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A NOVEL APPROACH TO THE Rh–Fe THERMOMETRIC CHARACTERISTICS APPROXIMATION

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Богданова Н.Б., Терзийска Б.М. E11-97-396
Новый подход к аппроксимации Rh–Fe термометрических характеристик

Рассматриваются новые аппроксимации $T = f(R)$ и $R = g(\log T)$ или $R = g(T)$ функций для Rh–Fe низкотемпературного термометра в диапазоне 1,8—372 K, построенные с использованием весового метода разложения по ортогональным полиномам (на основе рекуррентной формулы Форсайта). Весовые функции предложенного подхода относятся как к специфическим операционным условиям, так и к абсолютной чувствительности исследуемого низкотемпературного датчика. Представлено сравнение между предложенным подходом и другим полиномиальным описанием для Rh–Fe термометрических характеристик.

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Bogdanova N.B., Terzijska B.M. E11-97-396
A Novel Approach to the Rh–Fe Thermometric Characteristics Approximation

We consider a new approximation of the $T = f(R)$ and $T = f(\log R)$ or $R = g(T)$ functions for Rhodium–Iron (Rh–Fe) cryogenic resistance thermometer within 1.8—372 K, using weighted Orthonormal Polynomial Expansion Method (OPEM), based on Forsythe recurrence formula. The weighting functions of the proposed approach are referred to both the specified operating conditions and the absolute sensitivity of the investigated cryogenic resistance temperature sensor. A comparison between the OPEM and other polynomial descriptions of the Rh–Fe thermometric characteristics at the same test data is presented.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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1 Introduction

The choice of a numerical polynomial method for thermal data approximation, in particular for thermometer calibration test data description, is very significant step in the low temperature physical properties' investigations of solids, in analysing and clarifing the nature of the studied phenomena. The continuously refining cryogenic thermal experiment defines the concreate features of the method of approximation. In this sence, the routine temperature sensor calibration and automation of the thermal experiment determine the large number of demands to the mathematical fit of the thermometer test data, as the accuracy, simplicity and stability of the fit. The main requirement is to obtain lower errors of the mathematical fit than the thermometer calibration uncertainties. It is the reason to ask for more convenient and flexible method in comparison with conventional ones. The appropriate functional forms for approximating high precision thermal data are Chebyshev, exponential and ordinary polynomials [1, 2]. Here we propose a new numerical weighted method constructing orthonormal polynomials based on Forsythe three- term recurrence formula [3]. It is known as Orthonormal Polynomial Expansion Method (OPEM) and it is considered in [4, 5, 6, 7]. The algorithm and FORTRAN program realization of OPEM are developed for high-energy physics [5, 6, 7]. Recently, a number of studies on the weighted Orthonormal Polynomial Expansion Method (OPEM) application in cryogenic thermometry have been reported [8, 9, 10, 11, 12]. In three our reports, [8, 9, 10] we have briefly introduced the basic mathematical definitions and some preliminary results concerning these investigations. The main formulations concerning methodical and computational aspects of the OPEM application for the cryogenic thermometric characteristics' approximation as well as some interesting results for the OPEM global and interval description of the $T(R)$ and $R(T)$ functions of two type resistance temperature sensors in the whole useful temperature range, Germanium thermoresistor (GRT) $(1.4 - 100)K$ and Platinum thermoresistor (PRT) (14 – 325) K, are presented in [11] and [12]. The FORTRAN77 codes, PC IBM compatible versions are prepared for this study.

It is worth to note that some preliminary codes concerning Forsythe polynomials in cryogenic thermometry are offered in the paper of

Brights and Dawkins [13]. The criteria of a low temperature thermometer selection, the main characteristics and the temperature ranges of application are discussed elsewhere [14].

In this work a novel approach to the matematical description of the T and R functions for Rhodium-Iron (Rh-Fe) resistor by OPEM, using one- dimensional case approximation, has been presented. The useful temperature range of Rh-Fe thermoresistor is 1 to 800 K but the temperature range of use is 1 to 400 K. The Rh-Fe temperature sensor is also recommended for measurements above 77 K in Magnetic Fields because of its acceptable Magnetic Field-dependent Temperature error.

2 Orthonormal Polynomial Method

The Orthonormal Polynomial Expansion Method (OPEM), based on the recurrence Forsythe formula [3] generates orthonormal polynomials $\{p_i\}$ in the following form:

$$p_{i+1}^{(m)}(x) = \gamma_{i+1}\left((x-\alpha_{i+1})p_i^{(m)}(x) - (1-\delta_{i0})\beta_i p_{i-1}^{(m)}(x) + m p_i^{(m-1)}(x)\right). \quad (1)$$

Here $m = 0$ is for polynomials, $m > 0$ is for derivatives. The approximating function $f^{(m)\mathrm{appr}}(x)$ at $m = 0$ or approximating derivatives at $m > 0$ are presented as:

$$f^{(m)\mathrm{appr}}(x) = \sum_{k=0}^{n} b_k p_k^{(m)}(x) = \sum_{k=0}^{n} a_k x^k, \quad a_k = \sum_{i=k}^{n} a_k^{(i)} b_i, \quad (2)$$

where the coefficients b_k are for orthonormal expansion, a_k are for usual expansion and the relative coefficients $a_k^{(i)}$ are evaluated by recurrence relati on in [4].

Let the test data are the measured values of the argument x, the dependent quantity f and the standard deviation s in f $\{x_i, f_i, s_i, i = 1, \ldots M\}$. The orthonormal coefficients γ_{i+1}, recurrence factors α_{i+1} and β_i are presented by scalar products of the polynomials in the test data [4]. The scalar product is:

$$(p,q) = \sum_{i=1}^{M} w_i p(x_i) q(x_i),$$

where $w_i = 1/s_i^2$ are the weights and s_i^2 are the variances. For every pair of polynomial system $\{p_i, i = 1, \ldots, n\}$ it is fulfiled, that $(p_i, p_j) = \delta_{ij}$, where δ_{ij} is a Kronecker symbol. The coefficients b_k in orthonormal expansion according least square method can be defined as:

$$b_k = (f, p_k). \quad (3)$$

To select the optimum number of polynomials two criteria are used:

$$\sum_{i=1}^{M} w_i (f_i^{appr} - f_i)^2/(M-n-1) \rightarrow min; \quad (f_i^{appr} - f_i)^2 w_i \leq 1. \quad (4)$$

The first is the minimizing the normalized chi-square per degree of freedom and the second is the fulfilment of special inequality for any point in the interval $[1, M]$. The second criterion means, that the graph of the fitting curve passes through the error corridor of all experimental points $\{x_i, f_i \pm s_i, i = 1, \ldots M\}$. Preference is given to the second criterion and when it is satisfied, the search for minimum stops. In this way the computing time is reduced.

The new features of OPEM in comparison with Forsythe formula are: the stable telescoping trick for fitting series, described in details in [4]; the calculation of the inherited errors in b_k, shown in details in [4]; the evaluation of coefficients a_k in usual expression and their inherited errors according to relations, explained in [4]; the selection of the optimum number of orthogonal polynomials using two criteria, formulas (4); normalization of the iterative procedure with Eq.(1) by coefficient γ_{i+1}; stable and fast generation of the derivatives and integrals by Eqs.(1) and (2).

We do not discuss here in detail the main advantages of our algorithm as the absence of matrix inversion and unit numerical condition of involved matrices which allows high-degree fits, described in [4, 5, 6]. We only demonstrate these features in the cryogenic thermometry applications.

3 Approximation Details

In our study the commercialy available calibrating test data (54 experimental points) for Rh-Fe thermoresistor, Model TRRFLH-2, between

1.8 and 372 K, provided by VNIIFTRI, Russia and based on the International Temperature Scale ITS-90 have been used. The specified operating conditions are: the sensor current $I = 1$ mA, the absolute measurement system resolution $\Delta U_{\text{amsr}} = 0.001$ mV and the typical accuracy of VNIIFTRI calibration $\Delta T_{\text{cal}} = \pm 10$ mK.

For precise $R = g(T)$ and $T = f(\log R)$ description of this cryogenic temperature sensor a new type of weighting is proposed. The used weighting functions w^R and w^T were evaluated in each given point by the relation $w_i = 1/s_i^2$. The variance of the approximating functions, s^2, is accepted to be $(\Delta R_{arr})^2$ and $(\Delta T_{\text{atr}})^2$, respectively the absolute resistance and temperature resolutions of the sensor. Here ΔT_{atr} and ΔR_{arr} are the standard deviations s of R and T quantities in the OPEM notations. The values of the absolute resistance resolution $(\Delta R_{\text{arr}})_i$ and the absolute temperature resolutions $(\Delta T_{\text{atr}})_i$ are given by:

$$(\Delta R_{\text{arr}})_i = (\Delta U)_{\text{amsr}}/I_i \quad (\Omega). \tag{5}$$

where ΔU_{amsr} and $I_i (i = 1, ...M)$ are the absolute measurement system resolution and the sensor current, defining the specified operating conditions.

$$(\Delta T_{\text{atr}})_i = (\Delta R_{\text{arr}})_i / \mid (dR/dT)_i \mid \quad (\text{K}), \tag{6}$$

Here (dR/dT) is the absolute sensitivity of the thermoresistor.

For Rh-Fe temperature sensor: $(\Delta R_{\text{arr}})_i = 10^{-3}$ (Ω); and $(\Delta T_{\text{atr}})_i = 10^{-3}/(dR/dT)_i$ (K).

The absolute temperature resolution, ΔT_{atr}, and the relative temperature resolution, $\Delta T_{\text{atr}}/T$, are main characteristics of various thermometers. Following formula (6) the expression for the relative temperature resolution $\Delta T_{\text{atr}}/T$ can be written as:

$$(\Delta T_{\text{atr}}/T) = (\Delta R_{\text{arr}}/R) / \mid (T/R)(dR/dT) \mid . \tag{7}$$

Consequently, the relative temperature resolution $\Delta T_{\text{atr}}/T$ is depending on both the specified operating conditions and the material-specific parameter, $(T/R)(dR/dT)$, known as a specific sensitivity of the temperature sensor.

The weights w^R and w^T are estimated by the expressions:

$$w_i^R = 1/(\Delta R_{\text{arr}})_i^2 \quad (\Omega^{-2}); \tag{8}$$

$$w_i^T = 1/(\Delta T_{\text{atr}})_i^2 = (dR/dT)_i^2/(\Delta R_{\text{arr}})_i^2 \quad (\text{K}^{-2}) \tag{9}$$

Under the defined operating conditions

$$w^R = 10^6 = const. \quad (\Omega^{-2}).$$

Following Eqs. (7) and (8)

$$w_i^T = (dR/dT)_i^2 w_i^R = 10^6 (dR/dT)_i^2 \quad (\text{K}^{-2}).$$

It is clear that the weighting functions w^R and w^T for $R = g(T)$ and $T = f(R)$ or $T = f(\text{logR})$ approximations are related to the specified operating conditions (ΔU_{amsr} and I) and the absolute sensitivity (dR/dT) of the described thermometer.

The deviations $(\Delta R)_i$ between the experimental R_i and approximating R_i^{appr} values of the resistance, R, and their temperature equivalents $(\Delta R_{\text{te}})_i$ are calculated as follows:

$$(\Delta R)_i = (R_i - R_i^{\text{appr}}) \quad (\Omega); \quad (\Delta R_{\text{te}})_i = (R_i - R_i^{\text{appr}})/(dR/dT)_i \quad (\text{K}). \tag{10}$$

The deviations $(\Delta T)_i$ between experimental T_i and approximating T_i^{appr} values of the temperature, T, are obtained by:

$$(\Delta T)_i = (T_j - T_i^{\text{appr}}) \quad (\text{K}). \tag{11}$$

4 Results and Illustrations

The OPEM results for the Rh-Fe thermoresistor concerning R and T three- range approximation within 1.8 – 372 K are proposed in Figures 1, 2, 3 and in Table 1. Figure 1 illustrates the temperature dependences of the absolute sensitivity, dR/dT, the relative sensitivity, $(1/R)(dR/dT)$, the specific sensitivity, $(T/R)(dR/dT)$, the absolute temperature resolution, ΔT_{atr}, and the resistance, R, of the Rh-Fe temperature sensor. It is evident that the type of the temperature dependences of the absolute temperature resolution and the absolute, dR/dT, relative, $(1/R)(dR/dT)$, and specific, $(T/R)(dR/dT)$, sensitivities of the Rh-Fe sensor is persuasively identical, Fig. 1. Consequently, the only study of the temperature behaviour of the ΔT_{atr}

Table 1 OPEM approximation of $R(T)$ and $T(R)$ characteristics and approximation of $T(R)$ characteristic by exponential polynomials for the Rh-Fe thermoresistor

Range(K)	M	Formula	n	Δ_{max}(mK)	RMS(mK)	$W \times 10^6(OPEM)$
1.8÷24	20	$R = g(T)$	5	-3.31 [15.3K]	1.402	1
1.8÷24	20	$T = f(R)$	6	+5.84 [22.3K]	2.136	(0.43 ÷ 0.02)
24 ÷ 71	13	$R = g(T)$	5	-3.40 [36.5K]	1.429	1
24 ÷ 71	13	$T = f(\text{lgR})$	6	+4.47 [36.5K]	1.881	(0.02 ÷ 0.11)
71 ÷ 372	23	$R = g(T)$	9	+8.77 [122.6K]	2.981	1
71 ÷ 372	23	$T = f(\text{lgR})$	9	-6.38 [122.6K]	2.482	(0.11 ÷ 0.14)
1.8 ÷ 23	19	$T = T(R)$	9	+5.2 [15.3K]		
$R < Bk1$		$Bk1 = 13.$				
24 ÷ 61	12	$T = T(R)$	9	-3.5 [26.5K]	3.234	*VNIIFTRI*
$R < Bk2$		$Bk2 = 22.$				*(Russia)*
71 ÷ 372	23	$T = T(R)$	9	-6.3 [122.1K]		
$R > Bk2$		$Ak = 5.0$				

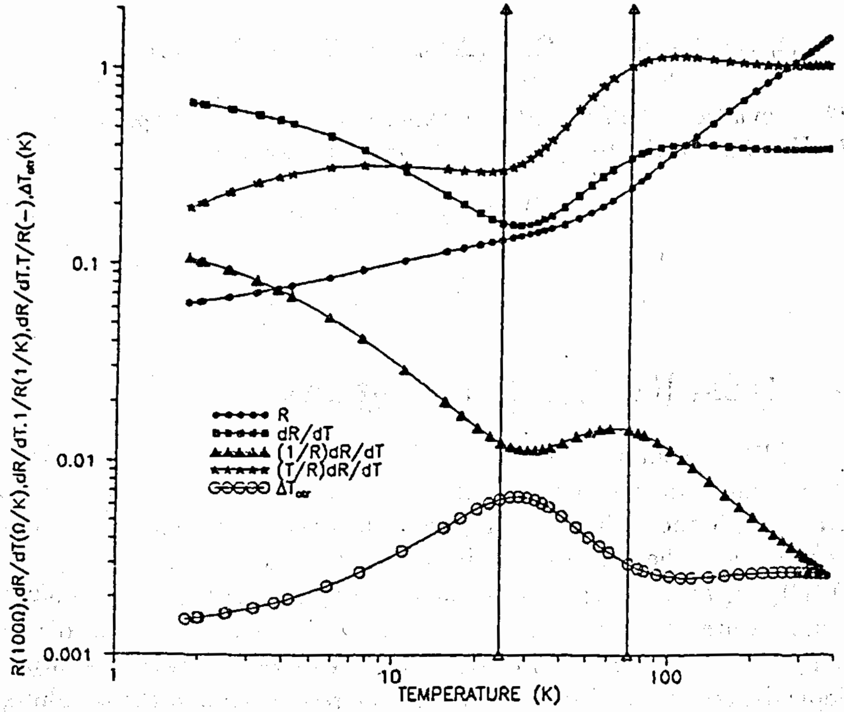

Figure 1 Log-log plot of the absolute temperature resolution ΔT_{rmatr}, the resistance R, the absolute dR/dT, relative $(1/R)dR/dT$ and spesific $(T/R)dR/dT$ sensitivities of the Rh-Fe thermoresistor vs temperature, T

$$w_i^T = 1/(\Delta T_{\text{atr}})_i^2 = (dR/dT)_i^2/(\Delta R_{\text{arr}})_i^2 \ (\text{K}^{-2}) \tag{9}$$

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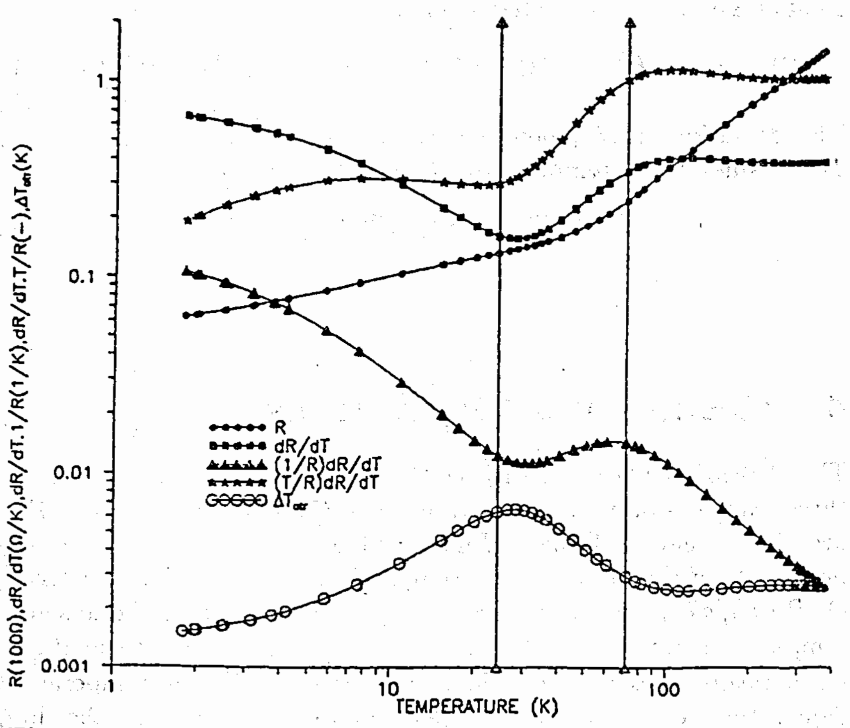

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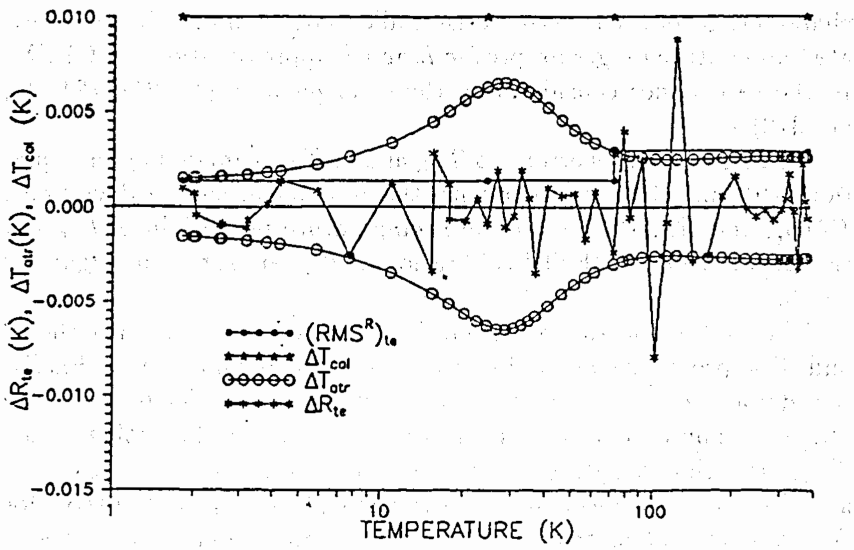


Figure 2 Temperature dependences of the OPEM differences ΔR_{te}, RMS^R_{te}, $\pm\Delta T_{atr}$ and ΔT_{cal} for the Rh-Fe thermoresistor

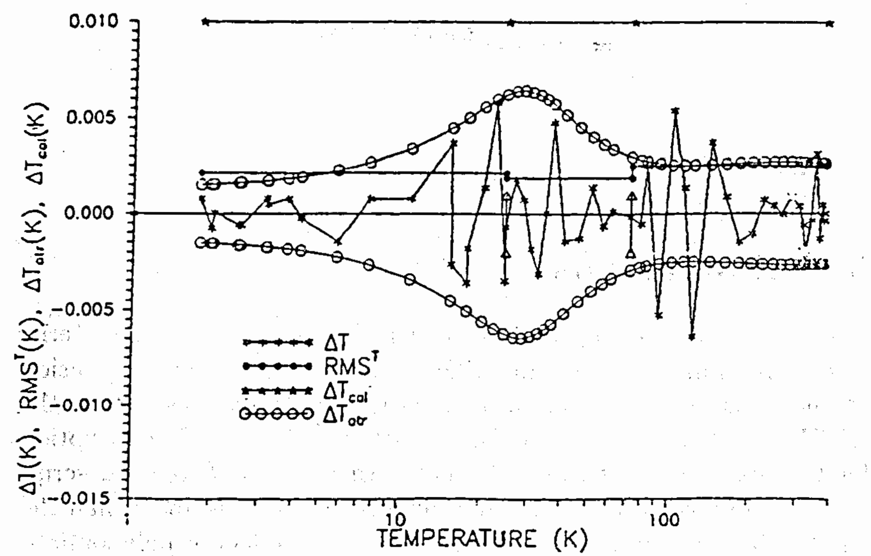


Figure 3 Temperature dependences of the OPEM differences ΔT, RMS^T, $\pm\Delta T_{atr}$ and ΔT_{cal} for the Rh-Fe thermoresistor

characteristic of each thermoresistor allows determining the appropriate temperature ranges for presise R and T approximations by OPEM. In the case under consideration these ranges are: (1.8-24); (24-71); (71-372).

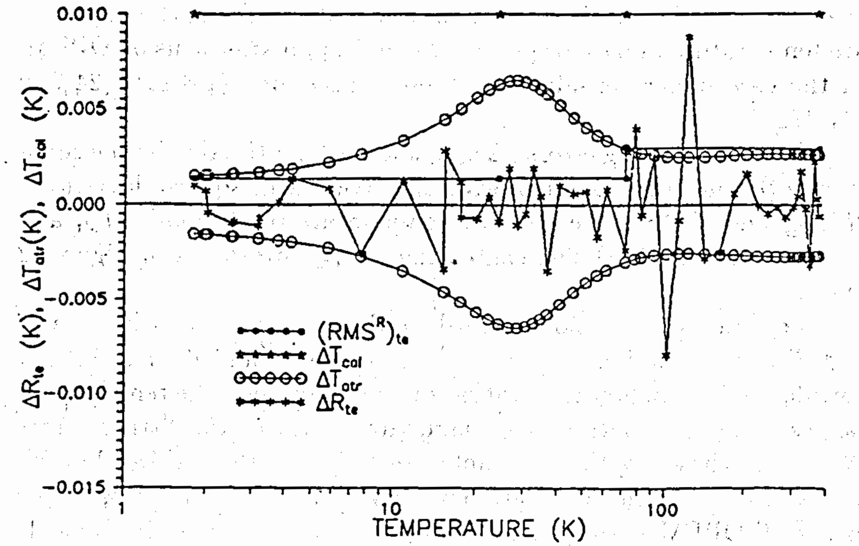
The OPEM fitting errors $(\Delta R_{te})_i$ and $(\Delta T)_i$ estimated by the equations (9) and (10), together with the root mean square deviations, RMS^R_{te} and RMS^T, the absolute temperature resolutioin, ΔT_{atr} and the accuracy of VNIIFTRI calibration, ΔT_{cal}, are shown in Figures 2 and 3.

The first point to note regarding the fitting errors with the R and T approximations is that the errors are bounded in a $\pm\Delta T_{atr}$ corridor and are bellow the calibration uncertainties. The temperature ranges of approximation, the maximum values of the fitting errors Δ_{max}, the chosen optimum number of polynomials, the weights w^R and w^T, the root mean square deviations $(RMS^R)_{te}$ and RMS^T for R and T OPEM matematical description, are presented in Table 1. For comparison the results of three- range exponential polinomials' description of the $T(R)$ function for the Rh-Fe resistor with the same calibrating test data proposed by VNIIFTRI, Russia, are also listed in Table 1. The used function is ($B1 = 13.$, $B2 = 22.$, $Ak = 5.$):

$$T = \exp^{\sum_{i=0}^{9} A_i \ln[(R-A_k)/(B_k-A_k)]^i}$$

5 Conclusions

In summary, a novel approach to the Rh-Fe thermometric characteristics' approximation by weighted Orthonormal Polynomial Expansion Method has been considered. It is important to emphasize that the OPEM number of polynomials, n, with the $T = f(R)$ description for the first subinterval (n = 6) and with the $T = f(\log R)$ description for the second and third subintervals (n = 6,9) is lower than the number of polynomials with the $T = T(R)$ exponential polynomials' description for these ranges carried out by VNIIFTRI, Russia, (n = 9,9,9), using the same calibrating test data, Table 1. Furthermore, the



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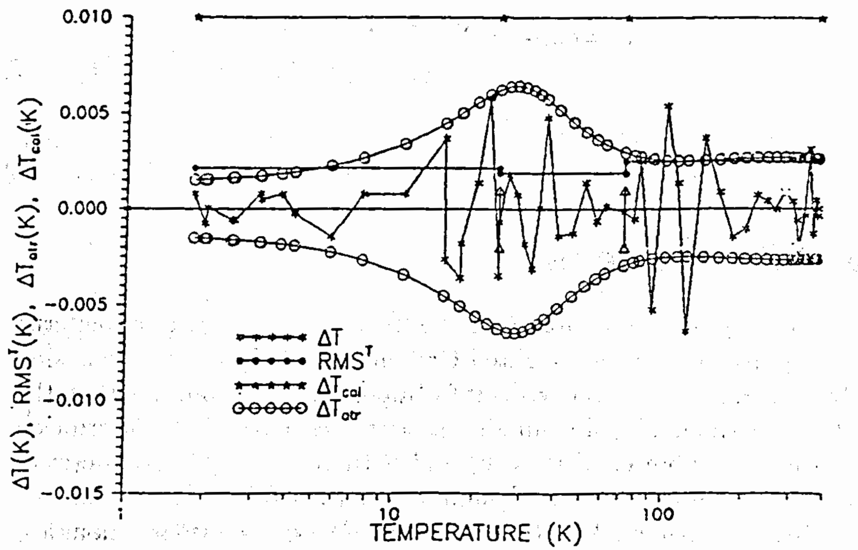

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OPEM fitting errors, expressed by RMS^T deviations are respectively 2.136 mK, 1.881 mK and 2.482 mK whereas the obtained by VNIIFTRI value for the RMS^T is 3.234 mK with the entire temperature range. Our main conclusion is that the OPEM proposes substantially better thermometric characteristics' approximation of T function for the Rh-Fe thermoresistor than the exponential polynomials' fit used by VNIIFTRI, Russia.

It is also clear that the proposed precise OPEM approximation of the $R = g(T)$ function (n = 5,5,9), Table 1, permits generating the interpolation tables for the Rh-Fe cryogenic temperature sensor.

Evidently, the present results strongly underline the advantages of the OPEM thermometric characteristics' description for the thermoresistors and imply the reliability of the OPEM application in cryogenic thermometry.

Finally, the successful use of the available OPEM in cryogenic thermometry, especially in calibrating the temperature sensors and automating the thermal experiment in the scientific laboratories, will give a direct benefit to reseachers.

6 Acknowledgements

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