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$\mathcal{D}$-TRANSFORMATION AND POLYNOMIAL TRACK RECOGNITION

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$\mathcal{D}$-преобразование и полиномиальный фильтр системы распознавания трека

Описывается аппарат $\mathcal{D}$-преобразования, который предлагается применять при разработке системы распознавания треков, использующей клеточные автоматы и другие виды нейронных сетей. Известные свойства полиномиального фильтра дают надежду на улучшение некоторых характеристик аппарата распознавания трека, реализующего этот метод. Одним из основных достоинств применения $\mathcal{D}$-преобразования является получение всех характеристик искомой кривой одновременно с построением образа полиномиальной фильтрации.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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$\mathcal{D}$-Transformation and Polynomial Track Recognition

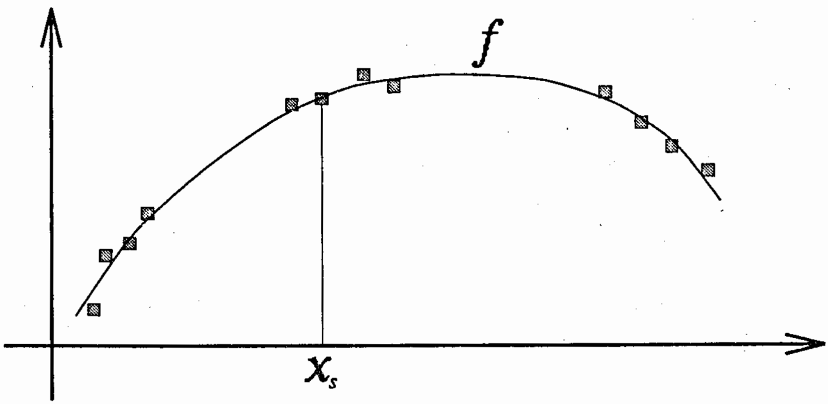
The apparatus of $\mathcal{D}$-transformation is described, which is offered to apply at engineering system of track recognition using cellular automatic devices and other kinds of neuron networks. The known properties of the polynomial filter give hope for improvement of some characteristics of the track recognition apparatus, realizing this method. One of the $\mathcal{D}$-transformation basic advantages is reception of all required curve characteristics simultaneously with construction of the polynomial filtration image.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 1996

We shall consider a polynomial construction problem, where the polynomial is approaching some set of experimental points in the best mean-square sense way. Number N designates quantity of experimental points, their meanings are set by numbers x_s, where $s = 1, 2, \ldots, N$, and the ordinates are designated by $\varphi(x_s)$. The approaching polynomial shall designate by the symbol f. It is convenient to introduce function of evasion

$$\chi^2\Big(\varphi, f\Big) = \sum_{s=1}^{N} \Big|\varphi(x_s) - f(x_s)\Big|^2 \quad \longrightarrow \quad \min.$$



In the space of polynomials which degree is not higher than n we shall consider basis $\{P_j\}_{j=0}^{n}$. In this basis any polynomial f is represented as

$$f(x) = \sum_{j=0}^{n} A_j \cdot P_j(x).$$

We consider, that this basis is formed by complete polynomial partition of the unit on dot basis $\{Y_j\}_{j=0}^{n}$.

By dot basis of dimension n we shall name the set of any $(n+1)$ various points $\{Y_j\}_{j=0}^{n}$. Complete polynomial partition of the unit on this dot basis is the such system $\{P_j\}_{j=0}^{n}$, that the following conditions are performed:

$$P_j\Big(Y_k\Big) = \begin{cases} 1, & j = k, \\ 0, & j \neq k. \end{cases}$$

The degree of this polynomial partition of the unit is equal to n.

The basic property of polynomial partition of the unit is the following identity

$$\sum_{j=0}^{n} P_j(x) \equiv 1.$$

Due to this identity the name of this construction was formed.

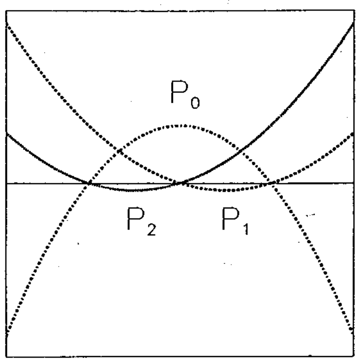
Any element of the complete partition of the unit over dot basis $\{ Y_j \}_{j=0}^{n}$ is written down as

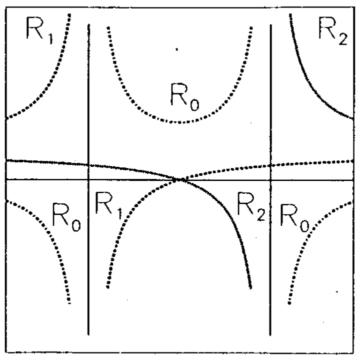
$$P_k(x) = \frac{\Pi_{j\neq k}(x - Y_j)}{\Pi_{j\neq k}(Y_k - Y_j)}, \quad j,\, k \in \{0,\, 1,\, \ldots,\, n\}.$$

Any complete polynomial partition of the unit forms corresponding complete rational partition of the unit. Any element of complete rational partition is determined from the following relations:

$$R_0(x) = \frac{1}{P_0(x)}, \qquad R_k(x) = \frac{-P_k(x)}{P_0(x)}, \quad k \neq 0.$$

The next drawings illustrate complete polynomial and rational partition of the unit on three–points dot basis.





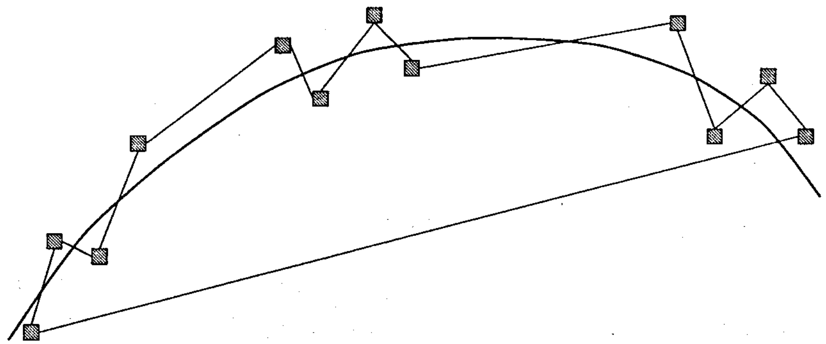
The $\mathcal{D}$–transformation of function f on dot basis $\{Y_j\}_{j=0}^{n}$ with parameters $\{F_j\}_{j=1}^{n}$ is determined by the following formula

$$\mathcal{D}\Big[\, f,\, \{F_j\}_{j=1}^{n}\, ;\, \{Y_j\}_{j=0}^{n} \Big](x) =$$

$$= \mathcal{D}[\, f\,](x) = f.(x) \cdot R_0(x) + \sum_{j=1}^{n} F_j \cdot R_j(x).$$

This method was designed by N. Dikoussar [1] in the case of three–points dot basis; multi–point variant of $\mathcal{D}$–transformation was developed by M. Nazarenko [2].

One of the methods of tracks recognition is the so-called rotor model of a neural network [3]. In this model any formal neuron is represented as individual vector, named as a rotor. Formal neuron is placed in experimental point, characterized in coordinate and direction, which reflects tangent direction of the track. Dynamic variable is the corner. These rotors cooperate into track by a chosen principle of affinity.



It is offered to use the $\mathcal{D}$–transformation apparatus for putting rotors into tracks. This transformation is a polynomial filter in the following sense. Let the dimension of dot basis equal to n. Any polynomial g, which degree is not higher than n, satisfying to relations

$$g(Y_j) = F_j, \qquad j \in \{1,\, \ldots,\, n\},$$

will be transformed to a constant. It is the basic property of the polynomial filter. The opportunity of work with nonlinear objects reflects more adequate peculiarities of researched object than earlier used linear models.

Parameters $\{F_j\}_{j=1}^{n}$ and the constant, which is the best approximation of $\mathcal{D}$–transformation value, are required values. The result of $\mathcal{D}$–transformation is obtained for one iteration and is steady to systematic polynomial smaller degree errors. The dot basis $\{Y_j\}_{j=0}^{n}$ is the object for optimization. The $\mathcal{D}$–transformation is represented in formal neuron basis, neural weight will form the appropriate partition of the unit. It is offered to apply $\mathcal{D}$–transformation at engineering system of tracks recognition, using cellular automatic devices and other kinds of neuron networks. The known properties of the polynomial filter give hope for improvement of some characteristics of the track recognition apparatus, realizing this method. One of the $\mathcal{D}$–transformation basic advantages is reception of all required curve characteristics simultaneously with construction of the polynomial filtration image.

References

[1] N. Dikoussar, *Comput. Phys. Commun.* 1994, **79**, 39.

[2] M. Nazarenko, *JINR Prerint* P5-96-151, Dubna, 1996.

[3] S. Baginyan, A. Glazov, I. Kisel, E. Konotopskaya, V. Neskoromnyi and G. Ososkov, *Comp. Phys. Comm.* 1994, **79**, 165.

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