СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E11-97-358

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ON SOME QUESTIONS OF THEORY OF THE ACOUSTIC LASER

1997

Завтрак С.Т. и др. E11-97-358
Некоторые вопросы теории акустического лазера

Предложена теоретическая схема акустического аналога лазера (сосера), аналогичная хорошо известной схеме лазера на свободных электронах, где электромагнитное излучение возникает вследствие самосинхронизации электронного пучка, движущегося через периодически меняющееся магнитное поле. Численная модель для сосера описывается системой трех нелинейных дифференциальных уравнений с частными производными. Проведено исследование модели посредством математического моделирования с помощью компьютера. Моделирование проведено в широком диапазоне физических параметров, описывающих активную среду и резонатор сосера. Результаты находятся в хорошем качественном согласии с теоретическими результатами для лазера на свободных электронах.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 1997

Zavtrak S.T. et al. E11-97-358
On Some Questions of Theory of the Acoustic Laser

A theoretical scheme of acoustic analog of laser (saser) was suggested and investigated numerically with the help of computer simulation procedure. The suggested scheme for the saser is analogous to a well-known scheme of free-electron laser (FEL) where an electromagnetic emission is created by self-synchronized electron beam moving through magnetic periodic systems. A computational model of the saser was described by a sytem of three nonlinear differential equations with partial derivatives. A simulation was performed in a wide range of physical parameters for active media and resonator. The obtained results are in good agreement with the results known for FEL.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 1997

I. THEORETICAL BACKGROUND

At the present time, the problem of creating an acoustic analogue of the laser (which will be referred below as 'saser' - sound amplification by stimulated emission of radiation or 'acoustic laser') is of great interest not only because of its evident fundamental significance but also because there are a variety of potential applications for such devices. As generators of directed shock waves, they could be used for direct underwater communication and impact action on underwater objects. Propagation into other dense media could give rise to medical, engineering and underground remote sensing applications. If propagation through appreciable distance of more rarefied media, notably air, could be achieved, many more applications would be opened up.

Recently, a theoretical scheme for a saser has been proposed by one of the present authors in Refs.[1-6], Fig.1-a. A liquid dielectric with uniformly distributed dispersed particles was suggested as the active medium. Different types of oils, liquefied gases or distilled water can be used as a liquid dielectric. Gas bubbles was suggested in Refs.[1-6] as dispersed particles due to, firstly, their very high compressibility and, secondly, their ability to give sound emission of the monopole type. The sound emission from solid corpuscles is of dipole type and much less efficient. The suggested scheme for a saser is analogous to that of a free-electron laser (FEL). It is well known that the useful electromagnetic radiation is created by an electron beam moving through magnetic periodic systems. These systems are called ondulators or wrigglers. Ondulators play the role of pumping. Inside such a system each electron oscillates and, hence emits electromagnetic waves. Initially the emission of each electron is added to the emission of others but different spatial phases. Thus, the resulting emission is equal to zero. In order to obtain non-zero emission we should put this system in a resonator to reflect back some part of useful energy. In optical lasers this is usually realised by means of half-silvered mirrors. In the FEL the reflection of the useful electromagnetic wave can be realised by means of a metal net. Then, under the action of a pump wave and the useful wave, electrons become grouped in so-called bunches. As a result, the emission becomes coherent. It leads to amplification of the electromagnetic field. This mechanism is well known as self-synchronisation. In the saser gas bubbles play role of the electrons in the FEL. Unlike common optical lasers, in which atoms can emit spontaneously, gas bubbles

can only oscillate under an external action, but not spontaneously. As has been shown in Refs.[1-6], external pumping of this active medium can be achieved by electrical (see, for example, Refs.[1-3,5]) or mechanical (see, for example, Refs.[4,6]) methods.

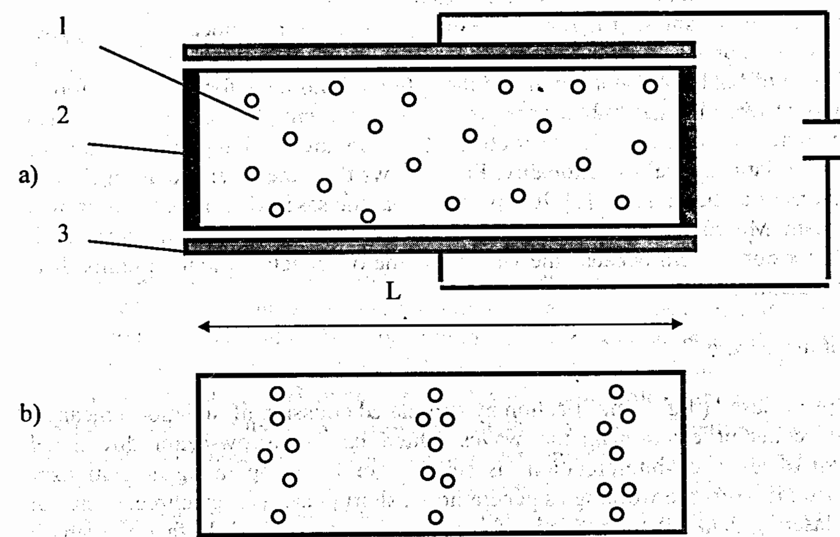

Fig. 1. Scheme of the acoustic laser.
1 - active medium;
2 - resonator;
3 - electrodes.

In the first case the electric field acting on such a system results in the deformation (electrostriction) of the dielectric and, hence, changes particle volumes. The value of the effective pressure acting on the particle is proportional to the square of electric intensity E and the difference between dielectric constants of liquid bubble and gas. For gas the dielectric constant is very close to 1. As for liquids, a high value of this constant is found in distilled water (about 81). It is clear that for electric pumping, distilled water is preferable with respect to other dielectrics. However, the electric pumped saser has one weak point. The electric intensity necessary to common saser generation in, for example distilled water with air bubbles, is of the order of a few tens of kV/cm. This is close to the breakdown potential. That is why in Refs.[4,6] a new simple scheme of saser with mechanical pumping has been proposed. In accordance with this scheme, the pumping can be achieved by a plane piezoelectric emitter of a piston type in the case of a rectangular resonator [4] or by radial mechanical pulsations of a cylinder in the case of cylindrical resonator [6].

In the saser the role of a laser mirror can be played by a wall of any material with acoustic impedance much greater (or much less) than that of the gas-liquid mixture. As has been shown in Refs.[1-6], the bubble bunching can be realised by means of well known acoustic radiation forces. These forces are similar to those which group the electrons in bunches in the FEL. In the saser, gas bubbles are grouped in planes in which their emision becomes coherent. Generation conditions for a saser were evaluated in Ref.[2]. It was shown that two types of losses must be overcome for generation to begin. The first type results from the energy dissipation in the active medium and the second one is caused by radiation losses at the boundaries of the resonator.

A further important step is made in Ref.[3] where the non-linear stage of saser operation and a saturation mode are investigated by numerical methods taking into account the role of bubble coagulation under the action of Bjorkens forces These forces can be important when the gas bubbles are grouped in coherent planes. It is well known that usual magnetostrictive and electrostrictive generators are working as generators of piston type with large spatial directionality. Fig.4 shows that the saser radiation is a set of many pistons or a phased array [3]. It is possible that the saser directional pattern will be narrow-beam. Moreover, the saser differs from the above systems particularly in that it is a three-dimension system because the whole volume of an active medium emits. It is new physical quality.

What does it mean 'saser'?

In optics, 'laser' (light amplification by stimulated emission of radiation) means a generator of coherent electromagnetic waves which has a narrow-beam directional pattern. Most of all, such interpretation is suitable for lasers operating as stationary generators. But, if lasers are working as generators of short pulses (for example, chemical single-pass lasers), than this interpretation leaves much to be desired. In fact, the length of short impulses may be of the same order as wavelength. In this case, what does coherency mean? In this case laser may be obviously defined as a generator of short impulses which has a narrow-beam directional pattern. It is clear that the last definition is more general than the first one. But, there is one additional very important matter. Laser is a device in which the mechanism of self-synchronisation of elementary emitters (for example, atoms in optical laser) is realised. In saser, small gas bubbles play the role of elementary emitters and the self-synchronisation takes place as well. Thus, saser may be defined as a sound generator which has a narrow-beam directional pattern and, the operation of which is based on the mechanism of self-synchronisation. This definition is very important for the following consideration because it can cause some misunderstanding. Let us imagine that we have a set of coherent small piezoelectric emitters. The number of them may be, for example, a few tens. These emitters may be synchronised but it is an artificial synchronisation, not self-synchronisation even if the directional pattern may be quite narrow-beam (if we increase this number then the directional pattern will improve). None the less, such a system is not a saser but only a phased array. It is clear that in the case of saser scheme suggested above there are millions of small gas bubbles which can never be synchronised artificially, but only using the mechanism of self-synchronisation. But one can ask, what are advantages of a saser with respect to a phased array? The answer will be given below. In addition, we will consider another example of a sound generator which could have a narrow-beam directional pattern (this scheme was suggested by Prof. F.V.Bunkin within one of our

discussions). Let us take a cylinder and wind it around by detonation flex. The velocity of detonation wave in flex is about 7.5 km/s. Let us select the turns (these system may not be a periodic one) from the condition of synchronisation of flex detonation wave velocity (in axial cylinder direction) with shock wave velocity in air (which is about 330 m/s). Thus, we will build a single-pass shock wave generator. But, it is clear that this device is not a saser as well. This is a source of directed explosion. It should be noted that using this device we may build a single-pass amplifier for shock waves. If we can synchronise a shock wave impulse with a start moment of detonation wave (in axial cylinder direction) i.e. at the entrance to this device then, it is evidently, we can obtain an amplification of this impulse at the exit.

Alternative schemes for saser

It should be noted that there is a number of papers in which theoretical schemes for an acoustic laser were suggested. We can distinguish, at least, four alternative approaches. First, the self-synchronisation (due to nonlinear effects) in a system of incoherent mechanical oscillators (monopoles, for example, gas bubbles in liquid) and the amplification of an acoustic field were considered by Kobelev et al [7]. Secondly, sound oscillations in a Helmholtz resonator with overcooled vapour were investigated by Kotusov and Nemtsov [8]. However, neither of these schemes have been realised experimentally because of the weak self-synchronisation mechanism. A third approach is developed by Prieur [9,10], Tucher [11], Hutson [12] at al. The active medium in this approach is a piece of solid (for example, pure silicon) at a temperature of 0.5 K. Authors of these works suggested phonon transitions to amplify sound pulses. However, such phonons have frequency of the order of tens GHz. At such the frequencies, phonons were absorbed very quickly in solid (thus to eliminate phonon absorption, all experiments were carried out at low temperatures). It is clear that such an approach is not useful to build a saser as a generator of shock waves, although the researches believe that, eventually, acoustic lasers will be used as sensitive particle detectors (New Scientist, 27 April 1996). Finally, the fourth approach very interesting for us, was developed by Prof. V.K.Kedrinskii et al. [13,14] who suggested the use of chemically active media to build an 'acoustic laser' (although they did not use this terminology). Kedrinsky's group would like to create a saser by means of so-called chemically active media, i.e. liquid (for example, water) with gas bubbles containing hydrogen - oxygen or hydrocarbon - oxygen mixture. A shock wave runs through this medium, compresses gas bubbles which take fire leading to the amplification of the shock wave amplitude and so on. Thus, we have a single-pass shock wave amplifier. In our opinion, this approach may be an alternative scheme of a saser (alike the scheme suggested by the present authors).

What are potential advantages of a saser ?

Let us consider some advantages of a saser with respect to other sources of sound. At the present time, five main types of acoustic generators exist: electrodynamic, electrostatic, magnetic, magnetostrictive and electrostrictive. The first three types are generally used in air while the latter two ones are applied in underwater acoustics because of their high mechanical self-impedance. In general, these generators act as two-dimension generators (piston type), because only a single two-dimension working surface effects emission in the medium. Besides, such types of generator have large spatial directivity. In addition, magnetostrictive and electrostrictive generators lack a discrete spectral distribution of sound emission. It is therefore of interest to create new generator types that do not suffer from these shortcomings. One possibility is the saser. First of all, it is envisaged that these enable large output powers for sound emission to be obtained due to the very high compressibility of gas bubbles (close to 1, as compared to usual piezoelectric crystals for which the maximum compressibility not resulting in damage is about 0.001). Secondly, in saser, under the action of both the pumping wave and a useful mode inside the resonator gas bubbles become grouped in planes (similarly to electrons in free-electron lasers) in which their emission becomes coherent . Thus, a saser acts as a phased array. Unlike the usual magnetostrictive and electrostrictive emitters (2-dimension working systems) the whole 3- dimension volume of the active medium emits. This is a new concept. Thirdly, the sound velocity in the gas-liquid mixture is a function of gas content. Using this fact, we can more easily change the frequency of saser emission than that of magnetostrictive and electrostrictive emitters. Fourthly, it is very difficult to build an effective generator of strong sound waves in air because of huge difference between the acoustic impedance of air and the self-impedance of the emitter. It is well known that by changing the gas volume content, we can easily obtain sound velocity in such a gas-liquid mixture to be less than the sound velocities both in pure liquid (without bubbles) and pure gas. At the first sight, this might seem strange, but a gas-liquid mixture is quite an unusual medium. Its density is almost completely determined by that of the liquid component, but its compressibility is determined only by that of the gas component (the liquid phase can be considered to practically incompressible). It is possible that one can adjust the impedance of air and self-impedance of the saser and, thus, built an effective generator of strong sound wave in air.

II. SCHEME OF THE ACOUSTIC LASER

Let us consider an acoustic resonator containing particles dispersed in a liquid dielectric as active medium. For example, we can use different types of oils or distilled water as a liquid dielectric. It is well known that the distilled water has a high dielectric constant. Static electric field acting on the system (3,Fig3) causes deformation (electrostriction) of the dielectric particles [16]. The value of the effective pressure acting on the particles is equal to [16,17]

$$\Delta P = \frac{3}{8\pi}\frac{\varepsilon_l E^2(\varepsilon_l - \varepsilon_p)}{(2\varepsilon_l + \varepsilon_p)} \qquad (1)$$

Here ε_l and ε_p are the dielectric constants for liquid and dispersed particles respectively, E is the electric field intensity. In the case of an air bubbles in water ($\varepsilon_p \approx 1, \varepsilon_l \approx 81$) and at an electric intensity $E = 10kV/cm$ the value of ΔP is of order of 0.5 kPa. Let us suppose that E is a periodic time function: $E = E_0 \cos(\Omega t)$. The electromagnetic waves propagate through the medium with the velocity of light (for this medium), which is

much greater than the velocity of sound. Consequently, the pumping pressure wave can be considered as being independent on the spatial co-ordinate,

$$P(t) = P_E \exp(i\omega t) \qquad (2)$$

Here, $\omega = 2\Omega$ is an angular frequency. The pressure amplitude of the pumping wave P_E can be easily calculated from (2). The constant term in (2) is omitted. Under the action of pumping wave, the particles oscillate and emit sound waves. The initial distribution of particles is spatially homogeneous. The waves created by the dispersed particles are summed with different phases and result zero pressure for the useful wave. However, for active medium in the resonator, an acoustic mode can appear. Then the particles would grouped to bunches by the acoustic radiation forces. Moreover, it is well known that the state of the medium with the spatially homogeneous bubble distribution is unstable not only for a steady but also for a traveling wave [18]. This leads to self-synchronisation of the oscillating particles and the amplification of a pressure wave.

III. PRINCIPAL EQUATIONS

For simplifications of calculations, we supposed that the dispersed particles are spherical. Their pulsation was investigated in numerous papers (see, for example, [18-20]). In a monopole approximation, the equation for the particle radius pulsation is:

$$R_l(t) = -\frac{A}{\rho_l R_0^2 \omega^2}[P_E \exp(i\omega t) + P(r,t)] \qquad (3)$$

The right hand side of this formula contains the resulting pressure on a particle. The first term corresponds to the pumping wave (2), the second one describes the pressure created by the oscillations of other particles; A is the scattering amplitude; r is the position vector of a particle in the liquid; R_0 is the mean particle radii; ρ_l is the liquid density. The monopole approximation holds true at the condition $k_l R_0 << 1$ (k_l is a wave number in the liquid). The case of a liquid with gas bubbles gives [19]

$$A = \frac{R_0}{(\omega_0 / \omega)^2 - 1 + i\delta} \qquad (4)$$

where $\omega_0 = \omega_0(R_0)$ is the resonance frequency of the bubble, δ is the absorption constant. The sound pressure wave $P'(r,t)$ is described by the known equation [18]

$$\Delta P - \frac{1}{c_l^2}\frac{\partial^2 P}{\partial t^2} = \rho_l \frac{\partial^2}{\partial t^2}\int_0^\infty 4\pi n(r, R_0, t) R_0^2 R_l(t)\, dR_0 \qquad (5)$$

where c_l is the sound velocity in the pure liquid (without particles), $n(r,R_0,t)$ is the particle size distribution function (n is equal to the number of the particles per unit liquid volume with mean radii between R_0 to $R_0 + dR_0$). Let us suppose that at t=0 the distribution of the particles is spatially homogeneous, i.e.,

$$n(r,R_0,0) = n_0(R_0) \qquad (6)$$

For sound pressure created by the external pumping, one can obtain

$$\Delta P - \frac{1}{c_l^2}\frac{\partial^2 P}{\partial t^2} - (\alpha + i\beta)P = (\alpha + i\beta)P_E \exp(i\omega t) \qquad (7)$$

where

$$\alpha = \alpha(r, t) = -4\pi \operatorname{Re} \int_0^\infty An(r, R_0, t)\,dR_0 \qquad (8)$$

$$\beta = \beta(r, t) = -4\pi \operatorname{Im} \int_0^\infty An(r, R_0, t)\,dR_0 \qquad (9)$$

In the case of the liquid with gas bubbles we have

$$\alpha = \alpha(r, t) = 4\pi \int_0^\infty \frac{R_0(1 - \omega_0^2/\omega^2)}{(1 - \omega_0^2/\omega^2)^2 + \delta^2} n(r, R_0, t)\,dR_0 \qquad (10)$$

$$\beta = \beta(r, t) = 4\pi \int_0^\infty \frac{R_0\delta}{(1 - \omega_0^2/\omega^2)^2 + \delta^2} n(r, R_0, t)\,dR_0 \qquad (11)$$

If the spatial distribution is invariable and homogeneous as time passed, then

$$P(r, t) = P_0(t) = \frac{(\alpha_0 + i\beta_0)P_E \exp(i\omega t)}{k_l^2 - (\alpha_0 + i\beta)} \qquad (12)$$

Here $\alpha_0 = \alpha(r,0), \beta_0 = \beta(r,0)$ are independent of r, and $k_l = \omega / c_l$. The resulting amplitude is also spatially homogeneous,

$$P_0(t) = P_E \exp(i\omega t) + P_0(t) = \frac{P_E \exp(i\omega t)}{1 - \left[\dfrac{\alpha_0 + i\beta_0}{k_l^2}\right]} \qquad (13)$$

The appearance of the factor

$$F = \left[1 - \frac{\alpha_0 + i\beta_0}{k_l^2}\right]^{-1} \qquad (14)$$

is caused by the presence of dispersed particles. The translation motion of the particle is given by the equation [21],

$$\frac{4\pi}{3}\left[\rho_l + \frac{1}{2}\rho_p\right]R_0^3 \frac{dU}{dt} = F_1 + F_2 + D + F_r + F_B \qquad (15)$$

The left part of this equation contains the usual mass of particle $m_p = \frac{4}{3}\pi\rho_p R_0^3$ (ρ_p is the density of particles) and apparent mass $m_l = \frac{2}{3}\pi\rho_l R_0^3$ (see for example, [22]): $F_1 = -4\pi(\rho_p + \frac{1}{2}\rho_l)R_0^2 U (dR_1/dt)$ is the drag force due to the particle volume oscillations (its time average $\langle F_1 \rangle = 0$); F_2 is the buoyant force which is small for small particles; D is the viscous drag force which for small Reynolds number $\mathrm{Re} = 2R_0 U\rho_l/\mu_l$ (μ_l is the liquid viscosity) is given by Stoke's low,

$$D = -6\pi\mu_l R_0 U f_v \qquad (16)$$

where f_v is the correcting factor which is given $f_v = 1$ for solid particles and $f_v = \frac{2}{3}$ for gas bubbles [22]; F_r is the time-average acoustic radiation force. The expression for F_r is very complicated but in the case being considered it can be represented as

$$F_r = -\frac{4\pi}{3}\langle R^3(t)\nabla P(r,t)\rangle f_r \qquad (17)$$

where $R(t) = R_0 + R_1(t)$ is the current particle radius, $P(r,t)$ is the resulting pressure acting on a particle, the numerical factor f_r is given as follows:

$$f_r = \frac{\left[1 + 2\dfrac{\rho_p}{\rho_l} - 3\dfrac{\rho_p^2 c_p^2}{\rho_l^2 c_l^2}\right]}{\left[1 + 2\dfrac{\rho_p}{\rho_l}\right]} \qquad (18)$$

F_B is the so-called secondary Bjerness force [23] which is caused by the interaction between particles (this force is created by the secondary radiation of the particles and is usually smaller with comparison to F_r). Substitution of all these terms into (15) and the time averaging gives the following equation:

$$\gamma U = -\alpha\nabla|P|^2 + i\beta(P^*\nabla P - P\nabla P^*) \qquad (19)$$

Here the functions α and β are given by the formulas (8) and (9) , respectively, $\gamma = 12\pi\mu_l R_0\rho_l\omega^2 N f_v / f_r$ where $N(r,t)$ is the full number of bubbles per unit volume of the liquid at the point r. To simplify the calculations, all particles are assumed to have equal radius, i.e.,

$$n(r, \tilde{R}_0, t) = N(r, t)\delta(\tilde{R}_0 - R_0) \qquad (20)$$

Equations (7) and (19) must be supplemented with the balance equation (we shall neglect the coagulation for particles)

$$\frac{\partial n}{\partial t} + \operatorname{div}(nU) = 0 \qquad (21)$$

IV. SOME SIMPLIFICATIONS OF EQUATIONS

Let us consider the equations

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nU) = 0$$

$$U = -\frac{\alpha}{\gamma}\frac{\partial |P|^2}{\partial x} + i\frac{\beta}{\gamma}\left(P^*\frac{\partial P}{\partial x} - P\frac{\partial P^*}{\partial x}\right)$$

where

$$\alpha = -4\pi \operatorname{Re}\int_0^\infty An(x, R_0, t)\,dR_0 = -4\pi n'(x,t)\operatorname{Re} A$$

$$\beta = -4\pi \,\mathrm{Im} \int An(x, R_0, t) dR_0 = -4\pi n'(x,t)\,\mathrm{Im}(A)$$

Neglecting by the terms of higher orders one can get:

$$U = -\frac{\alpha}{\gamma} P^* \frac{\partial P}{\partial x} - \frac{\alpha}{\gamma} P \frac{\partial P^*}{\partial x} + i\frac{\beta}{\gamma} P^* \frac{\partial P}{\partial x} - i\frac{\beta}{\gamma} P \frac{\partial P^*}{\partial x} = -i\frac{\alpha + i\beta}{\gamma} \rho \frac{\partial P^*}{\partial x} - \frac{\alpha - i\beta}{\gamma} P^* \frac{\partial P}{\partial x}$$

$$\approx -\frac{\alpha_0}{\gamma_0}\left\{P\frac{\partial P^*}{\partial x} + P^* \frac{\partial P}{\partial x}\right\}$$

Then the equation (21) takes the form

$$\frac{\partial n}{\partial t} - \frac{\alpha_0}{\gamma_0}\frac{\partial}{\partial x}\left(n\left[P\frac{\partial P^*}{\partial x} + P^*\frac{\partial P}{\partial x}\right]\right)$$

Representing the pressure as

$$P = P_0(t) + \Psi(x, t) \qquad (22)$$

we have obtained the following equation where unknown functions are $\Psi(z,t), n(z,t)$

$$\frac{\partial n}{\partial t} - \frac{\alpha_0}{\gamma_0}\frac{\partial}{\partial x}\left(n\frac{\partial}{\partial x}\left(P_0\Psi^* + P_0^*\Psi + \Psi\Psi^*\right)\right) = 0 \qquad (23)$$

Substituting (22) into (7) one can get the equation

$$\frac{\partial^2\Psi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} - (\alpha + i\beta)\Psi = (\alpha + i\beta)P_E e^{i\omega t} + (\alpha + i\beta)P_0(t) + \frac{1}{c^2}\frac{\partial^2 P_0}{\partial t^2}$$

and after simplifications one can find

$$\frac{\partial^2\Psi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} - (\alpha + i\beta)\Psi = (\alpha + i\beta)P_E e^{i\omega t} + (\alpha + i\beta)P_0(t) + \frac{1}{c^2}\frac{\partial^2 P_0}{\partial t^2} \qquad (24)$$

where the following substitution have been used $k = \frac{\omega}{c}$

$$\frac{\partial^2\Psi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} - (\alpha + i\beta)\Psi = (\alpha + i\beta)FP_E e^{i\omega t}$$

and given by

$$\alpha = \alpha_0\frac{n}{N_0}, \beta = \beta_0\frac{n}{N_0}$$

V. NUMERICAL SOLUTION OF THE EQUATIONS

Let us consider non-linear equation system of second order with complex coefficients:

$$\frac{\partial^2\Psi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} - (\alpha_0 + i\beta_0)\frac{N}{N_0}\Psi = (\alpha_0 + i\beta_0)\frac{N - N_0}{N_0}P_0$$

$$\frac{\partial N}{\partial t} = \frac{\alpha_0}{\gamma_0}\frac{\partial}{\partial x}\left(N\left(\frac{\partial}{\partial x}\left(P_0^*\Psi + P_0\Psi^* + \Psi\Psi^*\right)\right)\right), \qquad (25)$$

where $(x, t) \in \Omega$, $\Omega = [0, L] \times [0, \infty)$; L - resonator length; $\Psi(x, t)$-wave function ; $\Psi : \Omega \to C$;

$N(x, t)$ - particle density near x at moment t; $N : \Omega \to R$;

$P_0 = FP_E e^{i\omega t}$ - pressure created by external generator of frequency ω and intensity of P_E in the media characterised by F [24]; γ_0, ñ, α_0, β_0 R - are parameters [24].

Initial conditions are as follows:

$$\Psi(x, t)\big|_{t=0} = \Psi_0(x),$$

$$\frac{\partial\Psi(x, t)}{\partial t}\Big|_{t=0} = \Psi_1(x),$$

$$N(x, t)\big|_{t=0} = N_0(x) = \mathrm{const}.$$

Boundary conditions for function Ψ are as follows:

$$\frac{\partial\Psi(x,t)}{\partial x}\Big|_{x=0} = 0, \frac{\partial\Psi(x,t)}{\partial x}\Big|_{x=L} = 0.$$

After conversion the system (25) using the variable substitution as follows:

$$\Psi(x,t) = FP_E e^{i\tau}\varphi(\chi,\tau),$$

$$\frac{N(x,t)}{N_0} = \eta(\chi,\tau),$$

$$\kappa x = \chi, \omega t = \tau, \kappa = \frac{\omega}{c},$$

introducing rational variables φ_1, φ_2:

$$\varphi(\chi, \tau) = \varphi_1(\chi, \tau) + i\varphi_2(\chi, \tau),$$

and setting

$$\alpha_0' = \frac{\alpha_0}{\kappa^2}, \beta_0' = \frac{\beta_0}{\kappa^2}, q = \frac{\alpha_0\kappa FF^* P_E^2}{\gamma_0 c},$$

$$\Phi(\chi,\tau) = 2\varphi_1(\chi,\tau) + \varphi_1^2(\chi,\tau) + \varphi_2^2(\chi,\tau),$$

one can get the following system of three non-linear equations with unknown functions $\varphi_1(\chi, \tau)$, $\varphi_2(\chi, \tau)$, $\eta(\chi, \tau)$:

$$\frac{\partial^2\varphi_1}{\partial\chi^2} - \frac{\partial^2\varphi_1}{\partial\tau^2} + 2\frac{\partial\varphi_2}{\partial\tau} + \varphi_1 = \eta\alpha_0'\varphi_1 - \eta\beta_0'\varphi_2 + (\eta - 1)\alpha_0', \qquad (26)$$

$$\frac{\partial^2 \varphi_2}{\partial \chi^2} - \frac{\partial^2 \varphi_2}{\partial \tau^2} - 2\frac{\partial \varphi_1}{\partial \tau} + \varphi_2 = \eta\alpha'_0\varphi_2 + \eta\beta'_0\varphi_1 + (\eta - 1)\beta'_0, \qquad (27)$$

$$\frac{\partial \eta}{\partial \tau} = q\frac{\partial}{\partial \chi}\left(\eta \frac{\partial \Phi}{\partial \chi}\right), \qquad (28)$$

$\chi \in [0, \kappa L]$.

with following initial conditions:

$$\varphi_1(\chi,0) = \frac{\Psi_0}{FP_E}, \quad \varphi_2(\chi,0) = 0, \quad \frac{\partial \varphi_1(\chi,\tau)}{\partial \tau}\Big|_{\tau=0} = \frac{\Psi_1}{\omega FP_E},$$

$$\frac{\partial \varphi_2(\chi,\tau)}{\partial \tau}\Big|_{\tau=0} = -\frac{\Psi_0}{FP_E}.$$

and with following boundary conditions:

$$\frac{\partial \varphi_1(\chi,\tau)}{\partial \chi}\Big|_{\chi=0} = 0, \quad \frac{\partial \varphi_1(\chi,\tau)}{\partial \chi}\Big|_{\chi=\kappa L} = 0,$$

$$\frac{\partial \varphi_2(\chi,\tau)}{\partial \chi}\Big|_{\chi=0} = 0, \quad \frac{\partial \varphi_2(\chi,\tau)}{\partial \chi}\Big|_{\chi=\kappa L} = 0.$$

Computation were performed with next initial conditions for functions φ_1, φ_2:

$$\varphi_1(\chi,0) = 10^{-6}\cos(\chi);$$

$$\varphi_2(\chi,0) = 0;$$

$$\frac{\partial \varphi_1}{\partial \tau}(\chi,0) = 0;$$

$$\frac{\partial \varphi_2}{\partial \tau}(\chi,0) = 0.$$

Resonator length was choose as follows $L = \frac{2\pi}{\kappa}$. Discretisation of system (26)-(28) was performed using grids with step $\bar{\tau}$ for variable τ and with step h for variable χ:

$$\chi = hk, \quad k = 0..N$$

$$\tau = \bar{\tau}j, \quad j = 0...$$

We have used formulas of second order of accuracy to approximate derivatives of functions $\varphi_1, \varphi_2, \eta$ with variable χ and derivatives of functions φ_1, φ_2 with variables τ in the interval $(0, 2\pi)$ and the formulas of first order of accuracy to approximate derivatives of function η with variable τ in the interval $(0, 2\pi)$.

The initial values of functions φ_1' and φ_2' were approximated by formulas of second order of accuracy. Boundary conditions for function η one can obtain from

boundary conditions for functions φ_1 and φ_2. Boundary conditions obtained in such way are as follows:

$$\eta(0, \tau) = \exp\left(q\int_0^\tau \frac{\partial^2 \Phi}{\partial \chi^2}(0, \theta)d\theta\right),$$

$$\eta(kL, \tau) = \exp\left(q\int_0^\tau \frac{\partial^2 \Phi}{\partial \chi^2}(2\pi, \theta)d\theta\right)$$

If we replace the integrals in above expressions by their approximations calculated by trapezium formulas then we get boundary conditions of second order accuracy. Finally the following discrete equations were obtained:

$$\varphi^1_{k,j+1} - \bar{\tau}\varphi^2_{k,j+1} = A_{k,j}, \qquad (29)$$

$$\bar{\tau}\varphi^1_{k,j+1} + \varphi^2_{k,j+1} = B_{k,j}, \qquad (30)$$

$$\eta_{k,j+1} - q\frac{\bar{\tau}}{2h}\frac{\partial \Phi}{\partial \chi}\Big|_{k,j+1}\eta_{k+1,j+1} + q\frac{\bar{\tau}}{2h}\frac{\partial \Phi}{\partial \chi}\Big|_{k,j+1}\eta_{k-1,j+1} = \eta_{k,j}\left(1 + q\bar{\tau}\frac{\partial^2 \Phi}{\partial \chi^2}\Big|_{k,j+1}\right), \qquad (31)$$

j=1...,k=1...N-1,

where

$$A_{k,j} = \frac{\bar{\tau}^2}{h^2}\left\{\varphi^1_{k+1,j} - 2\varphi^1_{k,j} + \varphi^1_{k-1,j}\right\} + 2\varphi^1_{k,j} - \varphi^1_{k,j-1}$$
$$-\bar{\tau}\varphi^2_{k,j-1} + \bar{\tau}^2\varphi^1_{k,j} - \alpha'_0\eta_{k,j}\bar{\tau}^2\varphi^1_{k,j}$$
$$+\beta'_0\eta_{k,j}\bar{\tau}^2\varphi^1_{k,j} + \alpha'_0(\eta_{k,j} - 1)\bar{\tau}^2,$$

$$B_{k,j} = \frac{\bar{\tau}^2}{h^2}\left\{\varphi^2_{k+1,j} - 2\varphi^2_{k,j} + \varphi^2_{k-1,j}\right\} + 2\varphi^2_{k,j} - \varphi^2_{k,j-1}$$
$$+\bar{\tau}\varphi^1_{k,j-1} + \bar{\tau}^2\varphi^2_{k,j} - \alpha'_0\eta_{k,j}\bar{\tau}^2\varphi^2_{k,j}$$
$$-\beta'_0\eta_{k,j}\bar{\tau}^2\varphi^1_{k,j} - \beta'_0(\eta_{k,j} - 1)\bar{\tau}^2,$$

k=1...N-1,j=2...,

with the following initial conditions:

$$\varphi^1_{k,0} = 10^{-6}\cos(hk), \quad \varphi^2_{k,0} = 0,$$

$$\varphi^1_{k,1} = \varphi^1_{k,0}, \quad \varphi^2_{k,1} = \varphi^2_{k,0},$$

$$\eta_{k,0} = 1,$$

$$k = 0...N.$$

and with the following boundary conditions

$$4\varphi^1_{1,j} - \varphi^1_{2,j} - 3\varphi^1_{0,j} = 0,$$
$$4\varphi^2_{1,j} - \varphi^2_{2,j} - 3\varphi^2_{0,j} = 0,$$
$$4\varphi^1_{N-1,j} - \varphi^1_{N-2,j} - 3\varphi^1_{N,j} = 0,$$
$$4\varphi^2_{N-1,j} - \varphi^2_{N-2,j} - 3\varphi^2_{N,j} = 0,$$

$$\eta_{0,j+1} = \exp\left\{ q\left(0.5\bar{\tau}\frac{\partial^2\Phi}{\partial\chi^2}\Big|_{0,0} + \sum_{j'=1}^{jj}\bar{\tau}\frac{\partial^2\Phi}{\partial\chi^2}\Big|_{0,j'} + 0.5\bar{\tau}\frac{\partial^2\Phi}{\partial\chi^2}\Big|_{0,j+1} \right)\right\},$$

$$\eta_{N,j+1} = \exp\left\{ q\left(0.5\bar{\tau}\frac{\partial^2\Phi}{\partial\chi^2}\Big|_{N,0} + \sum_{j'=1}^{jj}\bar{\tau}\frac{\partial^2\Phi}{\partial\chi^2}\Big|_{N,j'} + 0.5\bar{\tau}\frac{\partial^2\Phi}{\partial\chi^2}\Big|_{N,j+1} \right)\right\},$$

where

$\Phi(\chi,\ \tau) = 2\varphi_1(\chi,\ \tau) + \varphi_1^2(\chi,\ \tau) + \varphi_2^2(\chi,\ \tau)$

A final equation system represents the three layers scheme in respect to pair functions $\varphi^1_{k,j}$, $\varphi^2_{k,j}$ and two layers scheme in respect to function $\eta_{k,j}$. The values of φ^1, φ^2 on j+1 -th level at k-th point was calculated using following system of equations:

$$\varphi^1_{k,j+1} - \bar{\tau}\varphi^2_{k,j+1} = A_{k,j},$$
$$\bar{\tau}\varphi^1_{k,j+1} + \varphi^2_{k,j+1} = B_{k,j},$$

The solutions of the system are the next:

$$\varphi^1_{k,j+1} = \frac{\Delta_1}{\Delta},\quad \varphi^2_{k,j+1} = \frac{\Delta_2}{\Delta},$$

where

$$\Delta = 1 + \bar{\tau}^2,$$

$$\Delta_1 = A_{k,j} + \bar{\tau}B_{k,j},$$

$$\Delta_2 = B_{k,j} - \bar{\tau}A_{k,j}.$$

The equation (29) was solved by factorisation method. So one needed the following steps to create the full procedure:

1) A given initial conditions allow one to calculate functions φ^1, φ^2 at 0-th and 1-th levels; using this values and values of the function η on 0-th level the values of η on 1-th level on boundary points one can successively calculate:
2) The values of η at the points k=1...N-1 on 1-th level;
3) The values φ^1, φ^2 on j+1-th level at the points k=1...N-1;
4) The values of φ^1, φ^2 on boundary points at j+1-th level;
5) The values of η at the boundary points on j+1-th level;
6) The values of η at the points k=1...N-1 on j+1-th level.

The constructed algorithm was tested on the model tasks with known exact solutions. A good agreement between exact and numerical solutions was obtained. The algorithm was used to obtain the solution of the formulated above task and investigation of the solution was performed using increasing density grids. The results were published in [25]

VI. NUMERICAL RESULTS

The constructed in this work procedure was realised using the following values for physical parameters:

$\alpha'_0 = 0.01$,

$\beta'_0 = 0.001$,

$q = 0.045$,

$\kappa L = 2\pi$,

A grid step of $\mathbf{h} = \frac{2\pi}{100}$ for x and a grid step of $\tau = \frac{\mathbf{h}}{12}$ for τ was used. Following problems were solved:

1.The task was solved with parameter PeF/Pst=10 and initial conditions $\varphi_1 = 10^{-6}\cos(x)$, $\varphi_2=0$. Values of the functions $\Phi(\chi,\ \tau)$ and $\eta(\chi,\ \tau)$ on layers numbers N=1000, 2000, 3000, 4000 ,5000 are represented in Fig. 2,3. The bunching is shown.

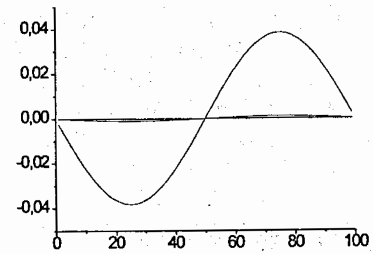

Fig.2 Values of the $\Phi(\chi,\ \tau)$ on the layers numbers numbers N=1000,2000,3000,4000 ,5000.

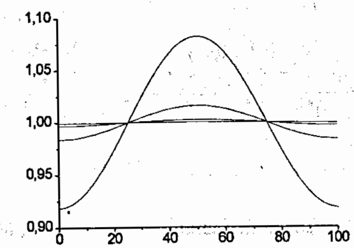

Fig.3. Values of $\eta(\chi,\ \tau)$ on the layers numbers N=1000,2000,3000,4000,5000.

2. The task was solved with parameter PeF/ P_{st} = 10, and initial conditions $\varphi_1=10^6\cos(3x)$, $\varphi_2=0$. A grid step of h=2pi/300 for x and a grid step of $\tau = \frac{\mathbf{h}}{12}$ for τ was used. The values of the function η at the layers 3000, 4500, 6000, 7500 are shown on the fig.4 . One can see that the bunching at this values of parameters don' t take place.

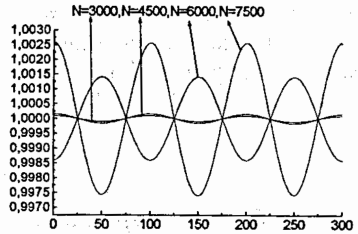

Fig.4. $\eta(\chi, \tau)$ on the layers numbers N=3000,4500,6000,7500.

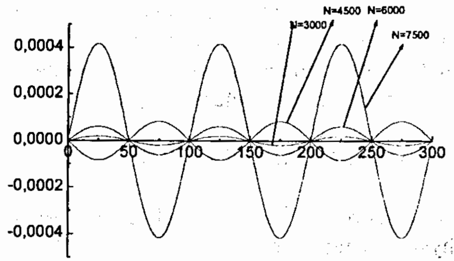

Fig.5. $\Phi(\chi, \tau)$ on the layers numbers N=3000,4500,6000,7500.

3. The task was solved on the following values of parameters: $P_e F/P_{st}=1$, and at the following initial conditions: $\varphi_1=\cos(x)$, $\varphi_2=0$. The results on the layers numbers N=100,200,300,400,500,600 are presented on the fig. 5. The bunching is shown. Due to lack of factor 10^{-6} nonlinear effect has appeared.

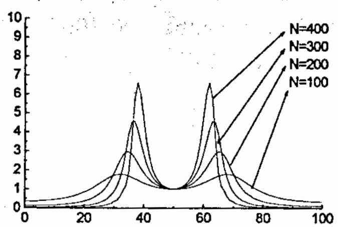

Fig.6 $\eta(\chi, \tau)$ on the layers numbers N=100,200,300,400.

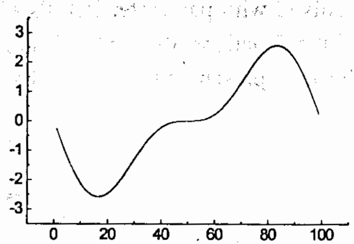

Fig.7. $\Phi(\chi, \tau)$ on the layers numbers N=100,200,300,400.

4. The task was solved on the following value of parameter $P_eF/P_{st}=0.5$, and initial conditions: $\varphi_1=\cos(3x)$, $\varphi_2=0$. The results on the layers numbers N=100,200, 300, 400, 500, 600 are presented on the fig. 8. The bunching is shown.

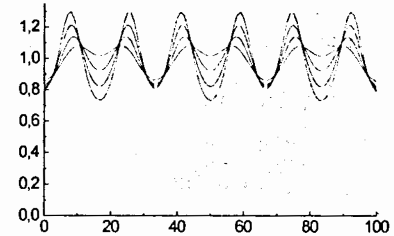

Fig.8. $\eta(\chi, \tau)$ on the layers numbers N=10,20,30,40.

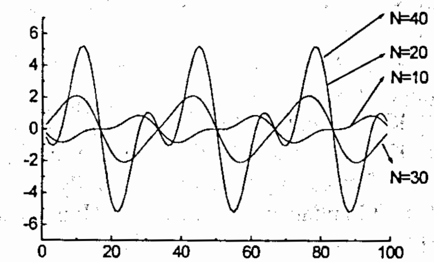

Fig.9. $\Phi(\chi, \tau)$ on the layers numbers N=10,20,30,40.

Considered examples show the bunching effect if saser parameters are in the definite range and absence of the bunching in other range. The bunching character also depends on initial conditions

This work was supported by RFFI grant 97-01-01040.

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