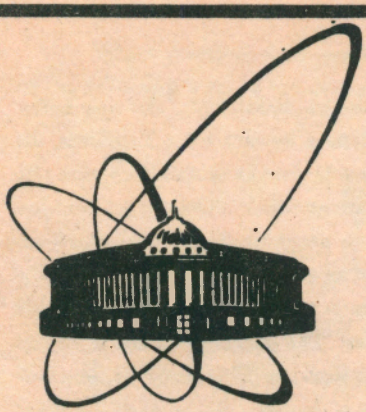

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REDUCE USAGE FOR CALCULATION OF LOW-ENERGY PROCESS AMPLITUDES IN CHIRAL QCD MODEL

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1 Introduction

The usage of computer algebra systems (CAS) for the automatization of doing bulky calculations in theoretical physics has not already been a novelty for a long time. In this paper we describe the usage of the REDUCE system [1] for the calculations with the non-linear chiral Lagrangians including higher derivatives which allow one to describe a wide region of the experimental data. However, their usage in the physical problems is connected with the need to make a great amount of calculations. It is especially difficult to obtain the explicit form of the Lagrangians in terms of the meson fields and to calculate the matrix elements of concrete processes. The latter operation is absent in the CAS we know, hence it is necessary not only to use some CAS or other, but also to expand its capabilities as well.

We have applied the REDUCE system for this problem, since it is available for a wide variety of computers, widely used by physicists, and can be easily extended. At the same time it has s sufficient set of mathematical operations and is convenient in use. One can distinguish several levels of its extension:

- by the substitutions and sets of commands; this level does not go out of the range of the standard set of REDUCE operations;
- inclusion of several procedures implementing the missing operations. They can be written in the LISP language, which REDUCE is based on. However, this method does not extend the available data types and the standard operations with them;
- creation of a "user package" extending the REDUCE syntax [2].

We have chosen the second method of extension as the minimal one. The described package of procedures has been written for REDUCE versions 3.0 and 3.3. Its size is approximately 2000 lines of code, there are 80 procedures including the auxiliary ones.

2 Physical model

2.1 Strong interaction

The notable achievements have been obtained in description of meson processes at energies $E < 1\,\mathrm{GeV}$ based on chiral Lagrangians. For the first time they were obtained in 1960s–70s as the group theory realization of symmetries of strong and weak interactions [3]. The more contemporary treatment of chiral Lagrangians as a low-energy limit of quantum chromodynamics (QCD) has been developed in the approaches based on the quark bosonization approach [4].

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Direct calculation of quark determinant leads to effective mesonic Lagrangian of the following form in the pseudoscalar sector:

$$\mathcal{L}_m^{QCD} = \mathcal{L}_0 + \mathcal{L}_Q + \mathcal{L}_{SB} \tag{1}$$

$$\mathcal{L}_0 = -\frac{F_0^2}{4}\,\mathrm{tr}\,(L_\mu L^\mu) \qquad \text{(kinetic term)} \tag{2}$$

$$\mathcal{L}_Q = \frac{1}{64\pi^2}\,\mathrm{tr}\left\{\frac{1}{2}\,[L_\mu, L_\nu]^2 + (L_\mu L^\mu)^2\right\} \qquad (p^4\text{-interaction}) \tag{3}$$

$$\mathcal{L}_{SB} = \frac{F_0^2}{4}\,\mathrm{tr}\,(MU + \mathrm{h.c.}) \qquad \text{(chiral symmetry breaking)}. \tag{4}$$

Here $F_0 = 93\,\mathrm{MeV}$ is the bare constant of $\pi \to \mu\nu$ decay; $L_\mu = (\partial_\mu U)U^+$; $U = \exp\left\{\frac{i\sqrt{2}}{F_0}\Phi\right\}$, $\Phi = \sum_{i=0}^{8}\frac{1}{\sqrt{2}}\lambda_i\varphi_i$ is the meson matrix for the nonet of pseudoscalar fields φ_i $(i = 0, 1, \ldots, 8)$,

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \overline{K}^0 \\ K^- & \overline{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} \end{pmatrix}, \tag{5}$$

λ_i are $U(3)$ group generators $(\lambda_0 = \sqrt{\frac{2}{3}}\,1)$, M is the mass matrix chosen in diagonal form $M = \mathrm{diag}\{\mu_u^2, \mu_d^2, \mu_s^2\}$.

For correct description of pseudoscalar nonet masses the following effective Lagrangian is also introduced here

$$\mathcal{L}_G = \frac{aF_0^2}{16N_C}\left[\mathrm{tr}\,\left(\ln U - \ln U^+\right)\right]^2 \tag{6}$$

accounting for gluonic $U(1)$ anomaly. Parameters μ_i^2 and a are fixed by the masses of π, K^0, K^+, η, η' mesons: $a = 0.729\,\mathrm{GeV}^2$, $\mu_u^2 = 0.0114\,\mathrm{GeV}^2$, $\mu_d^2 = 0.025\,\mathrm{GeV}^2$, $\mu_s^2 = 0.47\,\mathrm{GeV}^2$. The corresponding value of the $(\eta$–$\eta')$-mixing angle is $\varphi = 19°$:

$$\begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}\begin{pmatrix} \eta \\ \eta' \end{pmatrix}.$$

An additional interaction to the symmetry-breaking term $\mathcal{L}_{SB}$ is involved for an account of splitting of F_π, F_K constants:

$$\mathcal{L}'_{SB} = -\frac{F_0^2}{4\Lambda_\chi^2}\,\mathrm{tr}\,(M\,\partial^2 U + \mathrm{h.c.}), \tag{7}$$

with parameter $\Lambda_\chi^2 = 0.78\,\mathrm{GeV}^2$.

Lagrangians (2–4,6–7) accumulate all fundamental properties of the pseudoscalar meson nonet physics: spontaneous chiral symmetry breaking, quark condensate existence, Goldstone bosons and partial conservation of axial current (PCAC), and also reproduce the p^4-contribution, which reflects the fine structure of the meson interaction.

This model can also be extended to include vector and axial-vector meson nonets and also account for electromagnetic interactions. The details can be found elsewhere, e.g. see [5].

2.2 Weak interaction

Weak mesonic interactions are described by Lagrangians of the type (current × current). For example, leptonic decays of mesons are represented by the Lagrangian

$$\mathcal{L}_W^{(lh)} = \frac{G}{\sqrt{2}} \left(\cos\theta_c \, J_\mu^{1+i2} + \sin\theta_c \, J_\mu^{4+i5} \right) l^\mu \tag{8}$$

where J_μ^{1+i2}, J_μ^{4+i5} are the mesonic $(V-A)$ currents, which preserve and break respectively the strangeness, l^μ is leptonic current. Vainstein–Zakharov–Shifman Lagrangian is used for non-leptonic meson decays with the strangeness change $|\Delta S| = 1$

$$\mathcal{L}_W^{|\Delta S|=1} = \sqrt{2}\, G_F \, \sin\theta_c \, \cos\theta_c \sum_{i=1}^{6} c_i \mathcal{O}_i^{(q)} \tag{9}$$

where $\mathcal{O}_i^{(q)}$ are the 4-quark operators containing the products of left and/or right operators; $c_i(\mu)$ are Wilson coefficient functions. The bosonization procedure described in [5] allows one to formulate the certain prescription for quark currents j_μ entering the operators $\mathcal{O}_i^{(q)}$:

$$\begin{array}{ccccc} \textit{(quarks)} & & & \textit{(mesons)} & \\ \mathcal{O}_i^{(q)}[:\, j_\mu^+ \, j_\mu \,:] & \longrightarrow & \mathcal{O}_i^{(m)}[:\, J_\mu^+ \, J_\mu \,:], & i = 1, \ldots, 4; \\ :\, (\bar{q}_R q_L)(\bar{q}_L q_R) \,: & \longrightarrow & :\, (\partial_\mu A^\mu)\,(\partial_\nu V^\nu) \,:, & i = 5. \end{array} \tag{10}$$

Here $J_\mu = V_\mu - A_\mu$ are mesonic currents, and the symbol $: \ldots :$ denotes the normal ordering. The explicit expression for the corresponding mesonic Lagrangian describing the non-leptonic K decays have been given in [5].

Current matrix J_μ can be obtained from strong interaction Lagrangian by means of the procedure of variation on some external field:

$$J_\mu \equiv \lambda^k J_\mu^k = \frac{\delta \mathcal{L}_s(V_\mu \to V_\mu - i\eta_\mu)}{\delta \eta_\mu} \,. \tag{11}$$

Let us present the formulae for mesonic $(V-A)$ currents corresponding to Lagrangians (2), (3):

$$\mathcal{L}_0 \longrightarrow J_\mu^k = \frac{iF_0^2}{4} \,\mathrm{tr}\,(\lambda_k L_\mu) \tag{12}$$

$$\mathcal{L}_Q \longrightarrow J_\mu^k = -\frac{i}{64\,\pi^2} \,\mathrm{tr}\,(\lambda_k \{\{L_\mu, L_\nu\}, L^\nu\}) \,. \tag{13}$$

Direct emission of photons in the radiative decays of mesons is described by the electroweak current which can be obtained from the Lagrangian accounting for strong interaction with structural photons

$$\begin{aligned} J_\mu^{k(\gamma)} = & -e\frac{N_C}{32\,\pi^2} \,\mathrm{tr}\left\{ \lambda_k \frac{2}{3} F_{\mu\nu} \Big(U \left[Q, \partial^\nu U^+ \right] + \mathrm{h.c.} \Big) \right. \\ & + \frac{1}{\mu^2} \lambda_k \Big[\frac{1}{6} \Big(U \left(F_{\mu\nu} [Q, \partial^\nu \partial^2 U^+] + \partial_\alpha F_{\mu\nu} \, [Q, \partial^\alpha \partial^\nu U^+] \right) \\ & \quad - \partial_\mu F_{\nu\alpha} \, \partial^\alpha U [Q, \partial^\nu U^+] - F_{\nu\alpha} \, \partial^\nu U [Q, \partial_\mu \partial^\alpha U^+] \Big) \\ & \quad \left. - \frac{1}{90} F_{\mu\nu} \Big(\partial^\nu \partial_\alpha U^+ \, Q \, \partial^\alpha U + 2 \left\{ Q, \partial^\nu \partial_\alpha U^+ \, \partial^\alpha U \right\} \Big) + \mathrm{h.c.} \Big] \right\} \\ & - \frac{i}{15} \frac{e^2}{\mu^2} \frac{N_C}{32\,\pi^2} (F_{\alpha\beta})^2 \,\mathrm{tr}\left\{ \lambda_k \Big(\frac{1}{2} U \, Q \, \partial_\mu U^+ \, Q - U \left\{ Q^2, \partial_\mu U^+ \right\} - \mathrm{h.c.} \Big) \right\} . \end{aligned}$$

Here $F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$, $\mathcal{A}_\mu$ is electromagnetic field, $\mu = \frac{1}{3}\sum_{i=1}^{3} m_i \approx 380\,\mathrm{MeV}$ is the mean constituent mass of the u, d, s quark triplet. In this formula we neglected the terms depending on the current quark mass.

3 Calculation stages of process amplitudes

3.1 Transition to Lagrangians in terms of meson fields

Lagrangians introduced in the previous section are expressed in terms of matrix U, what reflects the chiral invariance of the theory. For calculation of some concrete process amplitude we need to express Lagrangians in terms of the real meson fields with the help of substitution

$$U \longrightarrow \exp\left\{ \frac{i\sqrt{2}}{F_0} \Phi \right\} = 1 + \frac{i\sqrt{2}}{F_0} \Phi - \frac{1}{F_0^2} \Phi^2 + \ldots$$

We should also have an opportunity to perform differentiation on space-time 4-vectors x_μ of product of non-commutative matrix-operator Φ, which appears yet as symbol, elements being not substituted. There is also necessity for a set of procedures to carry out Hermitian conjugation of such products and cyclic permutations of non-commutative operators under the trace operation.

The method of representation of required symbols in REDUCE is shown in Table 1. All transformation sequences shown in Table 1, can be realized by the substitutions defined in file "`utofi.red`". Kinetic Lagrangian calculation up to the 4th order on Φ is given in Example 1.

Table 1

Mathematical notation	REDUCE representation, transformation sequence
Φ	noncom fi; fi()
$U = \exp\left\{\frac{i\sqrt{2}}{F_0}\Phi\right\}$	u:=taylor(exp(i*sqrt(2)/fi0*fi()),fi(),<order>);
$\partial_\mu\Phi \equiv \Phi_\mu$	(fi() d mu) ⟶ fi(mu)
$\partial_\mu\partial_\nu\Phi$	(fi() d mu d nu) ⟶ fi(mu) d nu ⟶ fi(mu*nu)
$\partial_\nu\partial_\mu\Phi$	(fi() d nu d mu) ⟶ fi(nu) d mu ⟶ fi(mu*nu)
$\partial^2\partial_\mu\Phi \equiv \partial^\nu\partial_\nu\partial_\mu\Phi$	(fi() d mu d nu d nu) ⟶ (fi(mu) d nu d nu) ⟶ (fi(mu*nu) d nu) ⟶ fi(mu*nu**2)
$\partial_\mu(\Phi^2) \longrightarrow \Phi\Phi_\mu + \Phi_\mu\Phi$ [1]	(fi()**2 d mu) ⟶ fi()*fi(mu) + fi(mu)*fi()
matrix M	mm()
matrix Q	qq()

Example 1[2]

```
in "utofi.red"$
in "taylor.red"$
let ord**5=ord5, ord5=0;

u := taylor(exp(i*sqrt(2)/f0*ord*fi()),fi(),4);

           4    4          3              3                2    2   2
U := (FI() *ORD  - 2*FI() *SQRT(2)*ORD *FO*I - 6*FI() *ORD *FO  + 6

                            3        4        4
       *FI()*SQRT(2)*ORD*FO *I + 6*FO )/(6*FO )

w := hermit u;

           4    4          3              3                2    2   2
W := (FI() *ORD  + 2*FI() *SQRT(2)*ORD *FO*I - 6*FI() *ORD *FO  - 6

                            3        4        4
       *FI()*SQRT(2)*ORD*FO *I + 6*FO )/(6*FO )

lagr0 := f0**2/4*traceshift( (u d mu) * (w d mu) );

             2            2       2         2           2   2
LAGRO := (ORD *( - FI(MU) *FI() *ORD  + 3*FI(MU) *FO  + FI(MU)*FI()
```

[2]In this and the following examples lower case letters stand for REDUCE input and the upper case ones stand for REDUCE output.


```
                         2         2
      *FI(MU)*FI()*ORD ))/(6*FO )
```

Here the following functions from "utofi.red" are used:

taylor(X,Y,N) – expands expression X into Taylor series on Y up to the N-th order;

hermit(X) – calculates Hermitian conjugation of expression X;

traceshift(X) – transforms expression to the unique form using the possibility to perform cyclic permutation of the products of matrix symbols under the trace operation, e.g.

traceshift(fi(mu)*fi()*fi(nu)) ⟶ fi(mu)*fi()*fi(nu)
traceshift(fi(nu)*fi(mu)*fi()) ⟶ fi(mu)*fi()*fi(nu).

orderind(X) – transforms expression to the unique form using the possibility to redesignate the dummy indices, e.g.

orderind(fi(mu)*fi()*fi(mu)*fi()) ⟶ fi(mu)*fi()*fi(mu)*fi()
orderind(fi(nu)*fi()*fi(nu)*fi()) ⟶ fi(mu)*fi()*fi(mu)*fi().

variate(E,X,I) -variates expression E on non-commutative symbol X, which can optionally have some indices.dices. The last argument is the list of dummy indices in X. The first example below is illustrated by the corresponding mathematical notation:

$$\frac{\delta(\,\mathrm{tr}\,\eta_\mu\Phi_\mu\Phi)}{\delta\eta_\mu} \longrightarrow \Phi_\mu\Phi$$

variate(eta(mu)*fi(mu)*fi(), eta(mu), '(mu)) ⟶ fi(mu)*fi()
variate(fi(mu)*eta(mu)*fi(), eta(mu), '(mu)) ⟶ fi()*fi(mu)
variate(eta(nu)*fi(nu)*fi(), eta(mu), '(mu)) ⟶ fi(mu)*fi().

Since REDUCE does not allow argument specification for matrices, a special procedure fiprint is required to transform calculation results to the form suitable for substitution of elements of the matrices Φ, M, Q (see an example of its work in Example 2).

Example 2

```
in "utofi.red"$ symbolic (filist:=nil);

fiprint (3*ord**4*fi(mu)**2*fi(al**2*mu*nu)*fi())$

DEFFI(MU);
DEFFI(NU,AL,AL,MU);
DEFFI();

      2                  4
3*FIMU *FINUALALMU*FI*ORD
```

Output of procedure **fiprint** is designated to be included into the program text for generating matrices such as Φ, $\partial_\mu\Phi$, $\partial_\alpha\partial_\alpha\partial_\mu\partial_\nu\Phi$, etc. and their multiplication with some coefficients.

3.2 Calculation of process amplitudes

In order to obtain the explicit form of Lagrangians in terms of meson fields one should make substitution (5). While doing this, it is enough to retain only operators of particles taking part in the process specified, e.g. for the pion charge-exchange process $\pi^+\pi^- \to \pi^0\pi^0$ it is enough to set Φ as

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ & 0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

After definition of elements of Φ and its derivatives one can calculate Lagrangian using the standard REDUCE opportunities to evaluate products and traces of matrices. The obtained expression describes all processes with participation of specified mesons. For our example, these ones also will be $\pi^+\pi^0 \to \pi^+\pi^0$, $\pi^+\pi^+ \to \pi^+\pi^+$, etc. The concrete process is fixed by the procedure **setstate**, having for its argument a product of particle operators, which were "transformed to the final state" with the usage of crossing symmetry (i.e. each particle in the process initial state is replaced by the corresponding antiparticle with the opposite momentum sign). Therefore, the above example can be written as follows: $\pi^-(-p_1)\cdot\pi^+(-p_2)\cdot\pi^0(p_3)\cdot\pi^0(p_4)$. Since REDUCE requires operator argument to be a scalar one, each momentum has to be multiplied by a "unit" vector **ee**. Using particle names listed in Table 2 one should specify this process for REDUCE in the following way:

```
setstate(pim(-p1.ee)*pip(-p2.ee)*pi0(p3.ee)*pi0(p4.ee));.
```

After this, matrix Φ and its derivatives can be defined with procedure **deffi**. See example 3 for the listing.

Table 2

Particle	π^+	π^-	π^0	K^+	K^-	K^0	$\overline{K}^0$	η	η'	γ
REDUCE name	pip	pim	pi0	kap	kam	ka0	ak0	eta	etp	pho

Example 3[3]

```
on nero; off nat; in "mepmec.red"$ in "order.red"$

setstate(pim(-p1.ee)*pip(-p2.ee)*pi0(p3.ee)*pi0(p4.ee))$
deffi0()$ deffi()$ fi;

MAT(1,1) := (PIO(1)*SQRT(2))/2$
MAT(1,2) := PIP(1)$
MAT(2,1) := PIM(1)$
MAT(2,2) := ( - PIO(1)*SQRT(2))/2$

deffi(mu)$ fimu;

MAT(1,1) := (PIO(EE.MU)*SQRT(2))/2$
MAT(1,2) := PIP(EE.MU)$
MAT(2,1) := PIM(EE.MU)$
MAT(2,2) := ( - PIO(EE.MU)*SQRT(2))/2$

deffi(nu)$ finu;

MAT(1,1) := (PIO(EE.NU)*SQRT(2))/2$
MAT(1,2) := PIP(EE.NU)$
MAT(2,1) := PIM(EE.NU)$
MAT(2,2) := ( - PIO(EE.NU)*SQRT(2))/2$

deffi(mu,nu)$ fimunu;

MAT(1,1) := (PIO(EE.MU*EE.NU)*SQRT(2))/2$
MAT(1,2) := PIP(EE.MU*EE.NU)$
MAT(2,1) := PIM(EE.MU*EE.NU)$
MAT(2,2) := ( - PIO(EE.MU*EE.NU)*SQRT(2))/2$
```

In order to obtain Lagrangian describing the given process, one should select the Lagrangian terms with the particle operators in the required powers (in our example it is $(\pi^-\cdot\pi^+\cdot(\pi^0)^2)$). This operation is done in the procedure **evalfilter**. To calculate the amplitude one should match each operator in the Lagrangian term by the identical operator in the process state and replace the differentiation of the former by the momentum of the latter (with coefficient i). It is done by the procedure **evalfinal** having Lagrangian as its first argument. The rest arguments (if any) are indices to be contracted in the result. Hence, Lagrangian $\mathcal{L} = \pi^-(\partial_\mu\pi^+)(\partial_\nu\partial_\mu\pi^0)(\partial_\nu\pi^0)$ will give the amplitude $M = 1\cdot(-i\,p^2_\mu)\cdot(i\,p^3_\nu\cdot i\,p^3_\mu)\cdot(i\,p^4_\nu) = -(p^2\,p^3)(p^3\,p^4)$ (see Example 4 for the listing).

[3] **pip(1) is equivalent to pip().**

Example 4 (continuation of the previous one)

```
amp:=pim()*pip(mu.ee)*pi0(mu.ee*nu.ee)*pi0(nu.ee)$
amp1:=evalfinal(amp,mu,nu);
AMP1 :=  - P2.P3*P3.P4$
amp2:=evalfinal(amp,mu);
AMP2 :=  - NU.P3*NU.P4*P2.P3$
amp3:=evalfinal(amp);
AMP3 :=  - MU.P2*MU.P3*NU.P3*NU.P4$
```

If there are identical operators in the specified state, the obtained amplitude is to be symmetrized on these operators' momenta (see Example 5).

Example 5 (continuation of the previous one)

```
amp4:=symmetrize amp1;
AMP4 :=  - P3.P4*(P2.P3 + P2.P4)$
```

The procedures used above are defined in files "`ltome.red`" (Lagrangian TO Matrix Element) where general procedures are defined, and "`deffi.red`", where there are procedures for the work with matrix Φ of our particular model. Both files can be loaded by loading single file "`mepmec.red`".

Let us also mention other useful procedures in these files.

`diffind(E,I)` – differentiates expression `E` on index `I`:

$$\begin{array}{rcl} \partial_\nu(\pi^+\cdot\partial_\mu\pi^-) & \longrightarrow & \partial_\nu\pi^+\cdot\partial_\mu\pi^- + \pi^+\cdot\partial_\mu\partial_\nu\pi^- \\ \texttt{diffind(pip()*pim(mu),nu)} & \longrightarrow & \texttt{pip(nu)*pip(mu) + pip()*pim(mu*nu).} \end{array}$$

`evalwithindices(E,I1,I2,...)` – returns expression `E` with indices (`I1, I2,...`) contracted (any number of indices can be specified). This procedure allows one to satisfy the REDUCE requirement, that at any calculated expression all declared indices should be contracted: indices can be initially declared `vectors`, and then this procedure can be used, as it is shown in Example 6.

`matrixdefine()` - defines the explicit form of $SU(3)$ generators λ_i, and also the mass and charge matrices of quarks.

Example 6

```
in "ltome.red"$ vector p1,p2,mu;
part1:=p1.mu$ part2:=p2.mu$
evalwithindices(part1*part2,mu);
P1.P2
```

3.3 Photon inclusion to the calculation scheme

To expand the calculation scheme for the work with photons one should provide a place in REDUCE particle operator notation for specification of photon polarization (see Table 3). The Lorentz condition for photon ($\partial_\mu\mathcal{A}_\mu = 0$) and its masslessness ($\partial^2\mathcal{A}_\mu = 0$)

Table 3

Mathematical notation	REDUCE representation
$\mathcal{A}_\mu$	`pho(mu)`
$\partial_\alpha\partial_\nu\mathcal{A}_\mu$	`pho(mu,al*nu)`
$F_{\mu\nu}$	`f(mu,nu)`

can easily be defined by the corresponding substitutions. A small enhancement of procedures `setstate`, `evalfinal` and `symmetrize` is also required.

There is also a new procedure to determine the number of mesons and photons. Let us consider the process $K^+ \to \pi^+\pi^0\gamma\gamma$ in Example 7.

Example 7

```
in "mepmec.red"$
setstate(kam(-ka.ee)*pip(p1.ee)*pi0(p2.ee)*pho(p3.ee)*pho(p4.ee))$
setoperclass()$
NFI=3 NPHO=2
```

Expansion of the calculation scheme by vector and axial-vector mesons is also a straightforward one.

4 Conclusion

Summing up, we can recall the following features implemented for REDUCE:

- means for representation of the particle operators (with derivatives and polarization) and matrices of them;
- means for variation on some tensor structure, Hermitian conjugation, cyclic permutation of non-commutative symbols under the trace operation, Taylor series expansion, etc.;
- means for operating with matrices which depend on some arguments;
- means for selecting the terms responsible for some concrete process from Largrangian, consisting of various boson operators;
- means for calculation of amplitude from Lagrangian (with amplitude symmetrization on identical particles);
- our model specific definitions (matrices, particle properties, physical constants, decay kinematics, etc.);
- some other utilities (for calculations with 4-vectors, sorting procedures, etc.);
- using all this, we provide the procedures for calculation of amplitudes from strong, weak and electromagnetic Lagrangians in chiral QCD model and also of bosonized currents and Lagrangians.

The described package of procedures allows to calculate the amplitudes of various processes by simple specifying the particles involved and their momenta. The simplified version of procedure **strong** implementing the strong Lagrangian (1) for 4-meson processes is shown in Example 8. After the procedure definition one can calculate the amplitude of, say, $\pi^+\pi^- \to \pi^0\pi^0$ scattering by command

```
strong(pim(-p1.ee)*pip(-p2.ee)*pi0(p3.ee)*pi0(p4.ee));
```

The similar procedure can also be written for the weak and electromagnetic interaction.

Example 8

```
in "mepmec.red"$ linelength 79$
procedure strong arg;
```



```
%
% Evaluates matrix element of the strong transition.
% Included Lagrangians are: L0, LQ, LSB
%
% Not included: LTachyon, LWZW, L'SB, LG
%
begin scalar amp;
   setstate arg;
   deffi0 (); deffi (); deffi (mu); deffi (nu);
   amp:= 1/(6*f0**2)*trace(+(fimu*fi-fi*fimu)*fimu*fi
                           +1/2*mm*fi**4
                           +3/(8*(pi*f0)**2)*(fimu*finu)**2);
   return symmetrize evalfinal(amp,mu,nu);
end$   % Procedure STRONG is now defined. Let us use it.

factor f0; on time;

strong(pim(-p1.ee)*pip(-p2.ee)*pi0(p3.ee)*pi0(p4.ee));

     2   2
(2*F0 *PI *(4*P1.P2 + 2*P1.P3 + 2*P1.P4 + 2*P2.P3 + 2*P2.P4 + 4*P3.P4 +

           MUU2 + MUD2) + 3*( - P1.P2*P3.P4 + P1.P3*P2.P4 + P1.P4*P2.P3))

      4   2
/(12*F0 *PI )

TIME: 18721 MS
```

The described technique have been used for calculations of amplitudes of $K \to 2\pi$, $K \to 3\pi$ decays [5]. The results describe branchings and slope parameters of these decays, and also predict the parameters of direct CP violation — asymmetries for charged kaon decays, and the modification of Li-Wolfenstein relation for ϵ'_{+-0} parameter for $K^0 \to \pi^+\pi^-\pi^0$ decay. The direct CP violation has been fixed here by the results of NA31 experiment [7]

$$\mathrm{Re}(\epsilon'/\epsilon) = (3.3 \pm 1.5) \cdot 10^{-3}.$$

The usage of computer algebra has enabled us to take into account the following effects:

- p^4 contribution to strong interaction Lagrangian and the corresponding contributions to weak currents (utilization of the results of bosonization of quark Lagrangians);
- renormalization of Weinberg-Salam weak interaction by hard gluons leading to Shifman-Vainstein-Zakharov type Lagrangian;
- pole diagrams;
- violation of isotopic symmetry leading to (π–η–η')-mixing;
- meson rescattering by the method of superpropagator regularization of one-loop diagrams.

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Использование системы REDUCE для вычисления
низкоэнергетических процессов амплитуд
в киральной модели квантовой хромодинамики

Описывается расширение возможностей REDUCE для вычислений различных сильных и слабых процессов для нонета псевдоскалярных мезонов, описываемых киральными лагранжианами с высшими производными. Главными нетривиальными трудностями являются получение амплитуды процесса по лагранжиану, описывающему сильные и слабые процессы, а также преодоление некоторых недостатков REDUCE, таких как отсутствие аргументов у типа данных matrix и некоторых физических операций с операторами частиц. Описываемый пакет процедур позволяет вычислять амплитуды сильных и слабых процессов простым указанием участвующих частиц и их импульсов.

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REDUCE Usage for Calculation of Low-Energy
Process Amplitudes in Chiral QCD Model

We describe the extension of REDUCE capabilities for the calculations of various strong and weak processes in the pseudoscalar meson nonet region, governed by the chiral Lagrangians with higher derivatives. The main non-trivial difficulties are to obtain the process amplitude from the Lagrangian, describing the strong and weak interactions, and also to overcome some REDUCE deficiencies such as the lack of arguments in the matrix data type and of some physical operations with the particle operators. This package of procedures allows one to calculate the amplitudes of the strong and weak processes by simple specifying the particles involved and their momenta.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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