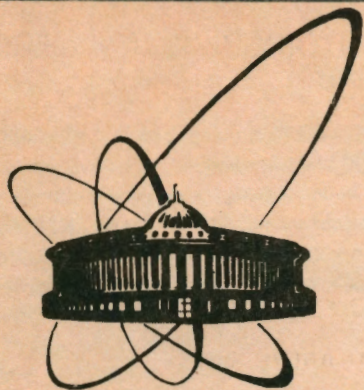


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СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
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NEWTONIAN ITERATIVE SCHEME
FOR SOLVING SCHWINGER-DYSON
EQUATION FOR A QUARK

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Introduction

Recently we have shown [1,2] that the continuous analog of Newton's method [3] together with the continuation method [4] can be used successfully for the computational investigation of dynamic properties of quarks in a hadron by means of the Schwinger-Dyson (SD) equation with the oscillator potential. The numerical scheme reduced the solution of the nonlinear differential equation to the solution of a sequence of boundary value problems for linear ordinary differential equations by Newton iterative scheme with special choices of iterative parameters and initial conditions for the iterations.

In this paper we apply a modification of Newtonian iterations [5] with the continuation method for solving SD integro-differential equation describing the self-energy of the quark with arbitrary values of both the current quark mass (m^0) and coupling constant (α) of the Coulomb potential that together with the oscillator potential contribute to the integral kernel of the equation. As a result, we obtain the computational scheme reducing the solution of the input problem to the solution of a sequence of linear differential boundary value problems. This scheme has a convenient algorithm for a assignment of initial conditions as a function of m^0 , α due to special choice of the iterative parameter as a step of integrating the evolutionary equation of the continuous analogy of Newton's method. *DYSON* program for IBM PC/AT computer written in FORTAN is presented. This program can be used for the study of the dynamics of quarks forming a hadron as well as for solving the Bethe-Salpeter BS equation describing the spectra of the bound states of quarks (as mesons, baryons and exotics)

2.Schwinger-Dyson equation for a quark

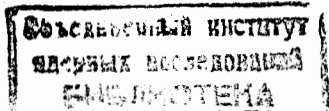
In the potential quark models inspired by quantum chromodynamics the SD equation is used as an effective tool for study of the constituent quark dynamics [1,2], [6]-[14]. Usually this equation has the following easy-to-solve form

$$(Z_m - 1)m^0 \cos\varphi(p) - (Z - 1)p \sin\varphi(p) = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V(|\mathbf{p} - \mathbf{q}|) [\sin\varphi(q) \cos\varphi(p) - \frac{p\mathbf{q}}{pq} \cos\varphi(q) \sin\varphi(p)] \quad (1)$$

where Z_m and Z are the renormalization constants providing finiteness of the equation in ultraviolet region ($p, q \rightarrow \infty$). An explicit form of the potential, $V(|\mathbf{p} - \mathbf{q}|)$, depends on the used model. For a massless ($m^0 = 0$) quark equation (1) has been solved using the oscillator potential,

$$V_{osc}(|\mathbf{k}|) = \frac{4}{3} V_0 \left(\frac{\partial}{\partial \mathbf{k}} \right)^2 \delta(\mathbf{k}), \quad (2)$$

with the phenomenological parameter V_0 defined from light meson spectra [7]. This potential yields no ultraviolet divergency that is $Z_m = Z = 0$. Equation (1) with $m^0 = 0$ and (2) takes the form of sine-Gordon type differential equation, the numerical solution of which has been obtained in ref [7]. This equation with nonzero bare quark



mass, $m^0 \neq 0$, in the oscillator potential approximation has been solved in refs [2,13] using the iterative scheme (with a continuation of the parameter m^0) based on the continuous analogy of Newton's method CAMEN.

Application of potential (2), indeed, is restricted to only light quarks, $m^0 \leq (\frac{4}{3}V_0)^{\frac{1}{2}}$. The "physical" potential adequated for describing all quarks has an additional Coulomb term,

$$V(|\mathbf{k}|) = V_{osc}(|\mathbf{k}|) + V_{col}(|\mathbf{k}|), \quad (3)$$

where

$$V_{col}(|\mathbf{k}|) = \frac{4}{3} \frac{4\pi\alpha}{k^2}.$$

Equation (1) with potential (3) is the integro-differential equation which can be written as

$$F_{m^0, \alpha}[\varphi(p)] \equiv F_{m^0, \alpha}^{osc}[\varphi(p)] + F_{m^0, \alpha}^{col}[\varphi(p)] \quad (4)$$

where

$$F_{m^0, \alpha}^{osc}[\varphi(p)] = \varphi''(p) + \frac{2}{p}\varphi'(p) - 2p\sin\varphi(p) + \frac{\sin 2\varphi(p)}{p^2} + 2m^0 \cos\varphi(p),$$

$$F_{m^0, \alpha}^{col}[\varphi(p)] = \cos\varphi(p) \left\{ \frac{2\alpha}{3\pi} \int_0^\infty dq \frac{q}{p} Q_0(p, q) \left[\sin\varphi(q) - \frac{m^0}{\sqrt{q^2 + m^0{}^2}} \right] \right\} - \sin\varphi(p) \left\{ \frac{2\alpha}{3\pi} \int_0^\infty dq \frac{q}{p} Q_1(p, q) [\cos\varphi(q) - 1] \right\}$$

$$Q_0 = \ln \left| \frac{p+q}{p-q} \right|, \quad Q_1 = \frac{p^2 + q^2}{2pq} Q_0(p, q) - 1$$

the prime means the derivative over $p = |\mathbf{p}|$. Here the following renormalization constants are used:

$$Z_m = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V_{col}(|\mathbf{p}-\mathbf{q}|) \frac{m^0}{\sqrt{q^2 + (m^0)^2}},$$

$$Z = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V_{col}(|\mathbf{p}-\mathbf{q}|) \frac{pq}{pq}.$$

The corresponding boundary conditions are

$$\varphi(0) = \frac{\pi}{2} \quad (5)$$

$$\varphi(\infty) = 0. \quad (6)$$

In this paper we represented CAMEN scheme and DYSOON program to solve problem (4)-(6) using the iteration scheme based on the continuous analogy of Newton's method combined with the continuation method.

2. Computational scheme.

In order to solve the boundary value problem (4)-(6) in the region $0 \leq p < \infty$ by means of a numerical method the asymptotical conditions (6) must be transferred to the finit boundary P_{max} of the interval $0 \leq p \leq P_{max}$, $P_{max} \gg 1$. This task requires redefinition of the asymptotic formula in the interval $(0 \leq p < \infty)$, and estimation of accuracy of this approximation. In general, one can carry out such an estimation only numerically using the results obtained for a range of values of the introduced parameter, P_{max} .

To redefine condition (6) in P_{max} , we can use a behavior of function $\varphi(p)$ at $p \rightarrow \infty$

$$\varphi(p \rightarrow \infty) \sim \frac{m^0}{\sqrt{p^2 + (m^0)^2}}. \quad (7)$$

So the modified for numerical calculations boundary conditions can be chosen as

$$\mathcal{N}[\varphi] = \begin{cases} \varphi(0) = \frac{\pi}{2} \\ \varphi(P_{max}) = \frac{m^0}{\sqrt{P_{max}^2 + (m^0)^2}} \end{cases}. \quad (8)$$

It is just these conditions that are used to solve the equation by using the Newton iterative scheme, with three parameters which are m^0 , and α and P_{max} .

2.1 Newton iteration scheme.

The Newton iterative scheme is based on the continuous analogy of Newton's method [3]. According to it, nonlinear equation (4) is replaced with the evaluation process over continuous parameter $t (0 \leq t < \infty)$

$$F'_{m^0, \alpha}[\varphi(p, t)] \frac{d\varphi(p, t)}{dt} = -F_{m^0, \alpha}[\varphi(p, t)]$$

$$\mathcal{N}'[\varphi(p, t)] \frac{d\varphi(p, t)}{dt} \Big|_{p=P_{max}} = -\mathcal{N}[\varphi(p, t)] \Big|_{p=P_{max}} \quad (9)$$

with $\varphi(p, 0) = \varphi_0(p)$ (in the initial approximation) where $F'_{m^0, \alpha}[\varphi(p)]$, $\mathcal{N}'[\varphi]_{p=P_{max}}$ are Frechet derivative, given in Appendix A.

In ref. [3] it is proved that evaluation process (9) in the limit of $t \rightarrow \infty$ converges, from the successfully chosen initial function φ_0 , to a solution of initial equation (4) if the following conditions are provided: i) in a local region there exists any nontrivial solution of the initial equation; ii) in a neighbourhood of this solution the function $F_{m^0, \alpha}[\varphi(p)]$ is "smooth"; iii) there exists the bounded operator $(F')^{-1}$.

Using Euler's method for discretizing process (9) $t_0, t_{k+1} = t_k + \tau_k, (k = 0, 1, 2, \dots)$ one gets the iteration scheme which is a generalization of classical Newton's method

$$F'_{m^0, \alpha}[\varphi_k(p)] \vartheta_k = -F_{m^0, \alpha}[\varphi_k(p)] \quad , \quad \varphi(p, t_k) = \varphi_k(p) \quad ,$$

$$\mathcal{N}' \vartheta_k = -\mathcal{N}[\varphi_k] \quad , \quad (10)$$

$$\varphi_{k+1} = \varphi_k + \tau_k \vartheta_k$$

where τ_k is the iteration parameters (the Euler method integration step) by choosing which one can provide an optimal condition for a convergence of the iterative process [3,14].

Thus, the Newtonian iteration for problem (4) and (8) consists in the solution of a linear problem (10) at every k -step.

2.2 Difference scheme

Equation (10) is a boundary value problem for linear in homogeneous integro-differential equation for ϑ_k . Straight application of a difference approximation with a quadrature formula of to this equation would lead to a linear algebraic equation with the completed matrix. However, such a way may require a large memory capacity of a computer and provide low speed of computations Mega flops. To avoid this defect we rewrite equation (10) in the following form

$$F_{m^0, \alpha}^{(1)}[\varphi_k(p)]\vartheta_k + F_{m^0, \alpha}^{(22)}[\varphi_k(p)]\vartheta_k = -F_{m^0, \alpha}[\varphi_k] - F_{m^0, \alpha}^{(21)}[\varphi_k(p)]\vartheta_{k-1}, \quad (11)$$

$$\mathcal{N}'\vartheta_k = -\mathcal{N}[\varphi_k] \quad (k = 0, 1, 2, \dots) \quad (12)$$

where the notations of Appendix A are used. Sufficient difference of this equation from equation (10) is that integral operator $F_{m^0, \alpha}^{(21)}$ is related to ϑ_{k-1} (i.e. to the solution of the previous step, $k-1$) but no to ϑ_k . As a result, equation (11) at every fixed k -step is a linear in homogeneous differential equation for $\vartheta_k(p)$.

There are several methods which may be used for solving boundary problem (11) and (12). One of the well known methods is the difference [15,16] Using in equation (11) the tree-point difference approximation of second order accuracy $O(h^2)$, h is the step of uniform grid: ($p_n = (n-1)h, n = 1, 2, \dots, N+1, h = \frac{E_{max}}{N}$) we get the following coupled algebraic equations with a three-diagonal matrix:

$$\begin{aligned} & \frac{1}{h^2} \left(\vartheta_{k(n+1)} - 2\vartheta_{k(n)} + \vartheta_{k(n-1)} \right) + \frac{2}{p_n} \left(\frac{\vartheta_{k(n+1)} - \vartheta_{k(n-1)}}{2h} \right) - 2p_n \cos \varphi_k(p_n) \vartheta_{k(n)} + \\ & + \frac{2}{p_n^2} \cos 2\varphi_k(p_n) \vartheta_{k(n)} - 2m^0 \sin \varphi_k(p_n) \vartheta_{k(n)} + F_{m^0, \alpha}^{(22)}[\varphi_k(p_n)]\vartheta_{k(n)} = \\ & - F_{m^0, \alpha}[\varphi_k(p_n), h] - F_{m^0, \alpha}^{(21)}[\varphi_k(p_n), h]\vartheta_{k-1} \end{aligned} \quad (13)$$

$$\mathcal{N}'\vartheta_k = -\mathcal{N}(\varphi_k) \quad (14)$$

where $F_{m^0, \alpha}[\varphi_k(p_n), h]$ and $F_{m^0, \alpha}^{(21)}[\varphi_k(p_n), h]$ are the discrete quantities obtained from the operators F and $F^{(21)}$, respectively (given in Appendix A) by using the Simpson formula. Whereas the problem (13) and (14) can be solved by means of the alternating direction implicit method.

Thus, to solve problem (4)-(6) with the physical parameters m^0 and α we arrived at the following procedure: by some manner one has to choose the initial approximation

$\{\varphi_0(p_n), n = 1, 2, \dots, N\}$ (which will be discussed in next Subsection), and solve coupled equations (13) with (14) to obtain the first step solutions which will be used as initial approximation in order to obtain the next step solutions $\{\varphi_1(p_n), n = 1, 2, \dots, N\}$. Then these solutions will be used as initial approximation in order to obtain the next step solutions $\{\varphi_2(p_n), n = 1, 2, \dots, N\}$, and so on.

Such an iteration process is continued to achieve either the given total number of iterations (in the case of convergence process) or a required accuracy, for example, controlled by the discrepancy condition

$$\delta_k = \| F_{m^0, \alpha}[\varphi_{kn}(p_n)] \| < \varepsilon, \quad (15)$$

where φ_{kn} is the difference scheme solution of problem (13) and (14) corresponding to k - iteration step, $\varepsilon > 0$ is the given norm of operator F .

Iteration parameter τ_k can be given optionally or chosen according to a definite rule. Here we represent some algorithms for choosing τ_k which are successfully used in practice [17] i) A simple algorithm is a choice of $\tau_k = \tau_0$ that corresponds to a constant step of the integration by Euler's method. This algorithm with a small enough τ_0 is usually used when one can not define the good initial approximation, for the purpose of obtaining a convergence from this approximation. Nevertheless, the expected convergence is too slow. Notice, when $\tau_0 = 1$ we have classical Newton's method. ii) The parameter τ_k can be chosen as

$$\tau_k = \begin{cases} \min(1, 2\tau_{k-1}) & \text{if } \delta_k < \delta_{k-1} \\ \max(\tau_0, \frac{\tau_{k-1}}{2}) & \text{if } \delta_k \geq \delta_{k-1} \end{cases}$$

where δ_k is the discrepancy with respect to a uniform grid metric. This algorithm is convenient for the successfully chosen initial approximations. It provides speed convergence, however, may be unstable far from the solution. iii) A more stable algorithm [17] which we used in *DYSON* program is a choice of τ_k

$$\tau_k = \begin{cases} \min(1, \frac{\tau_{k-1} \delta_{k-1}}{\delta_k}) & \text{if } \delta_k < \delta_{k-1} \\ \max(\tau_0, \frac{\tau_{k-1} \delta_{k-1}}{\delta_k}) & \text{if } \delta_k \geq \delta_{k-1} \end{cases} \quad (16)$$

Such an algorithm provides a convergence for a large class of initial approximations.

2.3 Method of extrapolation over a parameter

The Newtonian scheme provides a convergence to the solution of equation (4) if only the initial approximations are close to the solution. Therefore, one has to define a successful initial approximation.

In solving the problems depending on physical parameters (like m^0, α) the choice of the initial approximation can be simplified at the asymptotical values of the parameters. Such a continual transition [4] from a simple task to complicate one is allowed in the methods of extrapolation over parameter [15,18]. An application of these method can be considered as a possibility of extension of a convergence region of the iteration

method [18], that is as a manner of obtaining more success initial approximations by going over the parameters [1,2].

In more simple realization of a extrapolation method [1,2,18] one uses a natural dependence of the equations on the parameters of the investigated model. In our case, equation (4) contains two such parameters, m^0 and α (V_0 is fixed).

The solutions to problem (4)-(6) for the given values of the parameters, $m^0 = M$ and $\alpha = A$, in *DYSON* program we used the following way. The investigated intervals of the parameters, $0 \leq m^0 \leq M$ and $0 \leq \alpha \leq A$, are divided by nodes:

$$\{m_i^0; i = 1, 2, \dots, J, m_1^0 = 0, m_J^0 = M\}, \quad (17)$$

$$\{\alpha_j; j = 1, 2, \dots, J, \alpha_1 = 0, \alpha_J = A\}. \quad (18)$$

First, in integro-differential equation (4) with boundary conditions (5) and (6) one puts $\alpha = 0$ and solves the reduced differential equations for the range of parameters, (17), using difference scheme ((13),(14)). It is known that nodes m_i^0 one can by chosen so that for a given $m^0 = m_i^0$ the iterations converge to a solution from the initial approximation which is solution to problem ((13),(14)) in $m^0 = m_{i-1}^0$ [18]. The calculations end up when the corresponding subroutine of program *DYSON* obtains the solution φ_{kn} in $m^0 = M$ and $\alpha = 0$ with the required accuracy (15). Then, using the solution φ_{kn} as the initial approximation, one repeats such a procedure over a set of the parameter α to achieve the required accuracy.

2.4 Analysis of the calculation procedure and numerical results

In *DYSON* program the system controlling the calculation procedure is provided. The numerical solution of problem (4)-(6) involves the following parameters:

m^0 and α is the physical parameters included in the equation.

P_{max} is the parameter reducing a singular boundary problem to a finite region problem;

h is the step of the difference scheme

k is the number of the Newtonian iterations.

For estimation of an accuracy of the numerical solution the following notations are used:

$\varphi^*(p)$ is the exact solution to boundary problem (4)-(6);

$\varphi(p, P_{max})$ is the solution to the problem representing a finite region approximation to (4)-(8);

$\varphi(p, P_{max}, h)$ is the exact solution to difference problem ((13),(14));

$\varphi(p, P_{max}, h, k)$ is the approximation solution of problem ((13),(14)) obtained by means of k Newtonian iterations.

Then, for the grid metric the following estimations are valid

$$\|\varphi^*(p) - \varphi(p, P_{max}, h, k)\| \leq \|\varphi^*(p) - \varphi(p, P_{max})\| + \|\varphi(p, P_{max}) - \varphi(p, P_{max}, h)\| + \|\varphi(p, P_{max}, h) - \varphi(p, P_{max}, h, k)\|. \quad (19)$$

From ref. [3] it follows that

$$\begin{aligned} \|\varphi(p, P_{max}) - \varphi(p, P_{max}, h)\| &\leq C_1 h^2, \quad C_1 > 0 \\ \|\varphi(p, P_{max}, h) - \varphi(p, P_{max}, h, k)\| &\leq C_2 \delta_k, \quad C_2 > 0 \end{aligned} \quad (20)$$

with

$$\lim_{k \rightarrow \infty} \delta_k = 0$$

where δ_k is the discrepancy resulting from the substitution of the approximated solution $\varphi(p, P_{max}, h, k)$ into the grid boundary problem. Concerning the first term on the right side of estimation (19) it is natural to suggest that

$$\lim_{P_{max} \rightarrow \infty} \|\varphi^*(p) - \varphi(p, P_{max})\| = 0.$$

However, it is hard to get quantitative estimations, and the behavior of this quantity (the first term of r. h. s. of (19)) can be analyzed only by the results of calculations performed for a rising values of P_{max} . Consequently, it is advisably to choose the calculation scheme parameters P_{max} and k so that

$$\|\varphi^*(p) - \varphi(p, P_{max}, h, k)\| \approx Ch^2. \quad (21)$$

This allows one to correct the numerical solution, $\varphi(p, P_{max}, h, k)$, by Richardson's extrapolation [19]. In Tables 1-3 we quoted the numerical results displaying the convergence of the difference solutions for various values of α when other parameter, m^0 , being fixed. From the tables we see that the parameters of the calculation scheme (P_{max} , k , τ_k , and h) are fixed so that the convergence corresponds to estimation (21).

We have tested the convergence of the grid solution $\varphi(p, P_{max}, h, k)$ for the variations of P_{max} ($P_{max} = 6, 7, 8$), and obtain the agreement of the results up to order 10^{-2} .

In figs 1 and 2 the solutions of the SD equation and the "dressed" quark energies (defined Appendix B) for several quark masses and $\alpha = 0.2$ are depicted.

3. The structure of the program *DYSON*

The considered above scheme of the numerical solution of problem (13)-(14) is realized *DYSON* program, in FORTRAN, and calculations are performed by CDC-6500 and IBM PC/AT computers.

In the main program, *DYSON*, the following parameters should be indicated:

N - is the maximum dimension for all the arrays are being filled by values of lattice nodes with the given steplength, H .

EPS - A small quantity on that a iteration process of finding of the initial approximation is end up.

AM - is the physical parameter (the mass of a bare quark)

$ALPHAS$ - is the physical parameter (the coupling constants of the Coulomb potential)

The subroutine *INFUN* that should be called *CALL INFUN* gives the initial algorithm of calculation of the initial approximation, and it provided with the print options. To call this subroutine, with

CALL NEWW1 (T0, EPS, ITER, MPR, NHIT, LST, IBT)

the following parameters should be specified:

T0 - is the initial step, $\tau_0 = 0.1$.

ITER - is the number of ending of the Newton iterations.

MPR, NHIT - is the whole numbers indicating the print options

LST = 3 - is the choice of τ_k (sec.(16))

IBT - is the key for using the switching function $(1 - e^{\tau_0})$:

= 1, the function is included;

= 0, not

The subroutine *DELTA1* calculates the discrepancy and used as

CALL DELTA1 (N, H, DEL, RDEL),

which contains the following additional input parameters:

DEL - is the quantity of the discrepancy

RDEL - is the quantity of the node in which a maximal deviation is achieved.

The subroutine *PROGON1* realizes the method of "progonka", using as

CALL PROGON1 (BETA)

BETA - is the value of the switching function.

The subroutine *TAUKA* calculates the iteration parameter τ_k , using as

CALL TAUKA (T0, TK, TKM1, DELK, DELKM1, LST)

an additional input parameters are

TK - is the iteration parameter τ_k , (16)

TKM1 - is the iteration parameter τ_{k-1} .

DELK - is the discrepancy, δ_k .

DELKM1 - is the discrepancy, δ_{k-1} .

The subroutine *DERIVE* calculates a devivative of the function $\varphi(p)$ with accuracy of h^2 , using as

CALL DERIVE (N, X, Y, DY),

an additional input parameters are

X(N) - is the lattice nodes.

Y(N) - is the values of the function, $\varphi(p)$, at nodes.

DY(N) - is the derivatives of the function, $\varphi'(p)$.

The subroutine *SIM* calculates an integral by Simpson's rule with an accuracy of $O(h^4)$, using as

CALL SIM (H, N, Y, RES)

an additional input parameters are

H - is the step of the integration

Y(N) - is the integrand function

RES - is the value of the integral

The subroutine *ENERGY* calculates the quark energy, $E(p)$, defined in Appendix B, using as *CALL ENERGY*

4. Conclusions

We proposed the numerical method for solution of the SD equation with the potential $V_0 r^2 - \alpha/r$. This equation is transformed to the value boundary problem for the integro-differential equation depending on the physical parameters (m^0 and α). The method consists the iterative scheme of the continuous analogy of Newton's method. In this scheme in order to solve the problem (4)-(6) with the given accuracy one has to choose the grid parameters (h, P_{max}) by solving the problem for subsequent values of these parameters.

For the iterative parameter we used expression (16) which requires a minimal number of the iterations. We used the method of a continuation over the parameters (m^0 and α) which provides an extantsion of the convergence region of the iterative scheme.

We tested the proposed method on the exactly solvable models. The method can be used as a powerfull tool for study the nonperturbative phenomena of QCD.

The authors would like to thank T.Strizh for helpful discussions.

Appendix A

The kernel of integral operator $F_{m^0, \alpha}^2[\varphi(p)]$ is singular one. Therefore for numerical calculations it is convience to represent in the following form

$$F^{(2)} = F^{(21)} + F^{(22)}$$

where

$$F^{(21)} = \frac{2\alpha}{3\pi} \left\{ \int_0^{P_{max}} dq Q_0(p, q) [W_{22}(p, q) - W_{22}(p, p)] - \frac{1}{p} W_{33} \right\},$$

$$F^{(22)} = \mathcal{L}(p) W_{22}(p, p),$$

$$W_{22}(p, q) = \frac{1}{p} \{ \cos\varphi(p) [q \sin\varphi(q) - m_0] - \sin\varphi(p) \left[\frac{p^2 + q^2}{2p} (\cos\varphi(q) - 1) \right] \};$$

$$W_{33} = \int_0^{P_{max}} dq q (\cos\varphi(q) - 1);$$

$$\mathcal{L}(p) = \int_0^{P_{max}} Q_0(p, q) dq = (p + P_{max}) \ln(p + P_{max}) + (p - P_{max}) \ln(|p - P_{max}|);$$

Then the l.h.s. of equation (9) can be written as

$$F'(p, \varphi, m_0, \alpha) \vartheta = (F'_1 + F'_{21} + F'_{22}) \vartheta$$

where $\vartheta = \frac{d\varphi}{dt}$

$$F_1' \vartheta = \vartheta'' + \frac{2}{p} \vartheta' - 2p \cos \varphi(p) + \frac{2}{p^2} \cos 2\varphi(p) \vartheta - 2m_0 \sin \varphi(p) \vartheta ,$$

$$F_{21}' = \frac{2\alpha}{3\pi} \left\{ \int_0^{P_{max}} Q_0(p, q) [W_{221}(p, q) \vartheta(p) + W_{222}(p, q) \vartheta(q) - W_{223}(p) \vartheta(p)] + \frac{1}{p} W_{331} \right\} ;$$

$$F_{22}' y = W_{223}(p) \mathcal{L}(p) \vartheta(p) ;$$

$$W_{221} = -\frac{1}{p} \left\{ \sin \varphi(p) [q \sin \varphi(q) - m_0] + \cos \varphi(p) \left[\frac{p^2 + q^2}{2p} (\cos \varphi(q) - 1) \right] \right\} ;$$

$$W_{222} = \frac{1}{p} \left\{ q \cos \varphi(p) \cos \varphi(q) + \frac{p^2 + q^2}{2p} \sin \varphi(p) \sin \varphi(q) \right\} ,$$

$$W_{223} = \cos \varphi(p) + \frac{m_0}{p} \sin \varphi(p) ,$$

$$W_{331} = \int_0^{\infty} dq q \sin \varphi(q) y(q) ,$$

Appendix B.

The energy of "dressed" quark is defined by

$$E(p) = E_{osc}(p) + E_{col}(p)$$

$$E_{osc}(p) = p \cos \varphi(p) + m_0 \sin \varphi(p) - \frac{1}{2} [\varphi'(p)]^2 - \frac{\cos^2 \varphi(p)}{p^2} ,$$

$$E_{col}(p) = \frac{2\alpha}{3\pi} \left\{ \sin \varphi(p) \left(\int \frac{dq}{p} Q_0 [\Phi_1(q) - \Phi_1(p)] + \frac{1}{p} \Phi_1(p) \mathcal{L}(p) \right) + \cos \varphi(p) \left(\int \frac{dq}{p} Q_0 [\Phi_2(p, q) - \Phi_3(p)] + \frac{1}{p} \Phi_3(p) \mathcal{L}(p) \right) \right\} ,$$

$$\Phi_1(p) = p \sin \varphi(p) - m_0 , \quad \Phi_2(p, q) = \frac{p^2 + q^2}{2p} (\cos \varphi(q) - 1) , \quad \Phi_3(p) = p (\cos \varphi(p) - 1) .$$

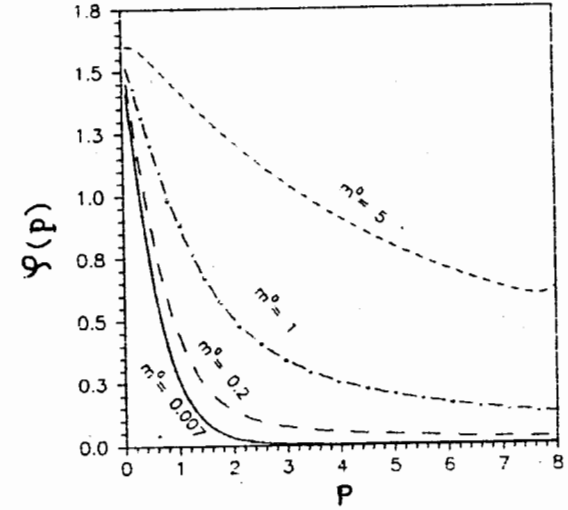


Fig. 1. The solutions SD equation (in $\alpha = 0.2$) for the several values of parameter m^0 (the quark mass).

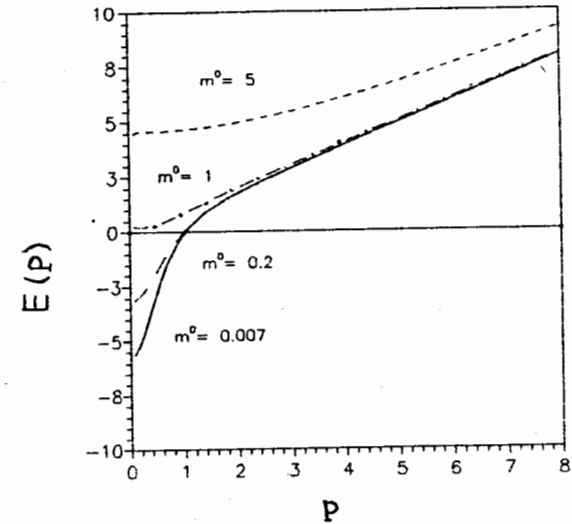


Fig. 2. The energy of the "dressed" quark (in $\alpha = 0.2$) for the variations of the quark mass (m^0).

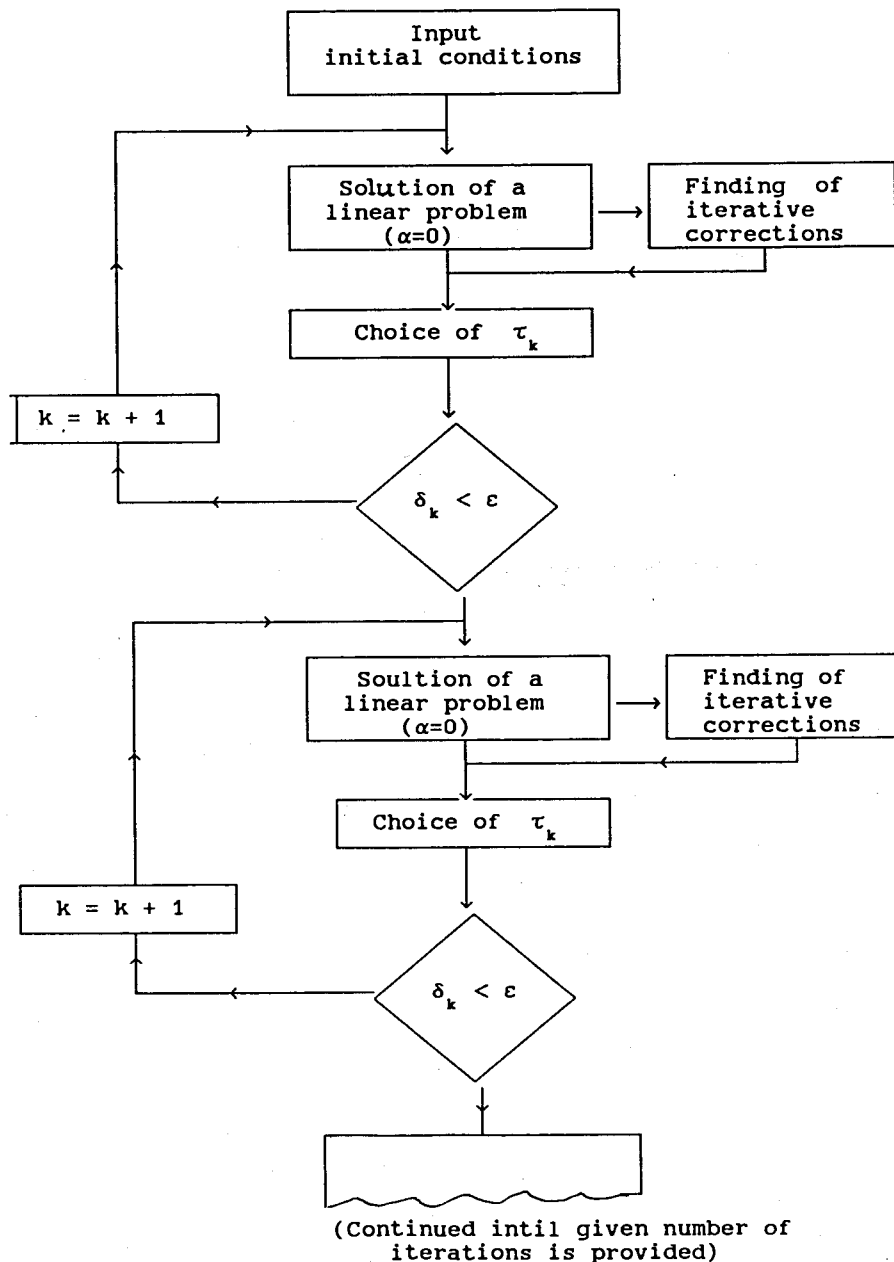


Fig. 3. The iterative scheme of the numerical realization of the solution of equation (9).

Table 1. The solutions, $\varphi(p)$, on gridirons $h, h/2$ and $h/4$, where h is the grid parameter, $h = 0.06$, $\sigma = (\varphi_h - \varphi_{h/2})/(\varphi_{h/2} - \varphi_{h/4}) \approx 4$, $P_{max} = 6$, $m^0 = 0.007, \alpha = 0$

p	φ_h	$\varphi_{h/2}$	$\varphi_{h/4}$	σ
0.06	1.45037198	1.45024562	1.45021689	4.3983
0.54	0.642634273	0.643247783	0.643400729	4.0113
1.02	0.239167869	0.239576370	0.239678353	4.0056
1.50	0.829140693E-01	0.830718428E-01	0.831112638E-01	4.0023
2.04	0.245793592E-01	0.246207397E-01	0.246310793E-01	4.0022
2.52	0.894773286E-02	0.895802770E-02	0.896059908E-02	4.0036
3.00	0.403698301E-02	0.403898628E-02	0.403948594E-02	4.0093
3.48	0.246771076E-02	0.246793963E-02	0.246799644E-02	4.0287
4.02	0.184914726E-02	0.184911222E-02	0.184910337E-02	3.9605
4.50	0.159100327E-02	0.159097661E-02	0.159096997E-02	4.0175
4.98	0.142095517E-02	0.142094598E-02	0.142094365E-02	3.9500
5.64	0.124689774E-02	0.124689913E-02	0.124689948E-02	4.0000
5.88	0.119283842E-02	0.119284017E-02	0.119284063E-02	3.7500
5.94	0.117971376E-02	0.117971480E-02	0.117971515E-02	3.0000
6.00	0.116666663E-02	0.116666663E-02	0.116666663E-02	0.0000

Table 2. The solutions, $\varphi(p)$, on gridirons $h, h/2$ and $h/4$, where h is the grid parameter, $h = 0.07$, $\sigma = (\varphi_h - \varphi_{h/2})/(\varphi_{h/2} - \varphi_{h/4}) \approx 4$, $P_{max} = 7$, $m^0 = 0.007, \alpha = 0$

p	φ_h	$\varphi_{h/2}$	$\varphi_{h/4}$	σ
0.07	1.43044317	1.43027508	1.43023765	4.4904
0.49	0.706986725	0.707791507	0.707991958	4.0149
1.05	0.224001244	0.224530503	0.224662602	4.0065
1.54	0.757063702E-01	0.759025291E-01	0.759515315E-01	4.0030
2.03	0.251110625E-01	0.251689311E-01	0.251833908E-01	4.0021
2.52	0.894277170E-02	0.895678997E-02	0.896029081E-02	4.0043
3.01	0.398248108E-02	0.398510927E-02	0.398576399E-02	4.0142
3.50	0.243176473E-02	0.243204250E-02	0.243211095E-02	4.0578
4.06	0.182166020E-02	0.182161015E-02	0.182159746E-02	3.9450
5.11	0.138297083E-02	0.138296059E-02	0.138295803E-02	4.0000
5.53	0.127415743E-02	0.127415417E-02	0.127415336E-02	4.0000
6.02	0.116829388E-02	0.116829388E-02	0.116829376E-02	0.0000
6.51	0.107874465E-02	0.107874663E-02	0.107874710E-02	4.2500
7.00	0.100000005E-02	0.100000005E-02	0.100000005E-02	0.0000

Table. 3. The solutions, $\varphi(p)$, on gridirons $h, h/2$ and $h/4$, where h is the grid parameter, $h = 0.06$, $\sigma = (\varphi_h - \varphi_{h/2})/(\varphi_{h/2} - \varphi_{h/4}) \approx 4$, $P_{max} = 6$, $m^0 = 0.007, \alpha = 0.2$

p	φ_h	$\varphi_{h/2}$	$\varphi_{h/4}$	σ
0.06	1.45386398	1.45373440	1.45370209	4.0111
0.54	0.661486323	0.662056327	0.662185788	4.4029
1.02	0.255236566	0.255638689	0.255728871	4.4590
1.50	0.929073021E-01	0.930754021E-01	0.931117386E-01	4.6262
2.04	0.297584292E-01	0.298122410E-01	0.298233032E-01	4.8645
2.52	0.117715644E-01	0.117921019E-01	0.117962351E-01	4.9689
2.94	0.609673560E-02	0.610732706E-02	0.610952498E-02	4.8189
3.00	0.563521404E-02	0.564502599E-02	0.564707769E-02	4.7823
3.54	0.328625250E-02	0.329212495E-02	0.329344603E-02	4.4452
3.96	0.254013925E-02	0.254466012E-02	0.254571741E-02	4.2759
4.02	0.246681343E-02	0.247118971E-02	0.247221743E-02	4.2583
4.50	0.203323620E-02	0.203668652E-02	0.203751400E-02	4.1697
5.04	0.171704323E-02	0.171971589E-02	0.172036455E-02	4.1202
5.52	0.148363062E-02	0.148555380E-02	0.148602459E-02	4.0851
5.88	0.126786297E-02	0.126861653E-02	0.126880279E-02	4.0456
5.94	0.122016622E-02	0.122057961E-02	0.122068194E-02	4.0398
6.00	0.116666663E-02	0.116666663E-02	0.116666663E-02	0.0000

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К проверке нарушения принципа относительности
путем исследования в накопительном кольце
локальных свойств спиновой прецессии
релятивистских электронов

Обсуждаются лоренцевская концепция физического вакуума, существование привилегированной системы отсчета, связанное с нарушением групповых свойств общих преобразований Лоренца без вращений, и возможность постановки эксперимента по обнаружению нарушения принципа относительности, обусловленного томасовской прецессией спина релятивистских электронов. Рассмотрены конкретные варианты постановки эксперимента, методы наблюдения и даны оценки требуемых рабочих параметров прибора.

Работа выполнена в Лаборатории сверхвысоких энергий ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1991

Перевод автора

Neganov B.S.

D1-91-96

On the Check-Up of the Violation of the Relativity
Principle by the Investigation of the Local Properties
of Spin Precession of Relativistic Electrons Conducted
at the Storage Ring

Lorentz's concept of physical vacuum, existence of a privileged reference frame related to violation of group properties of general Lorentz's transformations without rotation, and a possibility of setting up an experiment to detect violation of the principle of relativity due to Thomas spin precession of relativistic electrons are discussed. Specific variants of the experiment and methods of observation are considered, the necessary parameters of the device are estimated.

The investigation has been performed at the Laboratory of Superhigh Energies, JINR.

Communication of the Joint Institute for Nuclear Research, Dubna 1991