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CHARACTERISTICS OF A DOUBLE-CYLINDRICAL-MIRROR ELECTROSTATIC ENERGY-ANALYSER FOR ELECTRONS

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Recently [1], we described a new electron-spectrometer comprising a slowing down integral device put in series with a double cylindrical mirror analyzer. By a proper choice of analyzing voltages, the latter can operate at low energies, where the non-relativistic formulation is applicable.

Several types of electrostatic spectrometers have been described. The references to some of them are given in (2), while in (3) one may find a more complete review.

Here we present the analytical study of simple or double cylindrical-mirror analyzers.

The geometry is schematically given in fig.1, together with the definition of the symbols which enter in the calculations.

In the region between the two cylinders of radii R_1 and R_2 at the potentials V_1 and V_2 the electrostatic field $\mathcal{E}$ varies as r^{-1}.

The variables α and β are respectively the radial and azimuthal angles which define the trajectory of a particle of mass m and charge q.

A point of the trajectory is defined by the coordinates r and z, and the equation of the trajectory is:

$$(K\cos\beta\cos\alpha)^2(d^2r/dz^2) + (2r)^{-1} - (KR_1\sin\beta)^2 r^{-3} = 0, \qquad (1)$$

where K is a dimensionless parameter defined by:

$$K^2 = (qu)^{-1} E_a \log(R_2/R_1) \quad .$$

E_a is the energy when $r = R_1$; $u = V_2 - V_1$ is the analyzing voltage. In order to solve this equation, we use the auxiliary variable: $\phi = K\cos\beta\cos\alpha(dr/dz)$ whose value for $r = R_1$ is:

$$\phi_0 = K\cos\beta\sin\alpha.$$

A new variable can be introduced:

$$\Phi_0^2 = \phi_0^2 + (K\sin\beta)^2 = K^2(1-\cos^2\beta\cos^2\alpha).$$

For small β angles we obtain the following equations:

$$r/R_1 = \{1-(K\sin\beta)^2\exp[-2(\Phi_0^2-\phi^2)]\}\exp(\Phi_0^2-\phi^2),$$
$$z/R_1 = (K\cos\beta\cos\alpha)(a+b)\exp(\Phi_0^2), \qquad (2)$$

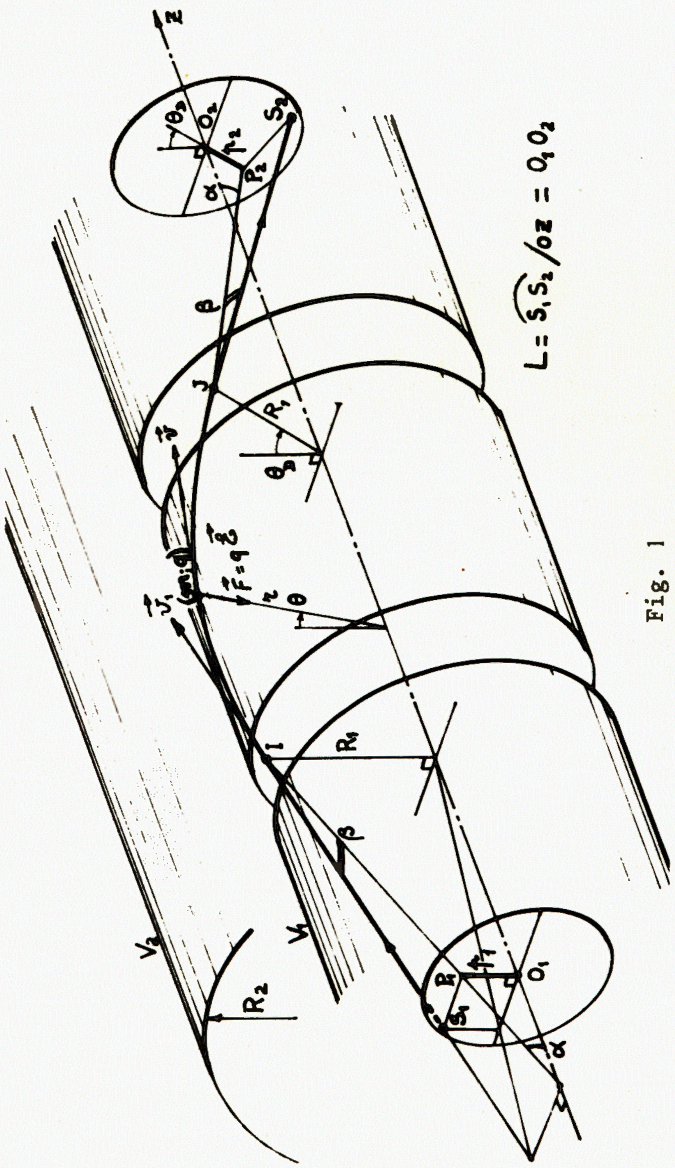


Fig. 1

where

$$a = \sqrt{\pi}\,\text{erf}\,\phi,$$

$$b = 2(K \sin\beta)^2 \exp(-2\Phi_0^2) \sum_0^n \frac{\phi^{2n+1}}{n!(2n+1)}.$$

The maximum distance of the trajectory of the axis is given by:

$$R_M/R_1 = [1 - (K \sin\beta)^2 \exp(-2\Phi_0^2)] \exp(\Phi_0^2).$$

Now, let S_1 be a point of the first straight part S_1I of the trajectory, characterised by β, a and p_1, whose projection on the initial (r, z) plane for $\theta = 0$ is $P_1(a, p_1)$, and let us consider any point $S_2(\beta, a, p_2)$ of the second straight path JS_2, whose projection on the (r, z) plane for $\theta = \theta_D$ is $P_2(a, p_2)$.

One may find the correspondence:

$$S_1(\beta, a, p_1) \xleftrightarrow{(\beta, a, \rho)} S_2(\beta, a, p_2)$$

which involves a new dimensionless parameter ρ defined as follows:

$$-1 \le \rho = (p_1 + p_2)/3R_1 \le +1 \qquad \text{with } |p_1| \text{ and } |p_2| \le R_1.$$

The projection L on the Oz axis of the trajectory S_1S_2 is a function of ϕ, a, ρ and K^2, derived from equation (2):

$$L/R_1 = 2[1 + \phi_0(a_0 + b_0)\exp(\Phi_0^2) - \rho](tg\,a)^{-1},$$

where a_0 and b_0 are the a and b values when $r = R_1$ and $\phi = \phi_0$.
With: $M = 1 + \phi_0(a_0 + b_0)\exp(\Phi_0^2)$

$$L/R_1 = 2(M - \rho)(tg\,a)^{-1} \qquad (3)$$

As the final result must be the focalization of an initially divergent beam, this means that the length L has to remain constant when a and β are close to a_0 and β_0 for given values of $\rho = \rho_0$ and K^2, corresponding to a point-to-point relation between the object S_1 and the image S_2.

That means that the two conditions:

$$(dL/d\beta)_{\beta = \beta_0} = 0, \qquad (4)$$

$$(dL/da)_{a = a_0} = 0 \qquad (5)$$

must be fulfilled.

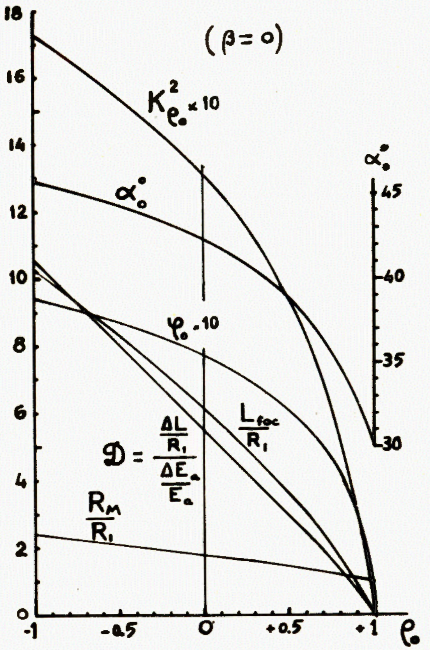


Fig. 2

The equation (4) may be reduced into a product of two terms: $(\sin^2\beta)[T(\beta, \alpha, K^2)] = 0$.

For small β angles which are of physical interest, the second term differs from zero. Then $\beta = 0$ is the only convenient solution.

The resolution of equation (5) gives the focalization conditions. We may solve it for $\beta = 0$, as justified a posteriori, if one introduces afterwards numerically small β values. The calculation of $(dL/d\alpha) = 0$ fixes the optimal values of the main parameters of the spectrometer as functions of ρ_0 (fig.2).

Indeed, the choice of ρ_0 determines the characteristics of the apparatus: the emission angle α_0, the focal length L_{foc}, the maximum of radial distance R_M, the dispersion $\mathfrak{D}$. The parameter ρ_0 determines also the mode of operation as the voltage u is function of $K^2_{\rho_0}$.

Figure 3 shows the allowed apertures $(\alpha_{max} - \alpha_{min})$ for different ρ_0 values.

Figure 4 shows schematically how two identical cylindrical mirrors operate in series. The focal length L_{foc} is simply doubled, and four cylindrical slits determine the allowed trajectories.

The different characteristics of such a double mirror, as regarding focal length, useful width of the slits, a.s.o., may be determined as follows: figure 5 gives the dependence of the length on the α angle for a given ρ_0 and different ρ values. Given α_0 and within the range of α_{min} to α_{max} values, one may deduce from the family of such curves the values of couples (α, ρ) which determine the family of trajectories inside a certain envelope which pass through the slits as is shown on figure 6.

Another important quantity is the radius R_2 of the outer cylinder. One must have: $R_2/R_1 > R_M/R_1 = \exp(\Phi_0^2)$, where we take into account the α_{max} value. That means that with such a cylindrical mirror, we analyse particles with an energy E_a equal to (in volts and atomic units):

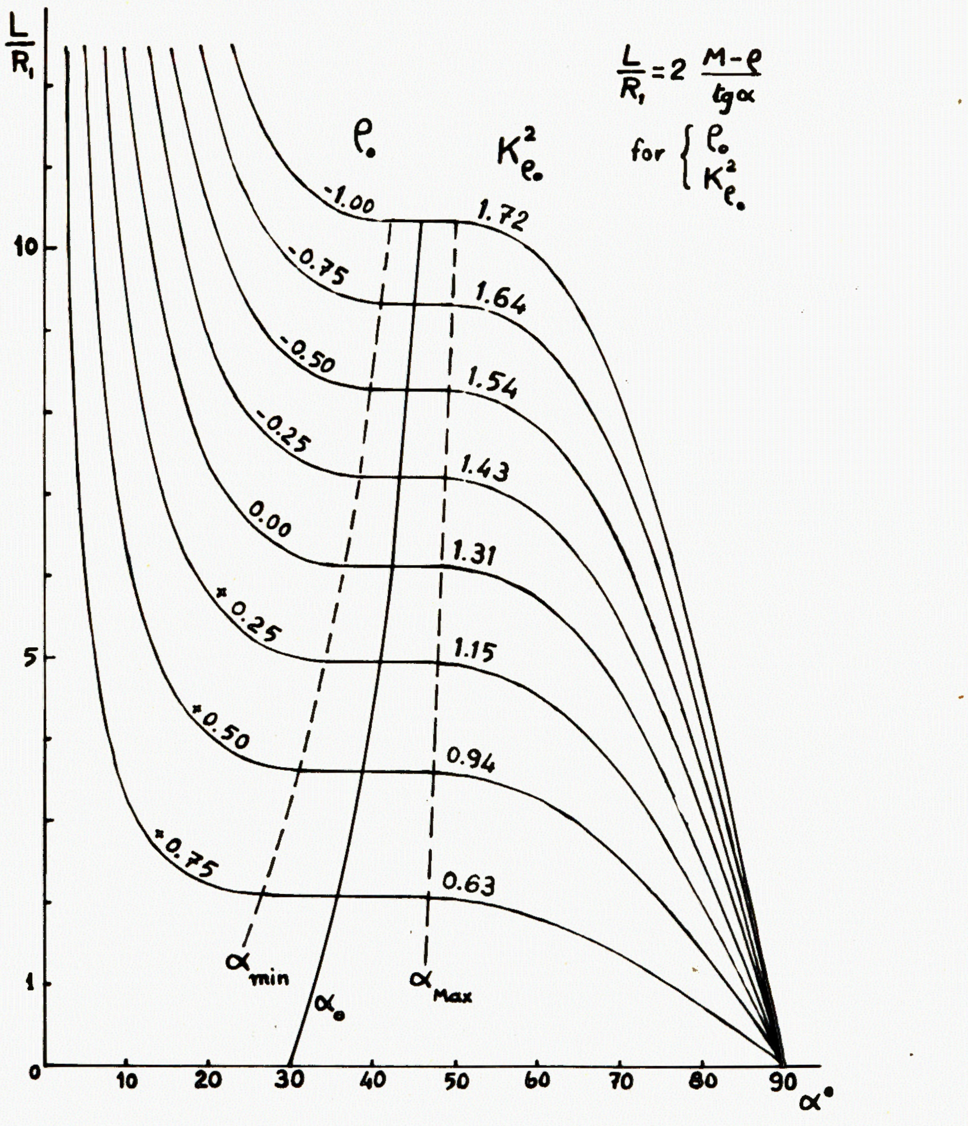
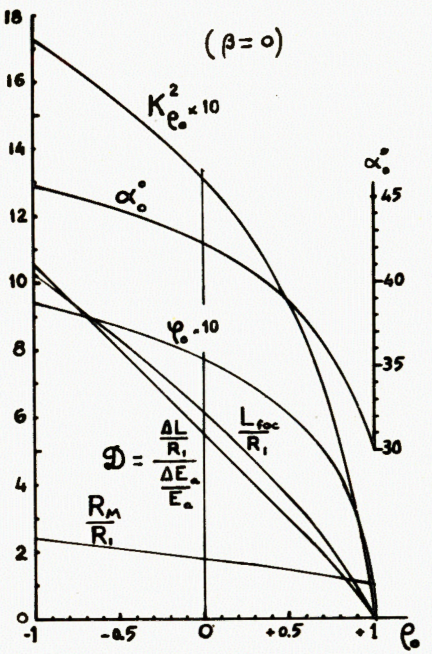


Fig. 3

$$E_a = K^2_{\rho_0}\, q\, u (\log R_2/R_1)^{-1} \sim K^2_{\rho_0}\, q\, u\, \Phi_0^{-2} =$$

$$= q\, u\, (1 - \cos^2\beta \cos^2\alpha_{max})^{-1} .$$

For $\beta = 0$ and $\alpha_{max} \sim 45^\circ$: $E_a = 2qu$.



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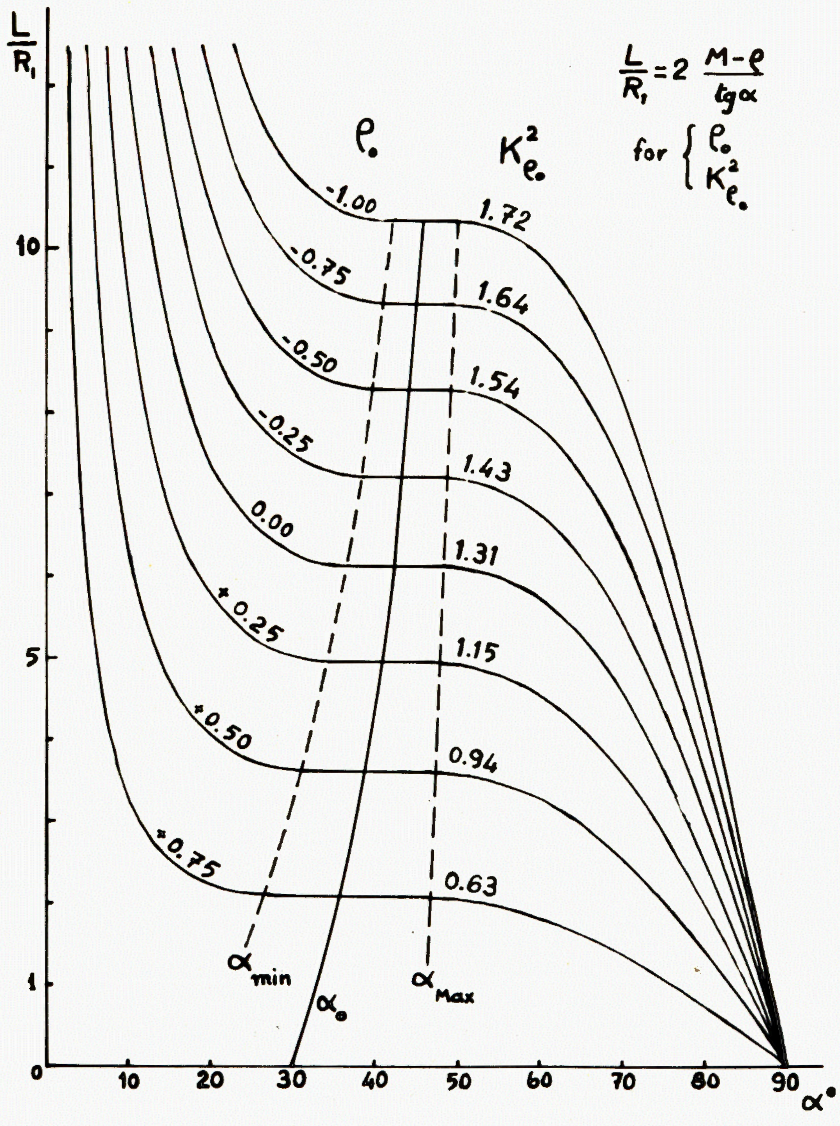

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$$E_a = K^2_{\rho_0}\, q\, u (\log R_2/R_1)^{-1} \sim K^2_{\rho_0}\, q\, u\, \Phi_0^{-2} =$$

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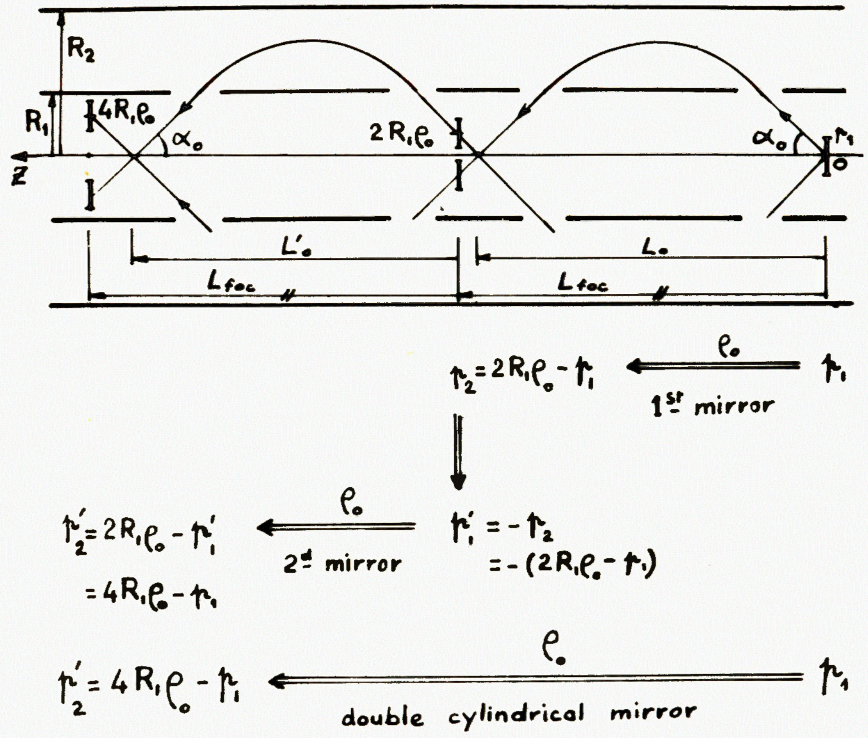

Fig. 4

We may write the quantity L/R_1 as:

$$L/R_1 = L_{foc}/R_1 + \Delta L/R_1 .$$

For $\beta = 0$, one obtains:

$$L/R_1 = L_{foc}/R_1 - 1.77 \cdot 10^{-6}\, \phi_0 \{3(1 + 4\sin^2 \alpha_0)(\sin\alpha_0)^{-2} \times$$
$$\times [2\phi_0 + (1 + 2\phi_0^2)\, a_0 \exp(\phi_0^2)] - 2\phi_0^2 (\mathrm{tg}\,\alpha_0)^{-4}\, [2\phi_0 (5 + 2\phi_0^2) +$$
$$+ (3 + 12\phi_0^2 + 4\phi_0^4)\, a \exp(\phi_0^2)]\} (d\alpha_0^\circ)^3 ,$$

which yields for the single mirror, with $\rho_0 = 0$:

$$L/R_1 = 6.13 - 0.8 \cdot 10^{-4} (d\alpha_0^\circ)^3$$

and for the " $\rho_0 = 0$ double mirror":

$$2L/R_1 = 12.26 - 1.6 \cdot 10^{-4} (d\alpha_0^\circ)^3 .$$

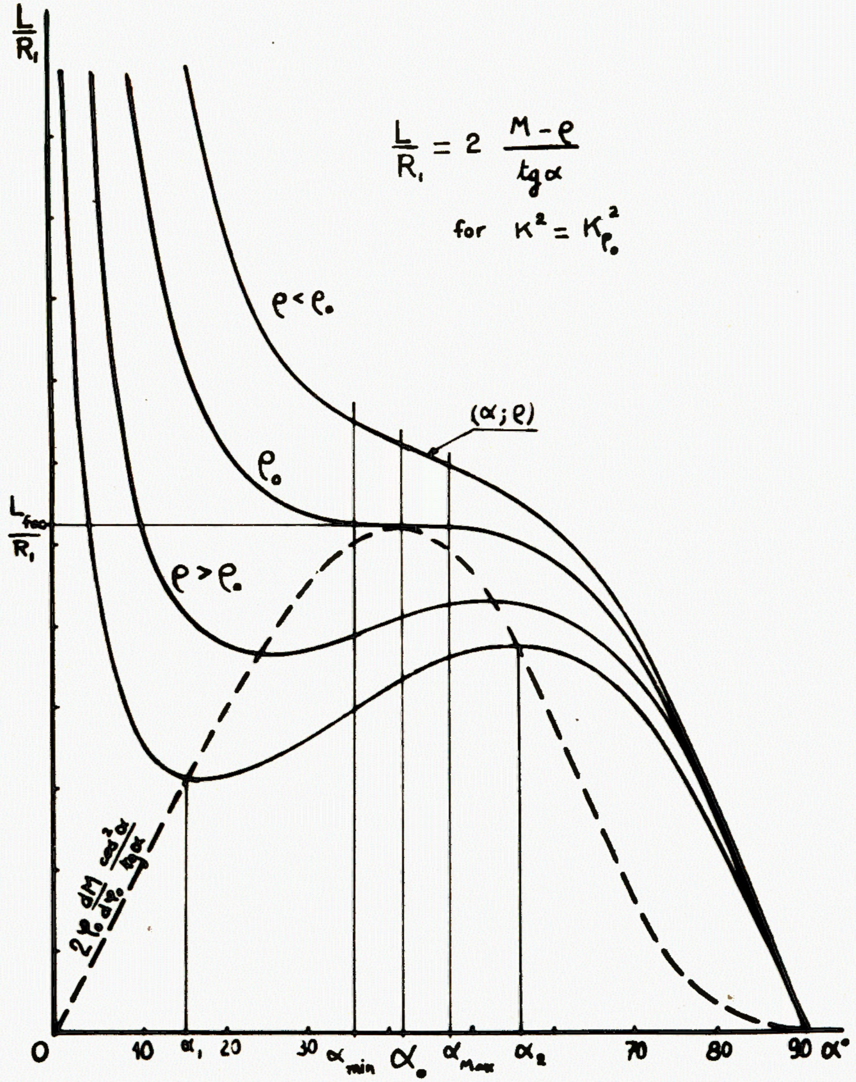

Fig. 5

Hence, if d is the diameter of the detector, we obtain the resolution:

$$\Delta E_a/E_a = (2\mathfrak{D})^{-1} (d/2R_1 + 2|\Delta L/R_1|) =$$
$$= 0.09 (d/2R_1 + 1.6 \cdot 10^{-4} |\Delta\alpha_0^\circ|^3).$$

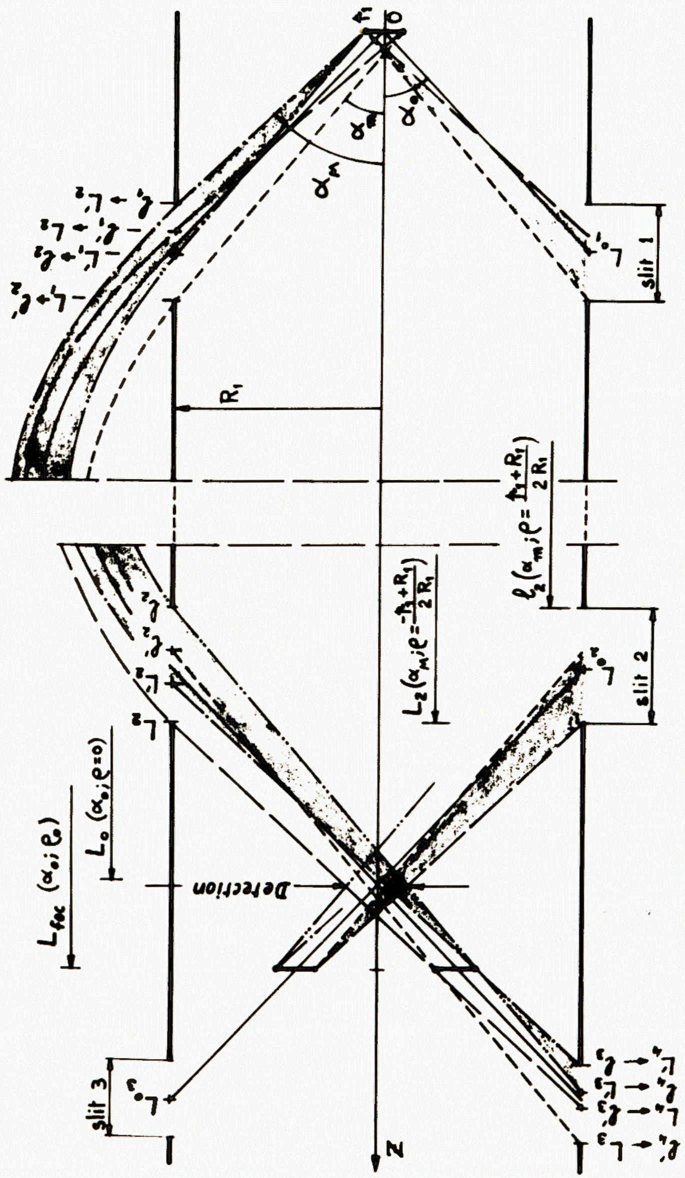

Fig. 6

On the table are given the solutions of eq. (2) for $\beta = 0$ and $\beta = 5°$.

Table

			L_{foc}/R_1	
ρ_0	$K^2_{\rho_0}$	α°_0	$\beta = 0$	$\beta = 5°$
-0.2	1.42	43.2	7.04	7.06
0.0	1.31	42.3	6.13	6.14
+0.2	1.19	41.2	5.18	5.19

One may see from these results that the difference in the focalization condition is insignificant. This shows the validity of our assumption when resolving eq. (5) for $\beta = 0$.

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Легран Б. E13-83-326

Характеристики электростатического анализатора электронов типа двойного цилиндрического зеркала

Представлено аналитическое исследование одиночного и двойного цилиндрического зеркального анализатора. Расчеты сделаны в нерелятивистском приближении. Приведены количественные соотношения между основными параметрами анализатора /радиальный угол, длина фокусировки, дисперсия и пр./, позволяющие оптимизировать выбор режима работы. Результаты расчета представлены в виде диаграмм так, чтобы их легко можно было использовать в практике. Два таких анализатора уже построены в Орсэ и в Дубне. Эти анализаторы работают в сочетании со сферическими замедлителями, и весь прибор используется как спектрометр, приспособленный для анализа низкоэнергетических электронов /1-50 кэВ/.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1983

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Characteristics of a Double-Cylindrical-Mirror Electrostatic Energy-Analyser for Electrons

The analytical study of single and double cylindrical mirror energy analysers is presented in nonrelativistic formulation. A number of relations between the main parameters of the analyser (radial angle, focalisation length, dispersion, etc.) allow one to optimize the choice of the operating conditions. The results of the calculations are presented in the form of diagrams in such a way that they can be easily used in any practical case. Two such analysers have been built in Orsay and in Dubna, and put in series with a spherical retarding, analyser, the whole device beeing used as a spectrometer well-suited for the analysis of low energy (1-50 keV) electrons.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1983