сообщения
объединенного
института
ядерных
исследований
дубна

E11-91-39

S.N.Dimova, D.I.Ivanova

FINITE ELEMENT METHOD WITH SPECIAL MESH REFINEMENT FOR ANALYSIS OF SINGLE POINT BLOW-UP SOLUTIONS

1991

Introduction. Computational experiment was undertaken in order to analyze the asymptotic behaviour near the finite blow-up time T_0 of the solutions of the initial value problem

$$(1) \qquad u_t = \frac{1}{r^{N-1}}(r^{N-1}u_r)_r + F(u) \quad \text{for } r \in \mathbb{R},\ t>0,$$

$$(2) \qquad u(r,0) = u_0(r) \geq 0 \quad \text{for } r \in \mathbb{R}.$$

This work is a continuation of paper [3], where the case

$$(3) \qquad F(u) = (1+u)\ln^{\beta}(1+u)$$

was considered. We were not perfectly satisfied from the numerical results for the case of a single point blow up ($\beta>2$). So we decided to improve our algorithm, to test it for the case (3) and to apply it to two other problems (1),(2) with

$$(4) \qquad F(u) = (A + u)^{\beta}, \quad \beta>1, \quad A\geq 0,$$

$$(5) \qquad F(u) = e^{u},$$

when the single point blow-up takes place too.

There are some works [1],[7],[8],[10],[15] where the asymptotic behaviour of the blow-up solutions of (1),(2),(4),(5) was analyzed numerically. We will mention here [1], where a rescaling algorithm for the case $F(u) = u^{\beta}$, N=1 was proposed and realized. It is based on a scale invariance of equation (1). By using the forward Euler finite difference scheme multiple grids, rescaling and refining only where the solution is large, they rich amplitudes of the solution u of the order of 10^{12} without fatal loss of accuracy.

Our algorithm, as it is in [3], is based on the finite element method (FEM) in space and on an explicit second order accurate in time scheme. Here we propose and realize a special mesh refinement, which is consistent with the space-time structure of the approximate self-similar solution (a.s.-s.s.) of the problems (1),(2),(3),(4),(5). More exactly, we refine the mesh in r so, that the step-length in the similarity variable ξ to be uniform and not greater than a given value h_{ξ} for every $t>0$.

Preliminaries. It is well known [12],[10] that the problem (1),(4) has not exact blowing-up self-similar solutions for N=1,2 and $\beta\leq(N+2)/(N-2)$, $N\geq3$. The same fact takes place for the problem (1),(5), N=1,2 [2],[10]. Herrero and Velazquez [14] have proved that if $u(r)$ blows up in finite time T_0, if $u_0(r)$ has a single maximum at $r=0$ and $u_0(r) = u_0(-r)$ for $r>0$, there holds for $N=1$:

If $F(u) = u^\beta$ with $\beta>1$, then

$$\lim_{t\to T_0} u(\xi((T_0-t)|\ln(T_0-t)|)^{1/2},t)\ (T_0-t)^{1/(\beta-1)} = \theta_a(\xi)$$

(6) $$\theta_a(\xi) = \{\beta-1 + [(\beta-1)^2/(4\beta)]\xi^2\ \}^{-1/(\beta-1)}$$

uniformly on compact sets $|\xi| \le \xi^*$ with $\xi^*>0$;

If $F(u) = e^u$, then

$$\lim_{t\to T_0} u(\xi((T_0-t)|\ln(T_0-t)|)^{1/2},t) + \ln(T_0-t) = \theta_a(\xi)$$

(7) $$\theta_a(\xi) = \ln((1 + \xi^2/4)^{-1})$$

uniformly on compact sets $|\xi| \le \xi^*$ with $\xi^*>0$.

As we know this is the first exact result in this direction. It means that the parabolic equation (1),(4) degenerates as $t\longrightarrow T_0$ into the Hamilton-Jacobi equation ([6],[7],[8],[9])

$$v_t + rv_r\ \{2(T_0 - t)|\ln(T_0 - t)|\}^{-1} = v^\beta,$$

which has an exact blow-up self-similar solution

$$v(r,t) = (T_0 - t)^{-1/(\beta-1)}\ \theta_a(\xi),$$

$\theta_a(\xi)$ given by (6), where

(8) $$\xi = r((T_0 - t)|\ln(T_0 - t)|)^{-1/2}.$$

The same is for the equation (1),(5) - it degenerates as $t\longrightarrow T_0$ into the Hamilton-Jacobi equation ([4],[8],[16])

$$v_t + rv_r\ \{2(T_0 - t)|\ln(T_0 - t)|\}^{-1} = e^V,$$

with a blow-up self-similar solution

$$v(r,t) = -\ln(T_0 - t) + \theta_a(\xi),$$

where $\theta_a(\xi)$ is given by (7) and ξ is defined above.

There are many qualitative results [5], [9], [11], [18], [13] which predict such asymptotic behaviour in the many dimensional case. We confirm this by numerical experiment.

The existence of effective localization of the process gives us possibility of considering initial-boundary value problem with Dirichlet or Neumann boundary conditions in the numerical solution. So the problem has the form:

$$(10)\qquad u_t = \frac{1}{r^{N-1}}(r^{N-1}u_r)_r + F(u) \quad in\ (0,R)\times(0,T_0),$$

$$(11)\qquad u_r(0,t) = 0 \quad \text{for } t \in [0,T_0),$$

$$(12)\qquad u(R,t) = 0 \ \text{ or } \ u_r(R,t) = 0 \quad \text{for } t \in [0,T_0),$$

$$(13)\qquad u(r,0) = u_0(r)\geq 0 \quad \text{in } [0,R],\ u_0 \in C([0,R]).$$

We do a change of variables $U = F(u)$ and get the following equations:

$$(14)\qquad U_t = \frac{1}{r^{N-1}}(r^{N-1}U_r)_r - \frac{\beta-1}{\beta}\frac{U_r^2}{U} + \beta\, U^{(2\beta-1)/\beta}$$

for $F(u) = (A + u)^{\beta}$, $U > 0$ when $u \geq 0$,

$$(15)\qquad U_t = \frac{1}{r^{N-1}}(r^{N-1}U_r)_r - \frac{U_r^2}{U} + U^2$$

for $F(u) = e^u$, $U > 0$ when $u \geq 0$.

The function U satisfies the corresponding boundary and initial conditions:

$$(16)\quad U_r(0,t) = 0 \quad \text{for } t\in[0,T_0),$$

$$(17)\quad U(R,t) = A \ \text{ or } \ U_r(R,t) = 0 \quad \text{for } t\in[0,T_0),\ F(u) = (A+u)^{\beta},$$

$$(18)\quad U(R,t) = 1 \ \text{ or } \ U_r(R,t) = 0 \quad \text{for } t\in[0,T_0),\ F(u) = e^u,$$

$$(19)\quad U(R,0) = U_0(r) = (A+u_0(r))^{\beta} \quad \text{for } r\in[0,R],$$

$$(20)\quad U(R,0) = U_0(r) = e^{u_0(r)} \quad \text{for } r\in[0,R].$$

After the same transformation we find the corresponding Hamilton-Jacobi equations and their solutions for $F(u)$ given by (4) and (5).

$$(21)\qquad V_t + rV_r\{2(T_0- t)|ln(T_0 - t)|\}^{-1} = \beta V^{(2\beta-1)/\beta}$$

$$V(r,t) = (T_0- t)^{-\beta/(\beta-1)}\, \Theta_a(\xi),$$

$$\Theta_a(\xi) = \{\beta-1 + [(\beta-1)^2/(4\beta)]\xi^2\ \}^{-\beta/(\beta-1)} .$$

$$(22)\qquad V_t + rV_r\{2(T_0- t)|ln(T_0- t)|\}^{-1} = V^2,$$

$$V(r,t) = (T_0- t)^{-1}\Theta_a(\xi),$$

$\Theta_a(\xi) = (1 + \xi^2/4)^{-1}$, where ξ is defined above.

We state a method of rescaling of the solutions $U(r,t)$ in order to show convergence to a.s.-s.s. $V(r,t)$ as $t \to T_0$. By usual approach the rescaled function has the form:

$$\Theta(\xi,t) = (T_0 - t)^{\frac{\beta}{\beta-1}} U(\xi[(T_0-t)|ln(T_0-t)|]^{1/2}, t) \tag{23}$$

$$\Theta(\xi,t) = (T_0 - t) U(\xi[(T_0-t)|ln(T_0-t)|]^{1/2}, t) \tag{24}$$

for $F(u)$ given by (4), (5) respectively. This is defined by the space-time structure of a.s.-s.s. (21), (22). The asymptotic stability of a.s.-s.s. is equivalent to the condition

$$\Theta(\xi,t) \longrightarrow \Theta_a(\xi) \text{ as } t \longrightarrow T_0. \tag{25}$$

For numerical calculations we also use another method of rescaling. Let $\gamma(t) = \sup\limits_r U/\Theta_0$, where $\Theta_0 = \Theta_a(0)$. Then:

$$\Theta(\xi,t) = U(\xi[\gamma(t)^{-(\beta-1)/\beta}|ln(\gamma(t)^{-(\beta-1)/\beta})|]^{1/2}, t)/\gamma(t) \tag{26}$$

$$\Theta(\xi,t) = U(\xi[\gamma(t)^{-1}|ln(\gamma(t)^{-1})|]^{1/2}, t)/\gamma(t) \tag{27}$$

In comparison with (23), (24) T_0 doesn't occur here. It is important, since T_0 is defined after finishing numerical calculations. One can see that (23) and (26), (24) and (27) are equivalent if (25) holds.

Numerical method. We solved numerically the original problems (10)-(13) and the reduced ones (14), (16), (17), (19) and (14), (16), (18), (20). In spite of the fact, that the first ones have a self-adjoint elliptic operator, and hence, they have many advantages in the algorithmic realization of the numerical method, we chose the second. In this way we can succeed better in approaching the blow-up time T_0, and in exhibiting the degeneracy and the convergence to a.s.-s.s. Thus, we describe below the numerical method for solving the initial boundary value problem:

$$U_t = AU \qquad \text{in } (0,R)\times(0,T_0) \tag{28}$$

$$U_r(0,t) = 0 \qquad \text{for } t \in [0,T_0) \tag{29}$$

$$U(R,t) = a \quad \text{or} \quad U_r(R,t) = 0 \qquad \text{for } t \in [0,T_0) \tag{30}$$

$$U(r,0) = U_0(r) = F(u_0) \qquad \text{for } r \in [0,R], \text{ where} \tag{31}$$

$$AU \equiv \frac{1}{r^{N-1}}(r^{N-1}U_r)_r - \frac{\beta-1}{\beta}\frac{U_r^2}{U} + \beta\, U^{(2\beta-1)/\beta},$$

$$a = A \qquad \text{for } F(u) = (A+u)^\beta,$$

$$AU \equiv \frac{1}{r^{N-1}}(r^{N-1}U_r)_r - \frac{U_r^2}{U} + U^2,$$

$$a = 1 \qquad \text{for } F(u) = e^u.$$

We use the lumped mass finite element method (FEM) [19],[20] with interpolation of the nonlinear coefficients.

The discretization is made on the basis of the problem (28)-(31) in weak form:

$$(32)\qquad (U_t,\chi) = A(t;U,\chi), \quad \forall\chi\in H^1_\alpha(0,R), \quad 0<t<T_0,$$

$$(33)\qquad U(0,\cdot) = U_0,$$

where $\displaystyle (\chi,\phi) = \int_0^R r^{N-1}\chi(r)\phi(r)dr,$

$$(34)\qquad A(t;\chi,\phi) = \int_0^R(-\chi_r\phi_r - a(\chi)\chi_r\phi + b(\chi)\chi\phi)r^{N-1}dr,$$

for $F(u) = u^\beta$: $a(U) = \dfrac{\beta-1}{\beta}\,\dfrac{U_r}{U}$, $b(U) = \beta\, U^{(\beta-1)/\beta}$

for $F(u) = e^u$: $a(U) = \dfrac{U_r}{U}$, $b(U) = U$,

$H^1_\alpha(0,R) = \{\chi{:}\,\chi,\ r^{(N-1)/2}\chi'\in L^2(0,R),\ (1-\alpha)\chi(R)=0\}$,

$\alpha = 0$ corresponds to the condition $U(R,t) = a$,

$\alpha = 1$ - to the condition $U_r(R,t) = 0$.

For the spatial discretization of (32), (33) we consider the standard piecewise polynomial Lagrangian finite element spaces. Let $\{\ 0 = r_1<r_2<\ldots<r_m = R,\ r_{i+1}-r_i\le h\ \}$ be a partition of the interval $[0,R]$ into elements $e_i = [r_i,r_{i+1}]$. Thus we denote by $S_{\alpha,h}$ the space of continuous functions on $[0,R]$ that reduce to polynomials of degree $\le$ k-1 on each element $e_i, i = 1,2,\ldots,m-1$:

$$S_{\alpha,h}=\{W(r)\in C([0,R]);\ W_{(r_i,r_{i+1})}\in P_{k-1};\ (1-\alpha)W(R)=0\}.$$

The approximation properties of $S_{\alpha,h}$ are well known [19]:

$$\|I_hW-W\|_{L^2(0,R)} + h\|\nabla I_hW-\nabla W\|_{L^2(0,R)} \le Ch^k\|W\|_{H^k},$$

$$\|I_hW-W\|_{L^\infty(0,R)} \le Ch^k\|W\|_{W^2_\infty(0,R)}.$$

Here I_h is the interpolation operator:

$I_h : C([0,R])\longrightarrow S_{\alpha,h}$, $(I_hW)(\eta_j) = W(\eta_j)$ for each of the nodes η_j, $j = 1,2,\ldots,M$, that define the degrees of freedom of $S_{\alpha,h}$.

Let $U_h(r,t)$ denote the approximate solution in $S_{\alpha,h}$. We pose the semidiscrete problem:

To find $U_h\in S_{\alpha,h}$ for each t, such that

$$(35)\qquad (U_{h,t},W) = A_h(t;U_h,W) \quad \text{for all } W\in S_{\alpha,h},$$

$$(36)\qquad U_h(0) = U_{0h}.$$

Let $\{\varphi_i\}_{i=1}^{M}$ be the standard Lagrangian nodal basis of $S_{\alpha,h}$. Representing $U_h(r,t)$ in the form

$$U_h(r,t) = \sum_{i=1}^{M} U_i(t)\varphi_i(r) \in S_{\alpha,h},$$

and using the lumped mass method our semidiscrete problem (35),(36) can be written in matrix form:

$$(37) \qquad \tilde{\mathbf{M}}\dot{\mathbf{U}}=\mathbf{K}(\mathbf{U})\mathbf{U},$$

$$(38) \qquad \mathbf{U}(0)=\mathbf{U}_0.$$

Here $\mathbf{U}=\mathbf{U}(t)=(U_1(t),U_2(t),\ldots,U_M(t))^T$, $\tilde{\mathbf{M}}$ is the lumped mass matrix,

$$\tilde{\mathbf{M}} = \mathrm{diag}\{\tilde{m}_{ii}\}, \quad \tilde{m}_{ii} = \sum_{j=1}^{M} m_{ij}, \quad m_{ij} = \int_0^R r^{N-1}\varphi_i\varphi_j dr, \ i,j=1,\ldots,M,$$

$$\mathbf{K}(\mathbf{U}) = \sum_e \mathbf{k}_e = \sum_e (\mathbf{k}_e^{(1)}+ \mathbf{k}_e^{(2)} + \mathbf{k}_e^{(3)}), \quad \mathbf{k}_e^{(l)}= (k_{ij}^{(l)}), \quad l=1,2,3,$$

$$(39) \qquad k_{ij}^{(1)} = -\int_e r^{N-1}\psi_i'\psi_j' dr, \qquad k_{ij}^{(2)} = -\int_e r^{N-1}a(U)\psi_i\psi_j' dr,$$

$$(40) \qquad k_{ij}^{(3)} = \int_e r^{N-1}b(U)\psi_i\psi_j dr,$$

$$a(U) = \frac{\beta-1}{\beta}\Big(\sum_{i=1}^{k} U_i\psi_i'\Big)/\Big(\sum_{i=1}^{k} U_i\psi_i\Big), \quad b(U) = \beta\Big(\sum_{i=1}^{k} U_i\psi_i\Big)^{(\beta-1)/\beta}$$

for $F(u) = u^\beta$, and for $F(u) = e^u$

$$a(U) = \Big(\sum_{i=1}^{k} U_i\psi_i'\Big)/\Big(\sum_{i=1}^{k} U_i\psi_i\Big), \qquad b(U) = \sum_{i=1}^{k} U_i\psi_i \ ,$$

ψ_i, $i=1,\ldots,k$ are the shape functions of the element e.

Let us note, that the matrix $\mathbf{K}$ is nonsymmetric one. When solving the system of ODE (37), (38), we don't calculate matrix $\mathbf{K}$ in explicit form - we calculate only the product $\mathbf{K}(\mathbf{U})\mathbf{U}$, accumulating it by means of the element matrices $\mathbf{k}_e$.

To solve the system (37),(38) of ODE we use a modification of the explicit Runge-Kutta method, which has second order of accuracy and an extended region of stability [17]. Moreover, the time-step τ is chosen automatically so as to guarantee relative stability and a desired accuracy ε at the end of the time-interval.

In computations we use linear finite elements on uniform and nonuniform grids. To approximate the integrals in (39), (40) we use the trapezoidal rule (N = 1) or the two-points Gauss rule (N = 2,3).

We make a special mesh refinement in consistency with the space-time structure of the a.s.-s.s. It is seen, as $t \to T_0$ the value of the self-similar variable $\xi = r[(T_0-t)|ln(T_0-t)|]^{-1/2}$ tends to infinity. So we choose the step-length in r such that the step-length in ξ to be uniform. We compute the values of the solution in the new included mesh-points using linear interpolation between the values in two old neighbouring points. It is clear that the number of the mesh-points increases as $t \to T_0$, so the computation process goes slowly and the computational error increases. To avoid this, after every change of the mesh we proceed the computations only in the interval $[0,R_k]$, where the solution grows. We suppose that the solution is established in the interval $[R_k,R]$ if the difference between the solution's values for $t=t_i$ and $t=t_{i+1}=t_i+\tau$ at the point R_k is less than a given constant $(=10^{-7})$. Using this mesh refinement and $\tau_{\min} = 10^{-16}$ we may compute sufficiently exactly the solution $U(r,t)$ when its amplitude is on the order of 10^{15}, since without mesh refinement we compute the solution $U(r,t)$ to amplitude of order 10^5.

4. Numerical results and interpretation. As it was said, the aim of the numerical experiments was:

- to analyze the space-time structure of the unbounded solutions of the problem (28)-(31);
- to confirm the degeneracy of the parabolic equation when $t \to T_0$ by showing convergence of its solution $U(r,t)$ to the a.s.-s.s. $V(r,t)$ in the sense of (23), (24), (25):

$$\Theta(\xi,t) \longrightarrow \Theta_a(\xi) \quad \text{as} \quad \|U(t)\|_{C_r} \equiv \sup_r U(r,t) \longrightarrow \infty. \tag{41}$$

The graph of $\Theta_a(\xi)$ is signed with ■ on Figures 1b-8b. The other symbols are used for the graphs of the solution and $\Theta(\xi,t)$ for different values of t.

First we show two results for the case $F(u) = (1+u)ln^{\beta}(1+u)$, $\beta>2$ (single point blow-up,[3]). Figures 1 a,b show the solution of the parabolic equation and the rescaled function $\Theta(\xi,t)$ for $N=1$ and $\beta=4$, figures 2 a,b - for $N=3$ and $\beta=2.5$. It is seen that the last two profiles of $\Theta(\xi,t)$ and $\Theta_a(\xi)$ coincide to within plotting resolution on compact sets of length $\xi^*=14$ for the first case and $\xi^*= 3$ for the second.

BETA N
4.000 1

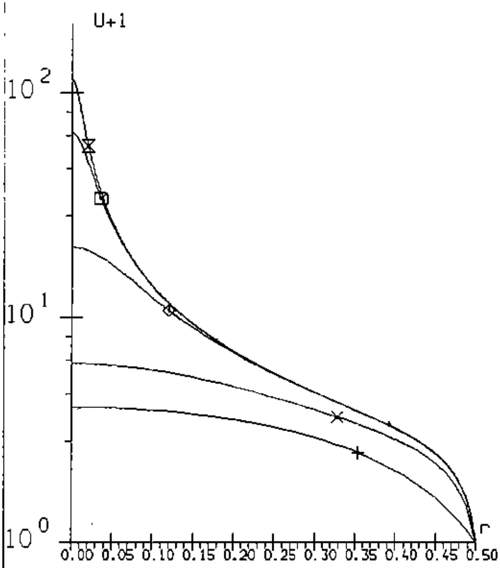

Fig. 1a

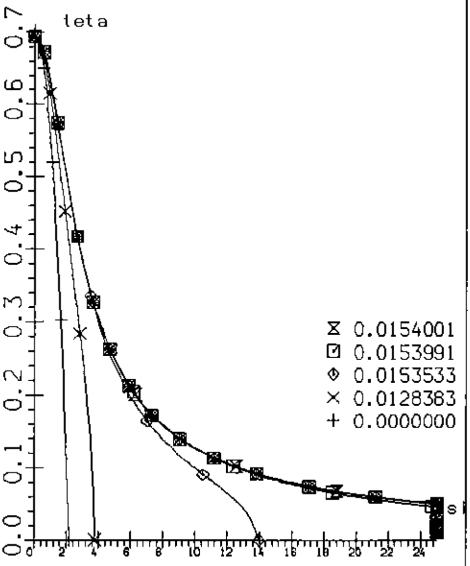

Fig. 1b

BETA N
2.500 3

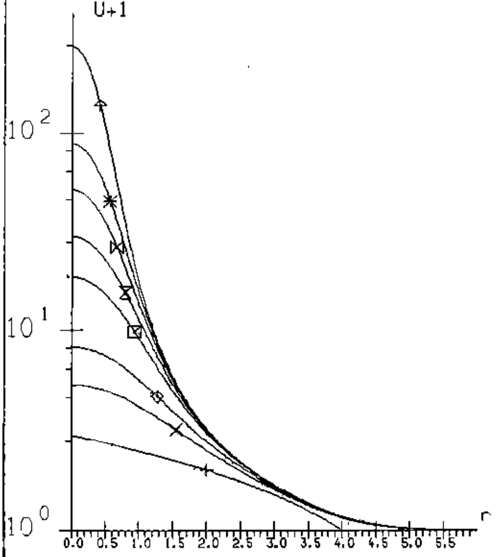

Fig. 2a

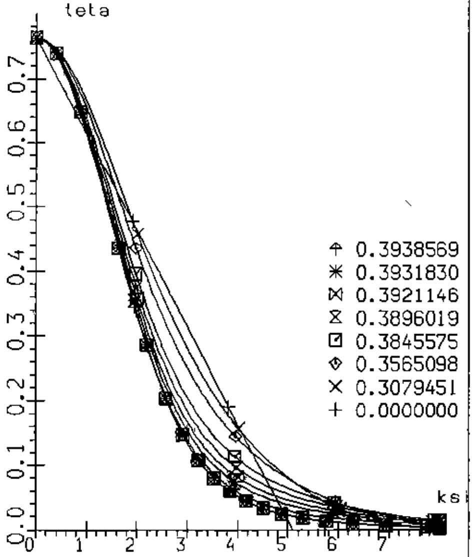

Fig. 2b

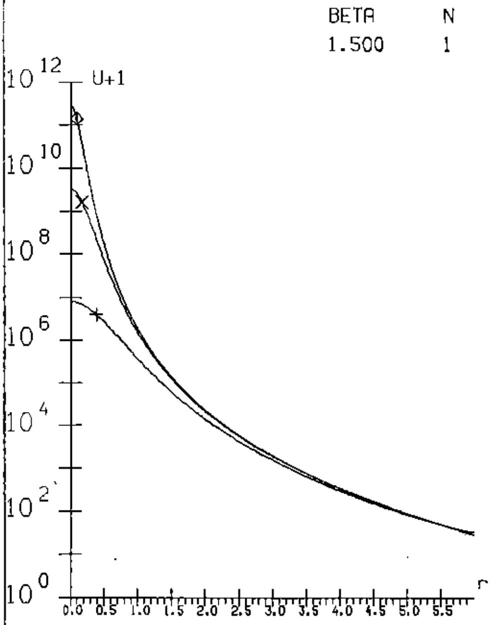

Fig. 3a

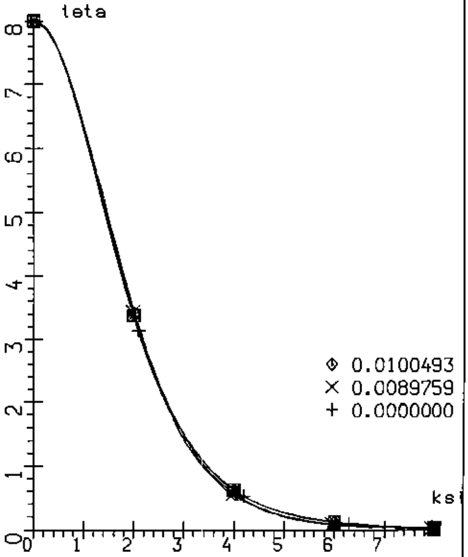

Fig. 3b

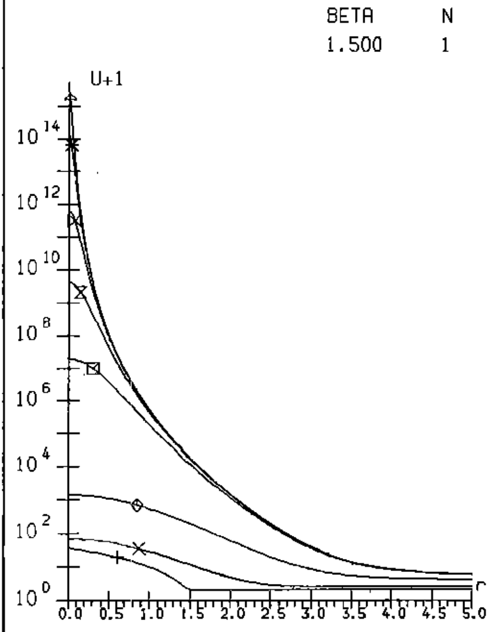

Fig. 4a

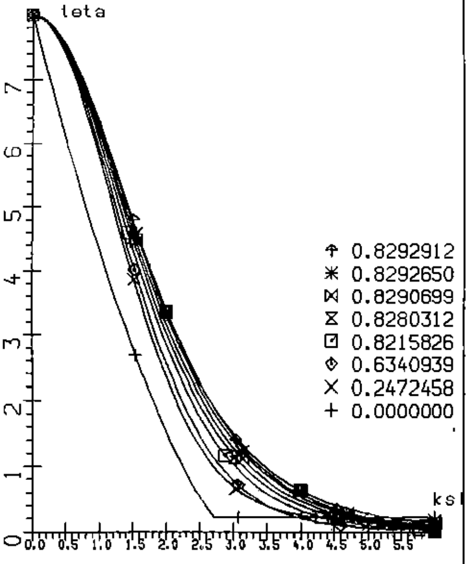

Fig. 4b

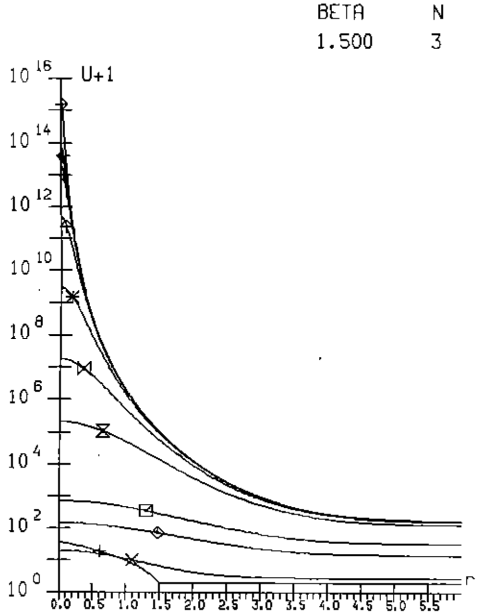

Fig. 5a

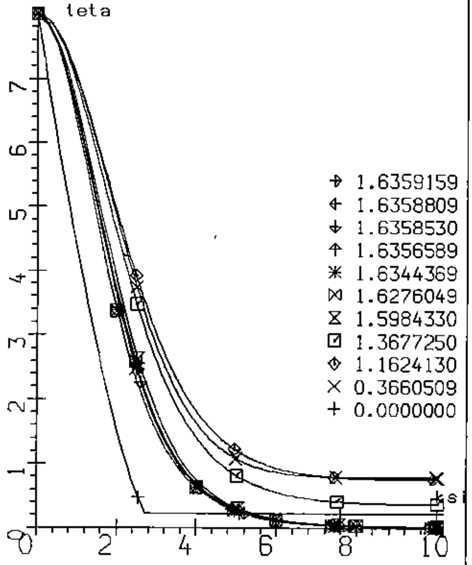

Fig. 5b

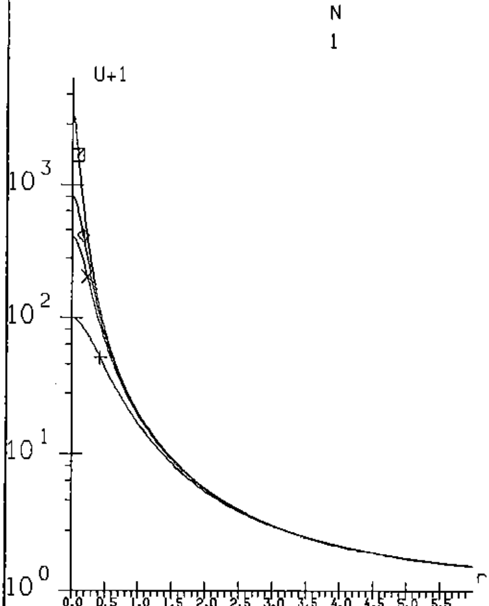

Fig. 6a

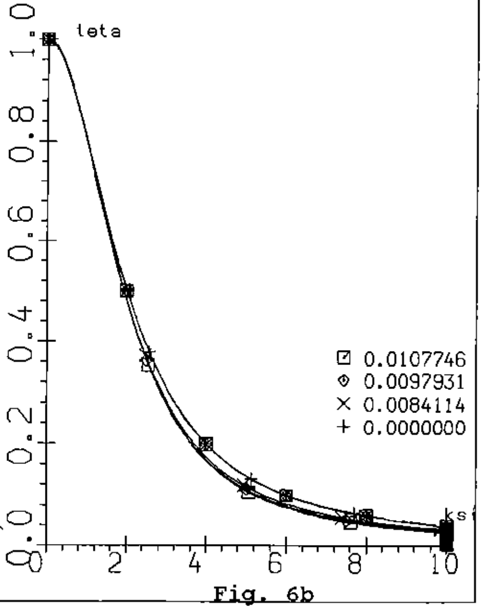

Fig. 6b

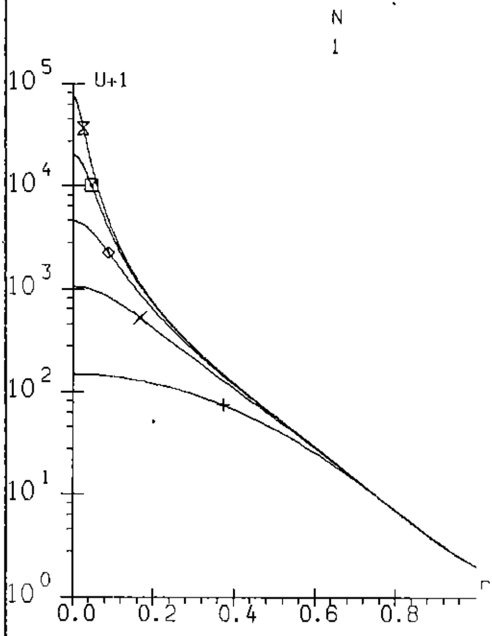

Fig. 7a

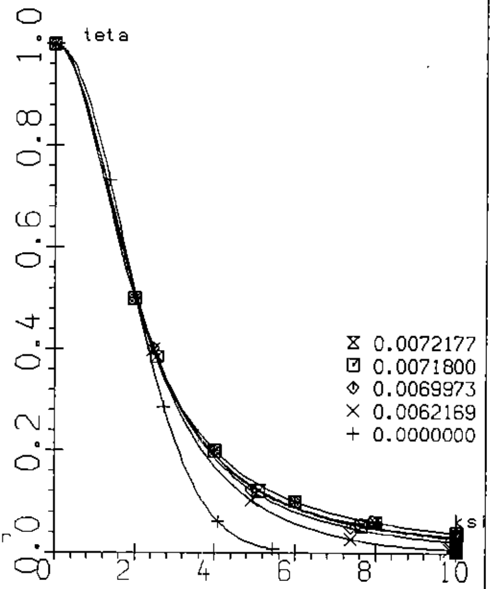

Fig. 7b

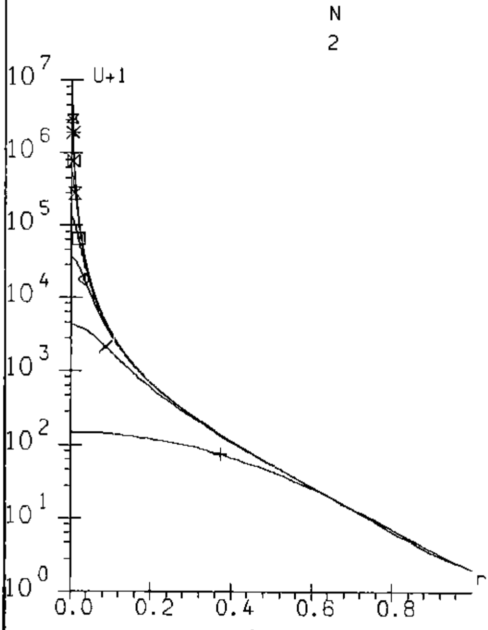

Fig. 8a

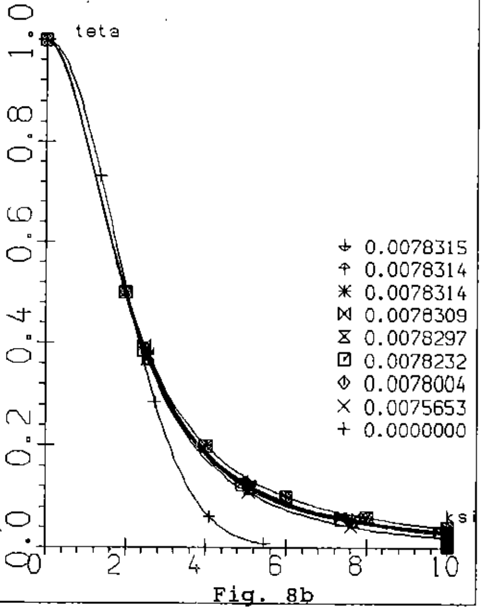

Fig. 8b

Figures 3-5 concern the case $F(u) = (A + u)^{\beta}$, $A = 1$. The evolution of the a.s.-s.s., which corresponds to blow-up time T_0=0.01, N=1 and β=1.5 is shown on figure 3a. One can find out a very good reconstruction of T_0 - in the computational process we get T_0= 0.01004. The rescaled function $\Theta(\xi,t)$ and the a.s.-s.s. coincide on the set of length 2.5 (figure 3b). The initial mesh has 121 points, the final one - 961 points; the minimal time-step is $\tau=10^{-16}$. Figures 4 a,b show the evolution of nonself-similar initial data for N=1, β=1.5. The initial mesh has 121 points, the last one - 1921 points.The amplitude of the solution U is on the order of 10^{15}. The profiles of $\Theta(\xi,t)$ and $\Theta_a(\xi)$ coincide on a set of length 2.5. The case N=3, β=1.5 is shown on figures 5 a,b.

Figures 6-8 are for the case $F(u) = e^u$. The evolution of self-similar initial data, corresponding to T_0= 0.01, N=1, and the rescaled function are shown on Figures 6 a,b. The evolution of nonself-similar initial data, given in the interval [0,1], for N=1 and N=2 are shown on figures 7a,b and 8a,b respectively. The results are unexpected even for us - the profiles of $\Theta(\xi,t)$ and $\Theta_a(\xi)$ coincide on a set of length 3.

Note , all computations are made with PC-AT, using double-precision arithmetic and memory not greater than 570K. So we think our results may compete with those of M.Berger and J.Kohn [1], done with Cray XMP.

Conclusions. Many other experiments, we have made, give us assurance, that the degeneracy of the semilinear heat equations (1),(4),(5) into corresponding equations of Hamilton-Jacobi type takes place in the many-dimensional case as well, when the first ones have not exact blow-up self-similar solutions. But this remains an open question.

Acknowledgments. The authors would like to thank Professor Sergey Kurdjumov, Professor Victor Galaktionov and Professor Sergey Posashkov for helpful discussions.

REFERENCES

[1] M. Berger and R. Kohn, *A rescaling algorithm for the numerical calculation of blowing up solutions,* Comm. Pure Appl. Math., 41(1988), pp.841-863.

[2] J.Bebernes, A.Bressan, D.Eberly, *A description of blow-up for the solid fuel model,* Indiana Univ. Math. J., 36(1987), pp.295-305

[3] S.N.Dimova, D.I.Ivanova,.V.A.Galaktionov, *Numerical analysis of blow-up and degeneracy of a semilinear heat equation, Preprint of JINR, Dubna,* E11-89-785, 1989.

[4] J. M. Dold, *Analysis of the early stage of thermal runaway,* Quart. J. Mech. Appl. Math., 38,(1985), pp.361-387.

[5] D.Eberly, W.Troy, *On the existence of logarithmic-type solutions to Kassoy problem in dimension* 3, Preprint,1987.

[6] V. A. Galaktionov, V. A. Dorodnitzyn, G. G. Elenin, S. Kurdjumov and A. A. Samarskii, *Quasilinear heat equation with source: localization, symmetry, explicit solutions, asymptotics, structures,* In: Modern Mathematical Problems, VINITI AN SSSR, Moscow, 1986, Vol.28, pp.96-206 (in Russian)

[7] V. A. Galaktionov and S. A . Posashkov , *Equation* $u_t=u_{xx}+u^\beta$. *Localization, asymptotic behavior of unbounded solutions,* Preprint Keldysh Inst. Appl. Math. Acad. Sci. USSR. No 97, 1985 (in Russian).

[8] V. A. Galaktionov and S. A. Posashkov, *On some properties of evolution of unbounded solutions of semilinear parabolic equations*, Preprint Keldysh Inst. Appl. Math. Acad. Sci. USSR. No 232, 1987 (in Russian).

[9] V. A. Galaktionov and S. A. Posashkov, *On a method of investigation of unbounded solutions of some quasilinear parabolic equations,* J. Vichisl. Math. Mathem. Fiz., 6(1988), pp.842-854 (in Russian)

[10] V. A. Galaktionv and S. A. Posashkov, On a *modeling* of *blow-up processes in the heat problems with nonlinear source,* J. Math.Model., 1(1989), pp.89-108.

[11] Y. Giga and R. V. Kohn, *Characterizing blowup using similarity variables,* Indiana Univ. Math. J., 36(1987), pp.1-40.

[12] Y. Giga and R. V. Kohn, *Asymptotically self-similar blow-up of semilinear heat equations,* Comm. Pure Appl. Math., 38 (1985), pp.297-319

[13] Y. Giga and R. V. Kohn, *Nondegeneracy of blow-up for semilinear heat equations,* Comm.Pure Appl.Math.,42(1989), pp.845-884.

[14] M. Herrero, J.J.L.Velazquez. *Blow-up behavior of one-dimensional semilinear parabolic equations,* to appear

[15] L. M. Hocking, K. Stewartson and J. T. Stuart, *A non - linear instability burst in plane parallel flow*, J. Fluid Mech., 51(1972), pp. 705 - 735.

[16] A. Lacey, *Global blow-up of a nonlinear heat equation*, Proc. Roy. Soc. Edinburgh., 104A(1986), pp.161-167.

[17] V. A. Novikov and E. A. Novikov, *Stability control of some explicit methods for integration of ODE*, Dokl. AN SSSR, 277 (1984), pp.1058-1062 (in Russian).

[18] A. A. Samarskii, V. A. Galaktionov, S. P. Kurdjumov, and A. P. Mikhailov, *Blow-up in Problems for Quasilinear parabolic equations*, Nauka, Moscow, 1987 (in Russian).

[19] V. Thomee, *Galerkin FEM for parabolic problems*, Lecture Notes in Mathem. 1054, 1984.

[20] Yi-Yong-Nie, V. Thomee, *A lumped mass FEM with quadrature for a nonlinear parabolic problem*, IMA J. Numer.Anal., 5(1985), pp. 371-396.

Received by Publishing Department
on January 23, 1991.