track, by the distance from the centre of the field of view and by its z-coordinate [16]. The nature of transformations

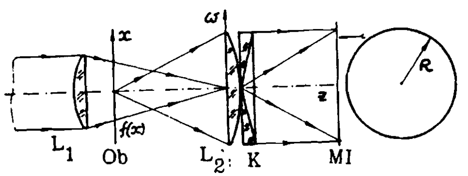
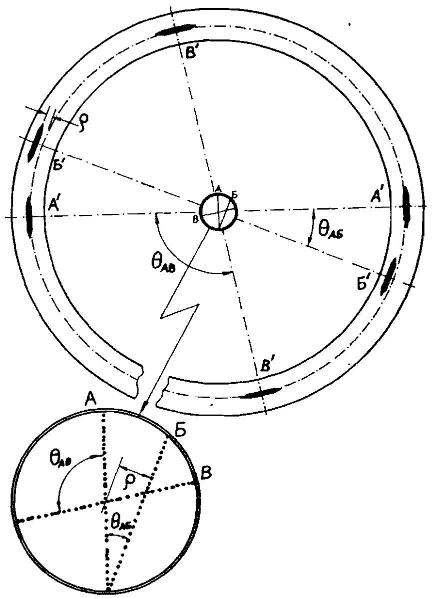


Fig. 2. Mesooptical imaging system with circular response: L_1 *– lens which produces the convergent light beam,* Ob *– object,* L_2 *– imaging lens,* K *– negative conical lens,* MI *– plane, where the mesooptical images of the straight line objects are displayed,* R *– radius of the focal circle.*

Fig. 3. Example of the transformations which the straight line particle tracks undergo in the mesooptical system with circular response (Fig. 2). In the centre is the field of view containing three-particle tracks: mesooptical images of the straight line particle tracks are shown on the focal ring of the width equal to the diameter of the field of view.



which the straight line particle tracks are subjected to in the mesooptical imaging system, can be explained with the help of Fig. 3. In the centre of Fig. 3 the optical images of three straight line particle tracks in the field of view of the system are shown. The corresponding three pairs of the mesooptical images of the straight line particle tracks are given on the periphery of Fig. 3 on the focal ring.

The transformations which the straight line particle tracks are subjected to along the radial coordinate in the meridional cross section of the mesooptical system are described directly by the Dirac δ_+-function. It is easy to see that the knife-edge filter with transmittance of Eq. (33) is virtually present in the mesooptical system. This can be explained by the fact that each mesooptical image of the straight line particle track is produced only by the positive (or negative) spatial frequencies and the virtual knife edge filter is always oriened parallel to the straight line particle track. In contrast to the Hilbert optics each straight line particle track has its own virtual knife-edge filter. Besides this, the Fourier transform of the straight line particle track produced by the convergent light beam and displayed in the region of the mesooptical element (Fig. 2) is additionally subjected to one operation, the multiplication by the function $i\omega$. To explain this, we must take into account the fact that the mesooptical image of each straight line particle track is focused from those parts of the mesooptical element which produce a narrow sector with the centre on the optical axis. Therefore the initial optical field $f_0(x)$ is subjected to the following chain of transformations in the mesooptical system:

$$f_0(x) \xrightarrow{\hat{\mathcal{F}}_x} F_0(\omega_x) \xrightarrow{G(\omega_x) = Y(\omega_x)} F_0(\omega_x) \cdot Y(\omega_x) \longrightarrow$$

$$\xrightarrow{i\omega} i\,\omega \cdot F_0(\omega_x) \cdot Y(\omega_x) \xrightarrow{\hat{\mathcal{F}}^{-1}}$$

$$\longrightarrow \frac{d}{dx}[\delta_+(x) \otimes f_0(x)] = \frac{d}{dx}[f_0(x) + i\chi(x)] = \tag{43}$$

$$= \frac{d}{dx} f_0(x) + i \frac{d}{dx}\chi(x).$$

The real and imaginary parts of the light field amplitude being differentiated. For example, if

$$f_0(x) = \Pi_A(x), \tag{44}$$

where $\Pi_A(x)$ is the rectangular pulse of width 2A /7/, then the output signal is described by the function

$$g(x) = \frac{d}{dx}\Pi_A(x) + \frac{d}{dx} i\hat{H}[\Pi_A(x)] = \frac{1}{2A}[\delta(x+A) - \delta(x-A)] +$$

$$+ \frac{i}{\pi}\frac{d}{dx}\ln\left|\frac{x+A}{x-A}\right| = \frac{1}{A}[\delta(x+A) - \delta(x-A)] - \tag{45}$$

$$- \frac{i}{\pi(x^2 - A^2)}.$$

This effect of differentiation in the mesooptics was observed experimentally /17/.

6. In some cases the straight line patricle tracks in nuclear research emulsion are practically parallel to each others within a small angular range (± 0°, 5), and the mesooptical element in the form of a torroid can be rep- laced by a cylindrical lens or a mirror (Fig. 4). Here differentiation with respect to the x coordinate is absent. As in the optical system with the knife-edge filter, the light field amplitude is equal to $f_0(x) + i\chi(x)$, Eq. (35).

It is instructive to note that the mesooptical system (Fig. 2) may contain more general mesooptical elements (Fig. 5). The difference between them is determined by location of the curvature centre of the mesooptical element. In the case of the traditional mesooptical system (Fig. 5a) the curvature centre of the mesooptical element coincides with the optical axis of the system. In the case of a cylindrical lens this centre is at infinity (Fig. 5c). For a general mesooptical element its curvature centre can be set at a distan-

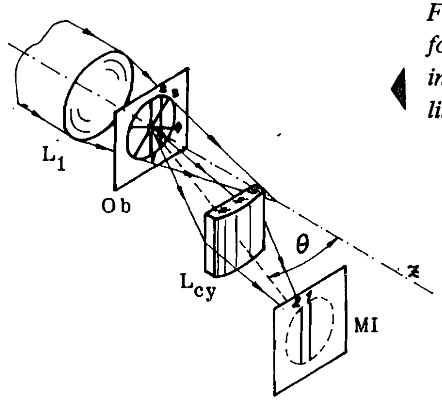


Fig. 4. Mesooptical imaging system constructed for observation of the collimated particle tracks in nuclear research emulsion has positive cylindrical lens L_{cy}.

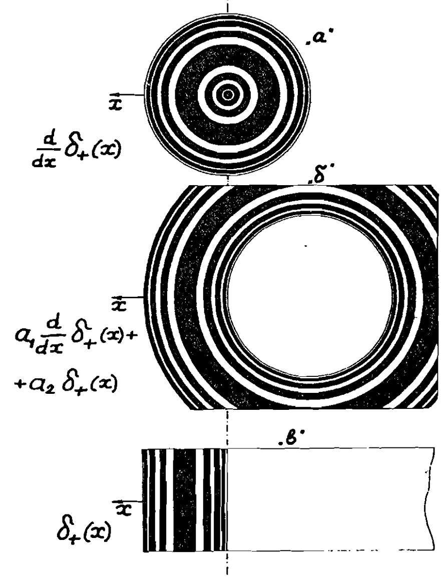


Fig. 5. Optical relief of three mesooptical elements which can be used for producing mesooptical imaging of straight-line particle tracks: a) a traditional mesooptical lens (mirror or kinoform), the convolution kernel of which has the form $(d/dx)\,\delta_+(x)$*; b) an intermediate scheme of the mesooptical lens, the curvature centre of which does not coincide with the optical axis of the system; the convolution kernel has form of superposition* $a_1(d/dx)\,\delta_+(x) + a_2\delta_+(x)$*; c) the optical relief of the cylindrical lens, the convolution kernel of which is equal to* $\delta_+(x)$*. The optical axis is perpendicular to the plane of figure and goes through the dot-and-dash line.*

ce from the optical axis equal to the width of the mesooptical element (Fig. 5b). It can be shown that the convolution kernel of the mesooptical system (Fig. 5b) has the form

$$a_1\delta_+(x) + a_2\frac{d}{dx}\delta_+(x), \tag{46}$$

i.e., is reduced to the superposition of $\delta_+(x)$ and its derivative $(d/dx)\,\delta_+(x)$.

Finally, the curvature centre of the mesooptical element may be on the mesooptical element's left instead of being on its right, as in Fig. 5. In this case the convolution kernel will contain the operator

$$\int \delta_+(x)\,dx, \tag{47}$$

which transforms the input function $f_o(x)$ into the output function $\int[f_o(x') + i\chi(x')]\,dx'$. (48)

We see that in Hilbert optics and in mesooptics we may construct a great number of the convolution kernels containing the Dirac delta-plus function, its derivatives and integrals.

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Сороко Л.М. E13-87-292

Дельта-плюс (или минус) функция Дирака
в оптике и мезооптике

Работа посвящена дельта-плюс функции Дирака и ее применению в гильберт-оптике и в мезооптике. Вначале дается введение и основные соотношения. Затем рассмотрены свойства когерентной изображающей системы, которая содержит различные фильтры пространственных частот. Дан анализ мезооптической изображающей системы, которая используется для наблюдения прямых следов частиц в ядерной фотоэмульсии. Показано, что в общем случае сверточное ядро мезооптической системы представляет суперпозицию вида $a_1\delta_+(x) + a_2 \times (d/dx)\delta_+(x)$.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

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Dirac Delta-Plus (or Minus) Function
in Optics and Mesooptics

The topic of this review article is Dirac delta-plus function and its application in Hilbert optics and in mesooptics. The article begins with the introduction of the backgrounds and of the underlying relations. Then the properties of the coherent imaging system containing various spatial frequency filters are treated. The analysis of the mesooptical imaging system used for observation of the straight line particle tracks in the nuclear research emulsion is presented. It is shown that in a general case the convolution kernel of the mesooptical system is a superposition of the form $a_1\delta_+(x) + a_2 \times (d/dx)\delta_+(x)$.

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