объединенный
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исследований
дубна

E11-92-338

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COMPUTER ALGEBRA, SYMMETRY ANALYSIS AND INTEGRABILITY OF NONLINEAR EVOLUTION EQUATIONS

Submitted to "Physics Computing´92 (4th International Conference on Computational Physics, Praha, 24-28 August, 1992)"

1992

1 Introduction

Partial differential equations of evolution type describe many natural processes and phenomena in different areas of physics: plasma physics, hydrodynamics, acoustics, quantum mechanics, nonlinear optics, etc.

There are two different groups of methods being used for solving nonlinear evolution equations (NEE) exactly. One is based on an explicit transformation from the nonlinear equation to a linear one. The other one uses the connection with an underlying isospectral linear eigenvalue problem. The well-known Burgers and Korteveg-de Vries (KdV) equations are prototypes of NEE solvable by the first and the second technique, respectively.

Exactly solvable, or integrable, NEE possess a number of remarkable properties reflecting their very rich internal algebraic structure: they have localized (soliton and multisoliton) solutions, Lax pair, infinite sequences of conserved quantities and higher or Lie-Bäcklund symmetries, nontrivial prolongation structures in Whalquist-Estabrook method, Bäcklund transformations, the Painlevé property, Hirota bilinear form, hereditary algebra, non-degenerated dispersion laws, etc.

It is very important to know whether a given NEE is integrable or not. To test any of above remarkable properties one has to perform, as a rule, very cumbersome algebraic transformations over mathematical expressions are generated by a given equation and to verify underlying algebraic identities. That is why modern computer algebra systems, such as Reduce, Maple, Mathematica, Macsyma, and Axiom are used now to develop a computer-aided test of integrability. A number of software packages written in different computer algebra languages have been created for verifying integrability of NEE (Table 1). Each package uses some criterion of integrability based on one or other remarkable property of integrable equations listed above.

Generally different criteria complement each other. However, for quasilinear (linear with respect to the highest spatial derivatives of dependent variables) evolution equations in one-spatial and one-temporal dimensions, the most efficient and constructive criterion is that one based on symmetry approach [12]. In this approach integrability conditions are nothing more than conditions of existence of high order conservation laws and symmetries.

Table 1. Computer tests of integrability.

Criterion	CAS	Equation	Author(s)	Comments
Lax pair	Reduce	$\vec{u}_t = \vec{F}[\vec{u}(x,t)]$	Ito M. [1] (1975)	Ansatz for $L - A$ pair
Whalquist-Estabrook method	Macsyma	$\vec{u}_t = \vec{F}[\vec{u}(\vec{x},t)]$	Reiman A. [2] (1979)	Closure of Lie algebra
	Reduce		Gragert P. [3] (1989)	Jacobi Identities & Closure of Lie algebra
	Reduce		Akselrod I., Gerdt V., Kovtun V., Robuk V. [4] (1991)	Construction of Lie algebra
Painlevé property	Macsyma	$F[\vec{u}(\vec{x},t)] = 0^{*)}$	Hereman W., Angenent S. [5] (1989)	Sufficient condition
	Reduce	$u_t = u_{kx} + f[u]^{*)}$ $u = u(x,t)$	Renner F. [6] (1991)	Interactive mode
Symmetry approach	Formac	$u_t = F[u(x,t)]^{*)}$	Gerdt V., Shvachka A., Zharkov A. [7] (1985)	Including Bäcklund transformation
		$\vec{u}_t = \Lambda\vec{u}_{kx} + \vec{f}[\vec{u}]^{*)}$ $\vec{u} = \vec{u}(x,t)$	Gerdt V., Shabat A. Svinolupov S., Zharkov A. [8] (1989)	$\Lambda = diag(\lambda_i)$ $\lambda_i \neq \lambda_j$ $(i \neq j)$ $\lambda_i \neq 0$
	mu-MATH	$\vec{u}_t = A(\vec{u})\vec{u}_{xx} + \vec{f}$ $\vec{f} = \vec{f}[\vec{u}]$ $\vec{u} = \vec{u}(x,t)$	Mikhailov A. (1988)	2 - component equations, unpublished
		$u_t = F[u(x,t)]$	Bakirov I., Sokolov V., Svinolupov S. (1990)	Unpublished
	DELiA		Bocharov A., Sokolov V., Svinolupov S. (1991)	Unpublished
	Reduce	$\vec{u}_t = \Lambda\vec{u}_{kx} + \vec{f}[u]^{*)}$ $\vec{u} = \vec{u}(x,t)$	Gerdt V., Zharkov A. [9] (1990)	$\Lambda = diag(\lambda_i)$ $\lambda_i \neq 0$
Hirota method	Reduce	$P_{ij}(D_{\vec{x}})f^i f^j = 0^{*)}$ $f^i = f^i(\vec{x},t)$	Hietarinta J. [10] (1991)	Three- and four-soliton solutions
Hereditary algebra	Maple	$u_t = F[u(x,t)]$	Wiwianka W., Fuchsstainer B. [11] (1990)	$F \in A(D,I,u)$ $D \equiv d/dx$ $I = D^{-1}$

$^{*)}$ Equation(s) may content arbitrary numeric parameters.

2 Symmetries and conservation laws

The symmetry approach allows to make in a completely algorithmic way the integrability analysis for the following form of NEE [8, 9, 13]

$$\begin{array}{l} \vec{u}_t = \vec{\Phi}(x,\vec{u},\ldots,\vec{u}_N) = \Lambda \vec{u}_N + \vec{F}(x,\vec{u},\ldots,\vec{u}_{N-1};\alpha_1,\ldots,\alpha_K), \quad \vec{u} = \vec{u}(x,t) \\ \vec{u}_i = D^i(\vec{u}), \quad D = d/dx, \vec{u} = (u^1,\ldots,u^M), \quad \vec{\Phi} = (\Phi^1,\ldots,\Phi^M), \\ \Lambda = diag(\lambda_1,\ldots,\lambda_M), \quad \lambda_i \alpha_j \in \mathrm{C}, \quad \lambda_i \neq 0, \quad \lambda_i \neq \lambda_j \ (i \neq j), \end{array} \tag{1}$$

where the nonlinear part $\vec{F}$ in r.h.s. is a polynomial in all its variables.

To be integrable, (1) must possess infinitely many time-independent higher symmetries, i.e. vector-functions $\vec{H} = (H^1,\ldots,H^M)$ in a finite number of differential variables $x,u,u_1,\ldots,u_n$ such that NEE $\vec{u}_\tau = \vec{H}(x,\vec{u},\vec{u}_1,\ldots,\vec{u}_n)$ is compatible with (1). In practice, however, the existence of M different higher symmetries of the order $n > N$ is sufficient for integrability of M-component NEE [14]. Symmetry $\vec{H}$ satisfies the differential equation $d\vec{H}/dt = \Phi_*(\vec{H})$ which is equivalent to the operator relation

$$\frac{dH_*}{dt} - [H_*,\Phi_*] = \frac{d\Phi_*}{d\tau}, \tag{2}$$

where Φ_* and H_* are matrix differential operators

$$\Phi_* = \sum_{i=0}^{N} \Phi_i D^i, \ \ [\Phi_i]_{kj} = \partial\Phi^k/\partial u^j_i, \ \ H_* = \sum_{i=0}^{n} H_i D^i, \ \ [H_i]_{kj} = \partial H^k/\partial u^j_i.$$

Integrability, i.e. solvability of (2) for as high order of $\vec{H}$ as desired, is equivalent [12] to solvability of the operator relation

$$L_t - [\Phi_*, L] = 0 \tag{3}$$

in terms of formal series

$$L = \sum_{k=-\infty}^{m} A_k D^k, \quad m > 0, \quad deg(L) \equiv m, \quad res(L) \equiv A_{-1}, \tag{4}$$

with matrix $M \times M$ coefficients A_k depending on a finite number of differential variables $x, \vec{u}, \vec{u}_1, \ldots$ and such that $det[A_m] \neq 0$.

The necessary integrability conditions have the form of conservation laws [8, 9]

$$\frac{d}{dt}(R(i,j)) \in ImD, \quad i \geq 0, \ j = 1,2,\ldots,M, \tag{5}$$

where $q \in Im\,D$ means that $q = D(\sigma(x,u,\ldots,u_k))$ and densities $R(i,j)$ are determined by formulas

$$R(i,j) = \begin{cases} \partial F^j/\partial u^j_{N-1}, & i = 0, \\ \partial/\partial\mu_j \left[trace(res(L))\right], & i > 0. \end{cases} \tag{6}$$

Here L is series (4) of degree $m = i$. Its coefficients can be computed by recurrence relations, which follow from (1),(3) and (4)

$$\begin{array}{ll} k > m & A_k = 0, \\ k = m & [A_m]_{ij} = \begin{cases} 0 & i \neq j, \\ \mu_i & i = j, \end{cases} \\ k < m & [A_k]_{ij} = \begin{cases} \frac{1}{\lambda_i - \lambda_j}\left\{\frac{d}{dt}([A_{N+k}]_{ij}) - [c_{N+k}]_{ij}|_{A_k=0}\right\}, \quad i \neq j \\ \frac{1}{N\lambda_i} D^{-1}\left\{\frac{d}{dt}([A_{N+k-1}]_{ii}) - [c_{N+k-1}]_{ii}|_{diag(A_k)=0}\right\} + \gamma^k_i, \\ i = j \end{cases} \end{array} \tag{7}$$

where $\mu_i, \gamma^k_i \in C$ are arbitrary constants and c_i are coefficients of commutator $[F_*, L] = \sum c_i D^i$. In addition to computation of conservation laws (6), recurrence relations (7) form the basis for computation of the m-order symmetry as a solution of (2) [8, 13].

3 Algorithm description

Heuristically optimal computational strategy of integrability analysis consists of the following three sequential steps [13]:

I. Verification of some first I necessary integrability conditions (5) ($i \leq I$) by the following procedure

$$\begin{array}{ll} 1) & i := 0 \\ 2) & for\ j := 1\ to\ M\ do\ R(0,j) := \partial F^j/\partial u^j_{N-1} \\ & \%\ computation\ of\ the\ firstdensities\ by\ formula\ (6) \\ 3) & if\ \frac{d}{dt}(R(i,j)) \in ImD\ then\ go\ to\ 4)\ else\ STOP \\ & \%\ testing\ conditions\ (5) \\ 4) & i := i + 1 \\ 5) & for\ k := i\ to\ -1\ step\ -1\ do\ compute\ A_k(j,l) \\ & \%\ computation\ of\ coefficients\ of\ series\ (4)\ by\ formulas\ (7) \\ 6) & R(i,j) := \partial/\partial\mu_j\, trace(A_{-1}) \\ & \%\ computation\ of\ densities\ by\ formula\ (6) \\ 7) & if\ i < I\ then\ go\ to\ 4)\ else\ STOP \end{array} \tag{8}$$

In the presence of the arbitrary parameters α_i in (1) conditions (5) are equivalent to some system of nonlinear algebraic equations in those parameters which can be produced by a completely automatic way [8, 9, 13].

II. Construction of a given order higher symmetry. Here in addition to the necessary conditions (5) one has to verify new ones which provide the existence of the symmetry and may lead to additional algebraic equations in arbitrary parameters α_i [13].

III. Solving the resulting system of the nonlinear algebraic equations obtained at steps I-II. An effective algorithmic method for doing this step is described in the next section.

4 Solving nonlinear algebraic equations

The systems of nonlinear algebraic equations which arise as integrability conditions for multiparametric NEE usually have infinitely many solutions. It follows from the fact that scale transformations of both independent and dependent variables in (1) do not impact at the property of integrability.

Therefore, to investigate and exactly solve those algebraic equations one needs an appropriate technique. We use that one [15] based on a Gröbner basis construction [16] for the polynomial ideal generated by the set of polynomials of the system under consideration. This technique allows in a completely algorithmic way to obtain the following information on the algebraic system:

- To verify its compatibility, i.e. the existence of common roots.
- To find the dimension of the solution space (algebraic variety) or, in other words, the dimension of the polynomial ideal generated by the algebraic system.
- To detect whether the system has finitely or infinitely many solutions.
- In finitely many solutions case (zero-dimensional ideal) to transform the system into an equivalent "triangular" form and therefore to reduce the initial multivariate problem to successive solving univariate equations.
- In infinitely many solutions case (positive dimensional ideal) to find all the maximal sets of (algebraically) independent variables modulo polynomial ideal [17] which could be considered as free parameters giving the parametrization of the solution space.

In addition to these general facilities in the framework of Gröbner basis technique one can effectively take into account such a property of polynomial systems resulting from integrability conditions as homogeneity [15] which follows from the above mentioned scale invariance. It leads to reduction a number of variables involved in Gröbner basis computation and makes the procedure much more efficient with respect to computing time and readability of output by splitting the initial systems into smaller subsystems with a reduced number of variables.

5 Implementation in Reduce: packages HSYM and ASYS

The algorithmic approaches of sections 3 and 4 have been implemented in the form of packages HSYM (Higher SYMmetries) [9, 13] and ASYS (Algebraic SYStems) [15], respectively, both written in the symbolic mode language Rlisp of computer algebra system Reduce [18].

Package HSYM uses the built-in recursive representation of polynomials in the Reduce "standard form". Algebraic operation over differential polynomials are realized by calls to the corresponding built-in procedures acting at "standard forms" and "standard quotients". Main procedures of the package are those ones which compute series (4), densities $R(i,j)$ by formulas (6), symmetries $\vec{H}$ as solution of (2) and reverse operator of total spatial derivative D. The latter operation (D^{-1}) plays the key role in verification of the necessary integrability conditions (5 as well as in recurrence relations (7). In both these cases the problem is reduced to solvability equations of the form $D(Q) = S \Leftrightarrow Q = D^{-1}S$ in terms of local functions, i.e. functions in a finite number of differential variables $x, \vec{u}, \vec{u}_1, \ldots$. Namely at the step of D^{-1} computation there arise restrictions on the r.h.s. of NEE (1), in particular, nonlinear algebraic equations in arbitrary parameters α_i if they are.

Package ASYS provides a user with all the set of facilities mentioned in section 4:

- Gröbner basis constructing by Buchberger algorithm [16];
- determination of the dimension of a solution space for a given polynomial system, computation of all sets of independent variables and reduction by these sets to zero-dimensional subsystems treating the independent variables as free parameters;
- verification of homogeneity properties and carrying out homogeneity reduction to a set of subsystems with reduced number of variables.

Unlike the HSYM package, the basic recursive polynomial representation used in REDUCE does not provide reasonable efficiency of a Gröbner basis construction. By this reason the ASYS package much like the REDUCE standard package GROEBNER [18] uses the distributive representation.

6 Examples

We illustrate the above technique and computer algebra software at two different examples: one gives integrability verification of the given equation and another belongs to a typical classification problem when all the integrable cases should be selected from some multiparametric family of NEE.

Example 1. In paper [19] the second-order NEE of the form (1)

$$u_t = u_2 + 2\,u^3 \tag{9}$$

was derived for which one remained to be investigated whether or not it is integrable. Verification of the first integrability condition from the canonical series (5) with the help of the HSYM package immediately shows its violation. Density $\rho_1 \equiv R(1,1)$ in formula (6) and its time derivative computed with HSYM are $\rho_1 = -3\,u^3$, $d\rho_1/dt = 6\,u_2\,u - 12\,u^4$. One can easily see that the latter expression does not satisfy condition (5). By this reason HSYM, doing all underlying computations completely automatically, will give as an output line the message

"non-integrable expression",

which means that equation (9) is not integrable.

Example 2. Seven-parametric family of the seven-order KdV-like NEE

$$u_t = u_7 + \lambda_1 u\, u_5 + \lambda_2 u_1 u_4 + \lambda_3 u_2 u_3 + \lambda_4 u^2 u_3 + \lambda_5 u\, u_1 u_2 + \lambda_6 u_1^3 + \lambda_7 u^3 u_1, \tag{10}$$

where $\lambda_i \in C$. Family (10) was first investigated in [20]. Here we give the complete solution in accordance with the scheme of section 3. Conditions (5) for $i = 1,3,5,7$ (all conditions with even i are identically satisfied for arbitrary λ_i) generate the following system of nonlinear algebraic equations

$$\begin{aligned}
&\lambda_1(\lambda_4 - \lambda_5/2 + \lambda_6) = (2/7\lambda_1^2 - \lambda_4)(-10\lambda_1 + 5\lambda_2 - \lambda_3) = 0,\\
&(2/7\lambda_1^2 - \lambda_4)(3\lambda_4 - \lambda_5 + \lambda_6) = 0,\\
&a_1(-3\lambda_1 + 2\lambda_2) + 21a_2 = a_1(2\lambda_4 - 2\lambda_5) + a_2(-45\lambda_1 + 15\lambda_2 - 3\lambda_3) = 0,\\
&2a_1\lambda_7 + a_2(12\lambda_4 - 3\lambda_5 + 2\lambda_6) = b_1(2\lambda_2 - \lambda_1) + 7b_2 = b_1\lambda_3 + 7b_2 = 0,\\
&b_1(-2\lambda_4 - 2\lambda_5) + b_2(2\lambda_2 - 8\lambda_1) + 84b_3 = 0, \qquad\qquad (11)\\
&b_1(8/3\lambda_5 + 6\lambda_6) + b_2(11\lambda_1 - 17/3\lambda_2 + 5/3\lambda_3) - 168b_3 = 0,\\
&15b_1\lambda_7 + b_2(5\lambda_4 - 2\lambda_5) + b_3(-120\lambda_1 + 30\lambda_2 - 6\lambda_3) = 0,\\
&-3b_1\lambda_7 + b_2(-\lambda_4/2 + \lambda_5/4 - \lambda_6/2) + b_3(24\lambda_1 - 6\lambda_2) = 0,\\
&3b_2\lambda_7 + b_3(40\lambda_4 - 8\lambda_5 + 4\lambda_6) = 0, \qquad\qquad (12)
\end{aligned}$$

where

$$\begin{aligned}
&a_1 = -2\lambda_1^2 + \lambda_1\lambda_2 + 2\lambda_1\lambda_3 - \lambda_2^2 - 7\lambda_5 + 21\lambda_6,\quad a_2 = 7\lambda_7 - 2\lambda_1\lambda_4 + 3/7\lambda_1^3,\\
&b_1 = \lambda_1(5\lambda_1 - 3\lambda_2 + \lambda_3),\quad b_2 = \lambda_1(2\lambda_6 - 4\lambda_4),\quad b_3 = \lambda_1\lambda_7/2\,.
\end{aligned}$$

By means of the ASYS package the solution space is found to be three-dimensional one [15] with a complete set of solutions collected in Table 2.

Table 2. Parametrization of the solution space for system (11).

Parameters	Solutions
λ_1	$\lambda_2 = 7/2\lambda_1, \lambda_3 = 6\lambda_1, \lambda_4 = 2/7\lambda_1^2, \lambda_5 = 9/7\lambda_1^2, \lambda_6 = 5/14\lambda_1^2, \lambda_7 = 4/147\lambda_1^3$
	$\lambda_2 = 3\lambda_1, \lambda_3 = 5\lambda_1, \lambda_4 = 5/14\lambda_1^2, \lambda_5 = 10/7\lambda_1^2, \lambda_6 = 5/14\lambda_1^2, \lambda_7 = 5/98\lambda_1^3$
	$\lambda_2 = 2\lambda_1, \lambda_3 = 3\lambda_1, \lambda_4 = 2/7\lambda_1^2, \lambda_5 = 6/7\lambda_1^2, \lambda_6 = 1/7\lambda_1^2, \lambda_7 = 4/147\lambda_1^3$
λ_2	$\lambda_1 = 0, \lambda_3 = 5\lambda_2, \lambda_4 = \lambda_5 = -2/63\lambda_2^2, \lambda_6 = 4/63\lambda_2^2, \lambda_7 = -10/1323\lambda_2^3$
λ_4	$\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_5 = \lambda_4, \lambda_6 = -2\lambda_4, \lambda_7 = 0$
λ_2, λ_4	$\lambda_1 = 0, \lambda_3 = 5\lambda_2, \lambda_5 = 1/14\lambda_2^2 + 9/2\lambda_4, \lambda_6 = 1/14\lambda_2^2 + 3/2\lambda_4, \lambda_7 = 0$
λ_3, λ_6	$\lambda_1 = 0, \lambda_2 = r_1, \lambda_4 = 0, \lambda_5 = -2/161r_1\lambda_3 + 40/23\lambda_6, \lambda_7 = 10/25921r_1\lambda_3^2 - 74/483r_1\lambda_6 - 16/3703\lambda_3\lambda_6, \quad r_1 = (5\lambda_3 \pm \sqrt{25\lambda_3^2 - 5152\lambda_6})/46$
	$\lambda_1 = \lambda_2 = \lambda_4 = \lambda_5 = \lambda_7 = 0$
$\lambda_2, \lambda_3, \lambda_5$	$\lambda_1 = 0, \lambda_4 = 0, \lambda_6 = 1/21\lambda_2^2 + 1/3\lambda_5, \lambda_7 = 0$

Verification of the existence of conservation laws (5) with numbers $i = 9, 11$ and higher symmetries for the solutions of Table 2 with package HSYM shows that only top three rows correspond to integrable equations (10). Those three equations are none others than the seven-order symmetries of the well-known low order NEE: the third-order KdV equation and the fifth-order Sawada-Kotera and Kaup-Kupershmidt equations, respectively [20, 21].

All computations took about 1 minute on a 25 Mhz 80386 DOS computer.

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Received by Publishing Department
on August 4, 1992.

Гердт В.П. E11-92-338
Компьютерная алгебра, симметрийный анализ
и интегрируемость нелинейных эволюционных уравнений

Представлены компьютерно-алгебраические аспекты симметрийного подхода к исследованию интегрируемости полиномиально-нелинейных эволюционных уравнений размерности 1+1. Подход основан на проверке существования законов сохранения и симметрий высших порядков. Если исследуемые уравнения содержат произвольные числовые параметры, тогда задача отбора всех интегрируемых случаев сводится к решению систем нелинейных алгебраических уравнений на имеющиеся параметры. Для этой цели использована техника базисов Гребнера, позволяющая эффективно упрощать и решать возникающие алгебраические уравнения, которые имеют, как правило, бесконечное множество решений.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1992

Gerdt V.P. E11-92-338
Computer Algebra, Symmetry Analysis
and Integrability of Nonlinear Evolution Equations

A computer algebra-aided symmetry approach to investigating integrability of polynomial-nonlinear evolution equations in one-temporal and one-spatial dimensions is presented. The approach is based on verifying the existence of higher conservation laws and symmetries. If the equations contain arbitraty numerical parameters, the problem of selection of all the integrable cases is reduced to the solving polynomial equations in those parameters. The Gröbner basis technique is used in order to simplify and to solve such polynomial systems which typically have infinitely many solutions.

The investigation has bee performed at the Laboratory of Computing Techniques and Automation, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1992