сообщения
объединенного
института
ядерных
исследований
дубна

E11-94-509

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NUMERICAL INVESTIGATION OF SCHWINGER — DYSON AND BETHE — SALPETER EQUATIONS WITH GAUSS AND OSCILLATOR POTENTIALS IN THE FRAMEWORK OF THE QUARKONIUM MODEL

1994

INTRODUCTION

Recently, in ref.[1] we have obtained the numerical solutions to the Schwinger - Dyson (SD) and Bethe - Salpeter (BS) equations for the QCD quark potential model developed in refs.[2]. In accordance with this model the SD equation describes the constituent quarks and allows one to search the spontaneous chiral symmetry breakdown. The solutions of the BS equation (eigenvalues and eigenfunctions) describe the masses and the wave functions of the free mesons composed of the quarks. For the effective potential in these equations we have used the approximation of the Gauss function.

We have shown that there is a subtraction scheme for the SD equation that gives a self consistent description of the pion mass and its leptonic decay constant (F_π). However, for the mass of the pion radial excited states we have obtained the values that are significantly lower than the available data [3].

On the other hand, it is known that the number of the radial excited states of mesons was described in the model with oscillator potential (see, particularly, refs.[4]).

In the present work we attempt to describe the spectra of pion and its radial excited states using the effective potential as the sum of the Gaussian and oscillator ones, and some subtraction schemes as in ref.[1]. This effective potential has the following form:

$$V = V_G + V_O, \qquad V_G = v_g \exp\left(-\mu^2 r^2\right) + C, \qquad V_O = -v_o r^2. \tag{1}$$

Here C is a constant, $v_g > 0$, $\mu > 0$ and $v_o > 0$ are parameters of the potential. Using the Fourier transformations, we obtain the potential in the momentum space

$$V(|\vec{p} - \vec{q}|) = \frac{v_g}{(\sqrt{\pi})^3} R^3 \exp\left(-R^2|\vec{p} - \vec{q}|^2\right) + C(2\pi)^3\delta(|\vec{p} - \vec{q}|) -$$

$$- v_o(2\pi)^3 \Delta_{\vec{p}} \delta(|\vec{p} - \vec{q}|), \quad R = 1/(2\mu). \tag{2}$$

The problem is to select such parameters for the potential and the subtraction schemes that lead to the numerical results for the system of SD and BS equations that are consistent with experimental data.

In part 1 of this work the boundary value problem for SD equation is formulated and some ways for quark wave function renormalization are suggested. In part 2 the boundary value problem for BS equation and the normalization condition for the eigenfunctions are formulated. In part 3 the numerical methods for the problem solution are discussed. The numerical results for Gaussian, oscillator and Gauss-plus-oscillator potentials are analyzed in part 4.

Note that for the different types of effective potentials the SD and BS equations are reduced to the different types of nonlinear functional equations. The effective method of the numerical solution to these equations is the continuous analogue of the Newton's method [5]. Particularly, newtonian iterative schemes are successfully used for numerical investigation of SD and BS equations in refs.[1],[4],[6]. In our investigation the continuous analogue of the Newton's method is also used.

1. SCHWINGER - DYSON EQUATION

The SD equation for an arbitrary potential $V(|\vec{p}-\vec{q}|)$ can be written as the coupled equations [4]:

$$\begin{cases} E(p)cos(2v(p)) = m_0 + \frac{1}{2}\int d\vec{q}V(|\vec{p}-\vec{q}|)cos(2v(q))/(2\pi)^3 \\ E(p)sin(2v(p)) = p + \frac{1}{2}\int d\vec{q}V(|\vec{p}-\vec{q}|)\xi sin(2v(q))/(2\pi)^3, \end{cases} \qquad (3)$$

where the integration over the three - dimensional vector space of $\vec{q}$ is supposed, $\xi = (\vec{p}/p, \vec{q}/q)$ is the scalar product of unique vectors, m_0 is the given constant (the current quark mass). $E(p)$ and $v(p)$ are the quark energy and wave function, respectively, that should be founded.

Integrating over angle $\Omega\vec{q}$ in these equations, we arrive at

$$\begin{cases} E(p)cos(2v(p)) = m_0 + I_1 \\ E(p)sin(2v(p)) = p + I_2, \end{cases} \qquad (4)$$

where

$$I_1 = \int_0^\infty dqV_1(p,q)cos(2v(q)), \qquad (5)$$

$$I_2 = \int_0^\infty dqV_2(p,q)sin(2v(q)), \qquad (6)$$

$$V_1 = \frac{1}{2}\frac{1}{(2\pi)^3}q^2\int d\Omega V(|\vec{p}-\vec{q}|), \qquad (7)$$

$$V_2 = \frac{1}{2}\frac{1}{(2\pi)^3}q^2\int d\Omega\xi V(|\vec{p}-\vec{q}|). \qquad (8)$$

Substituting the effective potential (2) into eqs.(3)-(5) and turning to the dimensionless values by relations

$$\tilde{E} = E/\hat{\alpha}, \tilde{m}_0 = m_0/\hat{\alpha}, \tilde{p} = p/\hat{\alpha}, \tilde{q} = q/\hat{\alpha}, \tilde{R} = R\hat{\alpha}, \tilde{C} = C/\hat{\alpha}, \qquad (9)$$

where

$$\hat{\alpha} = [2v_g/(8\pi^3\sqrt{\pi})]/\beta, \quad \beta = (\frac{4}{3}v_o)^{\frac{1}{3}}, \qquad (10)$$

we obtain:

$$\begin{cases} (\tilde{E}(\tilde{p}) - \tilde{C}/2)cos(2v(\tilde{p})) = \tilde{m}_0 + \tilde{I}_1 \\ (\tilde{E}(\tilde{p}) - \tilde{C}/2)sin(2v(\tilde{p})) = \tilde{p} + \tilde{I}_2. \end{cases} \qquad (11)$$

Here

$$\begin{cases} \tilde{I}_1 = [(sin(\phi(\tilde{p})))'' + \frac{2}{p}(sin(\phi(\tilde{p})))'] + \tilde{J}_1, \\ \tilde{I}_2 = [(cos(\phi(\tilde{p})))'' + \frac{2}{p}(cos(\phi(\tilde{p})))' - \frac{2}{p^2}cos(\phi(\tilde{p}))] + \tilde{J}_2, \end{cases} \qquad (12)$$

$$\tilde{J}_1 = \hat{\alpha}\int_0^\infty d\tilde{q}\tilde{V}_1(\tilde{p},\tilde{q})cos(2v(\tilde{q})), \qquad (13)$$

$$\tilde{J}_2 = \hat{\alpha}\int_0^\infty d\tilde{q}\tilde{V}_2(\tilde{p},\tilde{q})sin(2v(\tilde{q})), \qquad (14)$$

$$\tilde{V}_1 = \tilde{R}\frac{\tilde{q}}{\tilde{p}}[\exp(-\tilde{R}^2(\tilde{p}^2+\tilde{q}^2))sh(2\tilde{R}^2\tilde{p}\tilde{q})], \qquad (15)$$

$$\tilde{V}_2 = \frac{1}{2}\frac{1}{\tilde{R}\tilde{p}^2}\{\exp(-\tilde{R}^2(\tilde{p}^2+\tilde{q}^2))[2\tilde{R}^2\tilde{p}\tilde{q}ch(2\tilde{R}^2\tilde{p}\tilde{q}) - sh(2\tilde{R}^2\tilde{p}\tilde{q})]\}, \qquad (16)$$

$$\phi(\tilde{p}) = -2v(\tilde{p}) + \pi/2, \qquad (17)$$

In future the symbol $\sim$ will be omitted.

Taking into account the asymptotical conditions for the energy function $E(p)$ and the mass function $m(p) = E(p)cos(2v(p))$:

$$\lim_{p\to\infty} E(p) = p, \qquad \lim_{p\to 0} E(p) = const, \tag{18}$$

$$\lim_{p\to\infty} m(p) = m_0, \qquad \lim_{p\to 0} m(p) = const, \tag{19}$$

using substraction schemes considered in ref.[1], we obtain the following modifications of the SD equation:

$$1) \quad \begin{cases} E(p)cos(2v(p)) = m_0 + I_1 \\ E(p)sin(2v(p)) = p + \hat{I}_2 \end{cases} \tag{20}$$

where

$$\hat{I}_2 = [(cos(\phi(p)))'' + \frac{2}{p}(cos(\phi(p)))' - \frac{2}{p^2}cos(\phi(p))].$$

$$2) \quad \begin{cases} E(p)(cos(2v(p)) - C^*) = m_0 + I_1 \\ E(p)(sin(2v(p)) - C^*) = p + I_2, \end{cases} \tag{21}$$

where

$$C^* = -\lim_{p_M\to\infty} \int\limits_0^{p_M} dq V_2(p_M, q) sin(2v(q)).$$

The asymptotical behaviour of functions $v(p)$ and $\phi(p)$ has the form:

$$\lim_{p\to 0} v(p) = 0, \lim_{p\to\infty} v(p) = \pi/4, \lim_{p\to 0} \phi(p) = \pi/2, \lim_{p\to\infty} \phi(p) = 0. \tag{22}$$

2. BETHE - SALPETER EQUATION

The BS equation for the pseudoscalar meson as the quark-antiquark bound state has been obtained in ref.[4], and can be written as following coupled equations:

$$M L_{\binom{2}{1}}(\vec{p}) = E_t(p) L_{\binom{1}{2}}(\vec{p}) - \tag{23}$$

$$- \int \frac{d\vec{q}}{(2\pi)^3} V(|\vec{p} - \vec{q}|)[C_p^{\binom{-}{+}} C_q^{\binom{-}{+}} + \xi S_p^{\binom{-}{+}} S_q^{\binom{-}{+}}] L_{\binom{1}{2}}(\vec{q}),$$

where

$$C_p^{\binom{+}{-}} = cos(v_1(p) \pm v_2(p)), \qquad S_p^{\binom{+}{-}} = sin(v_1(p) \pm v_2(p)),$$

v_1, v_2 and E_1, E_2 are solutions of the SD equation for quark and antiquark. $E_t(p) = E_1(p) + E_2(p)$ is the total energy of the meson. M is the eigenvalue (mass of the coupled state). $L_{\binom{1}{2}}$ is the wave function. The normalization condition is:

$$\frac{4N_C}{M} \int \frac{d\vec{q}}{(2\pi)^3} L_1(\vec{q}) L_2(\vec{q}) = 1. \tag{24}$$

where $N_C = 3$ is the quantum number.

Using the solutions of the system (23), we can calculate the lepton decay constant for the pseudoscalar mesons:

$$F_\pi = \frac{4N_C}{M} \int \frac{d\vec{q}}{(2\pi)^3} L_2(\vec{q}) cos(v_1(q) + v_2(q)). \tag{25}$$

Let us look for a solution of the system (23) in the form:

$$L_{\binom{2}{1}}(\vec{p}) = \frac{1}{p} \sum_{l,m} U_{\binom{2}{1} lm}(p) Y_{lm}(\theta, \phi). \tag{26}$$

where $Y_{lm}(\theta, \phi)$ are spherical functions.

Let us substitute (26) in eqs.(23),(24) for $l, m = 0$ (the spherically symmetrical case). Then for the potential (2) with $C = 0$ we obtain:

$$M U_{\binom{2}{1}}(p) + U''_{\binom{1}{2}}(p) + W_{\binom{1}{2}}(p) U_{\binom{1}{2}}(p) = \tag{27}$$

$$= -2 \int\limits_0^\infty dq [C_p^{\binom{-}{+}} C_q^{\binom{-}{+}} \hat{V}_1(p, q) + S_p^{\binom{-}{+}} S_q^{\binom{-}{+}} \hat{V}_2(p, q)] U_{\binom{1}{2}}(q).$$

$$\frac{4N_C}{M} \frac{1}{(2\pi)^3} \int\limits_0^\infty dq U_1(q) U_2(q) = 1. \tag{28}$$

where

$$W_1 = -\{E_t + \frac{1}{4}(\phi_1' + \phi_2')^2 + \frac{2}{p^2} cos^2(\frac{\phi_1 + \phi_2}{2})\}, \tag{29}$$

$$W_2 = -\{E_t + \frac{1}{4}(\phi_1' - \phi_2')^2 + \frac{2}{p^2} sin^2(\frac{\phi_1 - \phi_2}{2})\}, \tag{30}$$

$$\phi_i = -2v_i + \pi/2, \quad i = 1, 2, \tag{31}$$

$$U_{\binom{2}{1} 00} = U_{\binom{2}{1}}, \tag{32}$$

$$\hat{V}_1(p, q) = \frac{p}{q} V_1(p, q), \quad \hat{V}_2(p, q) = \frac{p}{q} V_2(p, q). \tag{33}$$

$V_1(p,q)$ and $V_2(p,q)$ are determined by relations (15) and (16).

For expression (25) we have:

$$F_\pi = \frac{4N_C}{M}\frac{1}{(2\pi)^3}\sqrt{4\pi}\int_0^\infty dqqU_2(q)cos(v_1(q)+v_2(q)). \qquad (34)$$

The solution of the system (27) must satisfy the asymptotic conditions:

$$\lim_{p\to 0} U_{\binom{1}{2}}(p) = 0, \qquad \lim_{p\to\infty} U_{\binom{1}{2}}(p) = 0. \qquad (35)$$

Thus, we have an eigenvalue problem for the system of two equations (27) with normalization condition (28) and boundary conditions (35). It should be noted again that in the BS problem the solutions (v_1, E_1) and (v_2, E_2) for the SD system for two quarks with nonzero masses m_{01} and m_{02} were included.

3. THE NUMERICAL SOLUTIONS

3.1 The SD equation

Let us rewrite the system (11)-(17) in the form:

$$F_1 + F_2 = 0, \qquad (36)$$

where

$$F_1 = \phi''(p) + \frac{2}{p}\phi'(p) + \frac{1}{p^2}sin(2\phi(p)) + 2m_0cos(\phi(p)) - 2psin(\phi(p)), \quad (37)$$

$$F_2 = J_1cos(\phi(p)) - J_2sin(\phi(p)), \qquad (38)$$

J_1 and J_2 are defined by eqs.(13),(14), respectively.

The energy function in this case can be determined from the following formulas:

$$E(p) = E_1(p) + E_2(p) = 0, \qquad (39)$$

where

$$E_1(p) = m_0sin(\phi(p)) + p\cos(\phi(p)) - \frac{1}{2}(\phi'(p))^2 - \frac{1}{p^2}cos^2(\phi(p)),$$

$$E_2(p) = \frac{1}{2}[J_1sin(\phi(p)) + J_2cos(\phi(p))].$$

Nq.(36)-(38) is the nonlinear integro-differential equation. The modified newtonian iterative scheme is developed for its numerical solving. The modification proposed in ref.[7] is used.

For $v_g = 0$ we have a nonlinear differential equation. The program complex developed in refs.[4] is applied in this case.

For $v_o = 0$ the SD equation reduces to the system of two nonlinear integral equations that is solved with the help of the algorithm described in ref.[1].

3.2 The BS equation

The system (27) is the eigenvalue problem for two linear integro-differential equations with the normalized condition (28). Numerical solution of this problem is performed by using the modified algorithm based on the continuous analogue of Newton's method, suggested in ref.[8].

For $v_o = 0$ the program complex SYSINT(SYSINTM) [9] is applied.

For $v_g = 0$ we obtain the eigenvalue problem for two linear differential equations. For its numerical solving the program complex developed in refs.[4] is used and the program SLIPS2 [10] as well.

3.3 Common program realization

For the numerical investigation of the meson spectra and the decay constant on the base of the equations from part 1 and 2 the unique packaged programs on FORTRAN-77 were developed.

The solutions of SD equation for two masses m_{01} and m_{02} are calculated in accordance with one of two modifications described in part 1.

For the given solutions of SD problem (v_1, E_1) and (v_2, E_2), corresponding to the masses m_{01} and m_{02}, the BS problem (27)-(33) is solved. Using eq.(34), the decay constant F_π is calculated and the relation $\Delta = M_\pi/F_\pi$ is determined. The dimensional values can be calculated using eqs.(9)-(10).

The aim of the present investigation is to find parameters for each potential that lead to the numerical results consistent with experimental data for the pion given in refs.[3]:

$$M_\pi = 137MeV, F_\pi = 132MeV, M_{\pi'} = 1300MeV, M_{\pi''} = 1770MeV.$$

At first, the parameters for the description of the ground state of pion (M_π and F_π) are determined. Then for these values of parameters the pion radially excited states ($M_{\pi'}, F_{\pi'}$ and $M_\pi, F_{\pi''}$) are defined.

The calculations were performed on VAX-8350, CONVEX C120, PC AT-386/486.

The precision of the numerical solution of the SD and BS equations depends on the parameter p_M when we replace the semiaxis $[0,\infty)$ of an integration on the finite interval $[0,p_M]$ as well as a step h of the discrete mesh on this interval. The precision of the obtained results was controlled by the numerical calculations for a sequence of the twice compressible grids $h, h/2, h/4$ and increasing intervals $[0,p_M]$.

4. NUMERICAL RESULTS

In this part the results of the calculations for the masses and leptonic decay constants of pion and its radial excitations are presented for the oscillator potential, the Gauss potential and the sum of these potentials.

Note that for all cases the parameters of the model are fixed by using the following relation

$$\Delta = \frac{M_\pi}{F_\pi} \simeq \frac{137}{132} \simeq 1.04, \tag{40}$$

corresponding to the experimental data for the pion ground state.

It should be pointed out that the including of the oscillator potential allowed to find the solutions with the node number $N \geq 0$ whereas in the case of a pure Gaussian potential only finite number of solutions has been obtained [1].

In Table 1 the mass spectra and leptonic decay constants of the ground and radially excited states of pion are estimated by using the first modification of the S-D equation with the Gauss potential calculated in ref.[1].

Table 1.(modification 1)
$m_0 = 0.1$, $m_o(MeV) = 31MeV$, $R^2 = 8.2$, $137/M_\pi = 311$.

	M	F	M_{MeV}	F_{MeV}	M_{MeV}/F_{MeV}
π	0.441	0.438	137	132	1.04
π'	0.988	0.058	307	18	18.1
π''	1.404	0.063	437	20	22.2

Numerical results calculated for the case of oscillator potential are presented in Table 2.

Table 2.
$m_0 = 0.00078$, $m_o(MeV) = 1MeV$, $v_o = 1$, $137/M_\pi = 1159$.

	M	F	M_{MeV}	F_{MeV}	M_{MeV}/F_{MeV}
π	.118	0.124	137	132	1.04
π'	5.54	0.004	6420	0.4	$1.6 \cdot 10^4$
π''	8.11	0.002	9404	0.2	$3.9 \cdot 10^4$

In Tables 3-4 numerical results obtained for the sum of the oscillator and Gauss potentials are given for both variants considered in part 1. In all the cases $v_o = 1$.

Table 3(a). (modification 1)
$m_0 = 0.1$; $m_o(MeV) = 13MeV$; $\alpha = 1.95$, $R^2 = 3.$, $137/M_\pi = 134$.

	M	F	M_{MeV}	F_{MeV}	M_{MeV}/F_{MeV}
π	1.024	0.999	137	132	1.04
π'	3.794	0.057	507	8	66.44
π''	6.140	0.059	828	8	104.79

Table 3(b). (modification 1)
$m_0 = 0.05$, $m_o(MeV) = 8MeV$, $\alpha = 1.7$, $R^2 = 3.$, $137/M_\pi = 150$.

	M	F	M_{MeV}	F_{MeV}	M_{MeV}/F_{MeV}
π	0.914	0.888	137	132	1.04
π'	3.916	0.050	587	8	73.84
π''	6.318	0.034	948	5	188.49

Table 4(a). (modification 2)
$m_0 = 0.1$, $m_o(MeV) = 18MeV$, $\alpha = 0.36$, $R^2 = 3.$, $137/M_\pi = 181$.

	M	F	M_{MeV}	F_{MeV}	M_{MeV}/F_{MeV}
π	0.757	0.753	137	132	1.04
π'	5.131	0.020	928	4	258.18
π''	7.694	0.040	1392	7	194.72

Table 4(b). (modification 2)
$m_0 = 0.05$, $m_o(MeV) = 9MeV$, $\alpha = 0.16$, $R^2 = 3.$, $137/M_\pi = 184$.

	M	F	M_{MeV}	F_{MeV}	M_{MeV}/F_{MeV}
π	0.745	0.705	137	132	1.04
π'	5.374	0.012	989	2	439.33
π''	7.937	0.025	1460	2	322.02

From the Tables one can see that the estimated values of the pion radial excitations presented in Tables 3-4, consistent better with experimental data than the results of Tables 1-2. However, these estimations are smaller than the available data, i.e. the contribution of the oscillator potential cannot provide a self-consistent description of the ground and radial excited states of pion. Another issue of this article is that the subtraction scheme strongly depends on the form of the potential and, therefore, can be considered together with one as some new effective potential.

Thus, the numerical investigation shows that the addition of the oscillator potential to the Gauss potential in the subtraction schemes applied in the case of a pure Gaussian potential cannot solve the problem of self-consistent description of the mass spectra and the decay constants of the pion and its radial excitations.

I.V.Amirkhanov, E.V.Zemlyanaya, I.V.Puzynin, T.P.Puzynina and T.A.Strizh are grateful to Russian Foundation for Fundamental Research RFFR (Grant 94 - 01 - 01119) for support. V.N.Pervushin thanks RFFR (Grant 94 - 02 - 14411) for support.

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Received by Publishing Department
on December 28, 1995.

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Численное исследование уравнений Швингера — Дайсона и Бете — Солпитера с гауссовским и осцилляторным потенциалами в рамках модели кваркония

Рассматривается потенциальная модель кваркония на основе уравнения Швингера — Дайсона для массовой функции кварков и Бете — Солпитера для связанных состояний кварков. Разработано алгоритмическое и программное обеспечение для численного исследования этой модели и получены решения указанных уравнений с осцилляторным и гауссовским потенциалами для пиона. Рассматриваются модификации уравнения Швингера — Дайсона. Показано, что модель в рассматриваемом приближении описывает массы и константы лептонного распада основного состояния пиона. Однако для радиально возбужденных состояний пиона получены оценки существенно ниже имеющихся экспериментальных данных.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 1994

Amirkhanov I.V. et al. E11-94-509

Numerical Investigation of Schwinger — Dyson and Bethe — Salpeter Equations with Gauss and Oscillator Potentials in the Framework of the Quarkonium Model

The potential model of quarkonium is considered on the basis of the Schwinger — Dyson equation for the quark mass function and the Bethe — Salpeter equation for the bound states. The software for numerical investigation of the model is constructed. Solutions to these equations for the effective potential as the sum of the Gauss and oscillator potentials for pion are obtained. Some modifications for the Schwinger — Dyson equation are considered. As shown, the considered approximation describes the mass and the leptonic decay constant of the pion ground state. However, for the masses of the pion radial excited states this scheme gives the estimations that are smaller than the available experimental data.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 1994