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E11-90-599

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THE DEMAGNETIZING FACTORS FOR THE RECTANGULAR SAMPLES

1990

1. Introduction

A number of physical experiments,e.g. Muon Spin Rotation (μSR),Nuclear Magnetic Resonance(NMR) can directly measure the field inhomogeneity in materials by observing the distribution of the procession frequencies (ω) of host spin. The μSR investigations have been applied extensively to high-T_c superconductors and other magnetic systems /1-7/. We will conceder here some estimations which may be used in the analysis of μSR evidence.Usually the samples to be tested are placed in magnetic field.The time histogram of muon decay positron emitted from positive muons stopped in the specimen presents a sinusoidal oscillation due to the muon precession around $\vec{H}_{ext}$ as:

$$N(t) = N_0 e^{-t/\tau_\mu} \left(1 + A\, G(t)\cos(\omega_i t+\phi)\right) + C . \qquad (1)$$

where τ_μ=2.2 msec is the muon life time,A is the initial asymmetry,$G(t)$ is the muon spin depolarisation function, $\omega_i = \gamma_\mu B_\mu$ is the muon procession frequency,γ_μ=2π0.01355 MHz/G is the gyromagnetic ratio of the muon.The depolarization results from the dephasing effect of the local magnetic field inhomogeneities .To estimate the full width of the magnetic field distribution in the sample ΔB, the μSR data are usually analyzed for a combination of the Gaussian and Lorenzian distributions of the depolarization function:

$G(t)= e^{-\sigma^2 t^2/2}$, and $G(t)= e^{-\sigma t}$,with the relaxation rate σ_{tot} $\sigma^2_{tot}=\gamma^2_\mu\langle\Delta B^2\rangle$, <...> indicates the appropriate average.

The μSR signal also allows one to determine the average local field $B_\mu$seen by implanted μ^+ in the sample.Most generally the total magnetic field at the μ^+ can be expressed as follows(Schenck 1985.):

$$B_\mu=B_{ext}-B_{Dm}+B_L+B_C+B_{Dip} \qquad (2)$$

where B_{ext} is the external magnetic field,$B_{Dm}=\hat{N}\cdot M$ is the demagnetization field stemming from the magnetization of the finite sample,$\hat{N}$ is the demagnetization tensor, $B_L=4\pi/3M$ is

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the so-called Lorentz field which is produced by the empty Lorentz sphere inside a magnetized environment, B_C is a contact hyperfine field at the muon, B_{Dip} is the net field from magnetic dipoles inside the Lorentz sphere. We shall consider only the contribution of the macroscopic demagnetization field to the total magnetic field B_μ. In addition to various inhomogeneities in the sample there are also demagnetization effects. These effects will result in a macroscopic field broadening which can be distinguished from the microscopic effects. In this context it is worth-while consedering for example high T_c superconductors. Using the distribution of local fields B_μ in the vortex state of a type-II superconductor one can determine the magnetic field penetration depth λ by means of relation:

$$\lambda^4=0.00371\Phi_0^2/\langle\Delta B^2\rangle_{vort} \qquad (3)$$

where Φ_0 is the elementary flux quantum, $\langle\Delta B^2\rangle_{vort}$ originates from the vortex distribution. The total second moment of B inside the sample may be schematically written as:

$$\langle\Delta B^2\rangle_{tot}=\langle\Delta B^2\rangle_\Sigma + \langle\Delta B^2\rangle_{Dem} \qquad (4)$$

where $\langle\Delta B^2\rangle_{Dem}$ is the contribution from the demagnetization fields, $\langle\Delta B^2\rangle_\Sigma$ is the sum of the rest contributions (including the broadening from the vortex structure $\langle\Delta B^2\rangle_{vort}$). Let us consider the second term of eq.(4) and point out the path how this limb may be evaluated. In real μSR experimental set up with longitudinal or transverse field geometry the component of the magnetic induction $\vec{B}$ in direction of the external magnetic field is observed. The corresponding component of magnetic field strength $\vec{H}$ in every point ξ_i of the sample Ω (see Fig.1) is expresed as:

$$H^z(\xi_i)= H^z_{ext}-4\pi N^{z\beta} M^\beta(\xi_i) \qquad (5)$$

where $\vec{H}_{ext}=|\vec{H}_{ext}|\vec{z}$. Taking the average over all samples points, (5) will be :

$$\langle H^z(\xi_i)\rangle= H^z_{ext}-4\pi\langle N^{zz}\rangle\langle M^z(\xi_i)\rangle \ ,\ \langle H^x\rangle=\langle H^y\rangle=0 \qquad (6)$$

where symmetry of the sample has been taken into account. It is known that in the magnitostatics the magnitude of the magnetic induction is defined in the following way:

$$B^\alpha(\xi_i)= H^\alpha(\xi_i)+4\pi M^\alpha(\xi_i) \ . \qquad (7)$$

Combining eqs.(5),(6),(7) we will obtain :

$$4\pi\langle N^{zz}\rangle\langle M^z(\xi_i)\rangle= H_{ext}-B^z(\xi_i)+4\pi\langle M^z(\xi_i)\rangle \qquad (8)$$

To calculate $\langle N^{zz}\rangle$ we used the ideal diamagnetic approximation. ($\vec{B}\equiv 0$ in the sample). In this case let us define the mathematical expectation of N^{zz} as:

$$\langle N^{zz}\rangle= \left\langle\left(1 - \frac{Hext}{H^z(\xi_i)}\right)\right\rangle \qquad (9)$$

By neglecting nondiagonal terms of tensor $\hat{N}$, the dispersion of N^{zz} may be evaluate for N^{zz} distribution in the specimen. Such approach is justified in our case as long as components of magnetization $M^x(\xi_i)$ and $M^y(\xi_i)$ are more less than $M^z(\xi_i)$. The smallness of X and Y magnetization components will be seen from farther computer simulation. In type-II superconductors the magnetic field H, magnetization M, demagnetization tensor $\hat{N}$ and the averaged magnetic induction (i.e. internal field seen by μ^+) B_μ in macroscopic approach are related as $H=B_\mu-4\pi M=H_{ext}-4\pi\hat{N}M$. If the μ^+ Knight shift in the probe is negligibly small (as, for example, in high-T_c compounds), then the observed frequency shift of μ^+ precession is related directly to the demagnetization field:

$$F=B_\mu-H_{ext}=4\pi(1-\langle N^{zz}\rangle)M \qquad (10)$$

The subsequent broadening of F due to distribution of $\hat{N}$ inside the specimen is $4\pi\Delta N^{zz}M$. Thus $\langle\Delta B^2\rangle_{Dem}$ may be expressed as:

$$\langle\Delta B^2\rangle_{Dem} = (4\pi M)^2 \langle(\Delta N^{zz})^2\rangle . \qquad (11)$$

The bulk magnetization of the sample M may be estimated from relation (10) using the value of $\langle N^{zz}\rangle$.Taking into account the contribution of $\langle\Delta B^2\rangle_{Dem}$ to $\langle\Delta B^2\rangle_{tot}$ one can refine the penetration depth value. The purpose of this paper is to estimate the mathematical expectation and the dispersion of N^{zz} for rectangular samples in real field geometry for μSR set up.However,in oder to find $\langle N^{zz}\rangle$ from (9),it is necessary to compute $H^z(\xi_i)$. For calculating the magnetic fields in the specimen the boundary integral equations method was used.It is noteworthy that the mathematical expectation and the dispersion of N components obtained for the ideal diamagnetic approximation are valid for various substances,which have the same form.

2.The method of boundary integral equations.

In order to calculate the strength of the magnetic field ,it is necessary to solve the three-dimensional Maxwell equations for a sample placed in a homogeneous magnetic field:

$$\begin{cases} \mathrm{div}\ \vec{B} = 0 \\ \mathrm{rot}\ \vec{H} = 0 \\ \vec{B} = \mu\vec{H} \end{cases} \qquad (12)$$

Besides,at the phase boundary the vectors $\vec{B}$ and $\vec{H}$ should satisfy the following conditions:$B_{1n}=B_{2n}, H_{1t}=H_{2t}$. The integral analog of (12) is

$$\vec{H}(\vec{a}) = \vec{H}_{ext} + \frac{\vec{\nabla}_{\vec{a}}}{4\pi} \int_{\Omega} (\vec{B}(\mathrm{x}) - \vec{H}(\mathrm{x})), \nabla_{\vec{a}}\left(\frac{1}{|\vec{x}-\vec{a}|}\right)\, d\,\Omega, \qquad (13)$$

where $\vec{H}_{ext}$ is the external field ,$\vec{a}$ is the observation point.Assuming that the magnetic permeability μ in Ω is constant, eq.(13) is reduced to the boundary integral equation:

$$\vec{H}(\vec{a}) = \vec{H}_{ext} - \frac{\vec{\nabla}_{\vec{a}}}{4\pi} \int_{d\,\Omega} \left(\frac{((\vec{B}(\vec{x}) - \vec{H}(\vec{x})), d\vec{S}_{\vec{x}})}{|\vec{x}-\vec{a}|} \right) \qquad (14)$$

For the ideal diamagnetic $\vec{B}(\vec{a})=0$ holds in Ω. Therefore $\mu=0$ in this case,and (14) becomes:

$$\vec{H}(\vec{a}) = \vec{H}_{ext} + \frac{\vec{\nabla}_{\vec{a}}}{4\pi} \int_{d\,\Omega} \frac{(\vec{H}(\vec{x}), d\vec{S}_{\vec{x}})}{|\vec{x}-\vec{a}|} \qquad (15)$$

Introducing the new variable $\sigma(\vec{x}) = (\vec{H}(\vec{x}), \vec{n}(\vec{x}))$,where $\vec{n}(\vec{x})$ is the external normal to the boundary $d\,\Omega$ at point $\vec{x}$ and multiplying (15) by $\vec{n}(\vec{a})$ we obtain :

$$\sigma(\vec{a}) = (\vec{H}(\vec{a}), \vec{n}(\vec{a})) + \frac{\partial}{\partial n_{\vec{a}}} \left(\frac{1}{4\pi} \int_{d\,\Omega} \frac{\sigma(\vec{x})\, dS_{\vec{x}}}{|\vec{x}-\vec{a}|} \right) . \qquad (16)$$

For the discretization of (16) the boundary $d\,\Omega$ is divided into boundary elements $\{\Omega_i\}$ so that $d\,\Omega = \bigcup_{i=1}^{L} \Omega_i$.Assuming that $\sigma(\vec{a})$ is constant in the element Ω_i:$\sigma = \sigma_i$,and choosing the observation points a_i as the center of Ω_i, we obtain a set of equations

$$\sigma_i = (\vec{H}_{ext}(\vec{a}_i), \vec{n}_i) + \sum_{j=1}^{L} \frac{\partial}{\partial n_{\vec{a}}} \cdot \frac{1}{4\pi} \int_{d\Omega} \frac{\sigma_j \, dS}{| \vec{x}-\vec{a} |} \Bigg|_{\vec{a} = \vec{a}_i} \quad (17)$$

where $\vec{n}_i$ is the external normal to $\Omega_i, i=1,\dots,L$. Shortly, expression (17) may be rewritten as :

$$\hat{\sigma} = \hat{H} + [\,A\,]\,\hat{\sigma} \quad (18)$$

where $\hat{\sigma} = (\sigma_1,\dots,\sigma_L)^T$, $\hat{H} = \left[(\vec{H}_{ext}(\vec{a}_1),\vec{n}_1),\dots,(\vec{H}_{ext}(\vec{a}_L),\vec{n}_L)\right]^T$,

and $[\,A\,]$ is an $L \times L$ full matrix.
For solving (18) the following iterative process was used:

$$\hat{\sigma}_{k+1} = \hat{H} + [\,A\,]\,\hat{\sigma}_k\,, \quad \hat{\sigma}_0 = 0., \; k=0,1,\dots,L. \quad (19)$$

3. The numerical results.

The numerical simulations were performed by using a program code realizing the above algorithm .The rectangular specimen placed in a homogeneous magnetic field is shown in figure 1. The values of $\vec{H}$ and $\vec{B}$ were calculated inside and outside the sample.

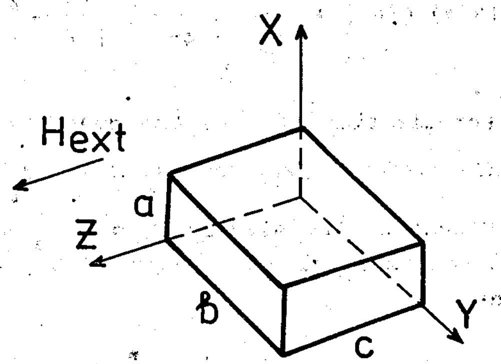

Figure 1.Schematic arrangement of the rectangular sample in magnetic field.

Due to symmetry or antisymmetry of the magnetic field of the sample the simulation was carried out for the 1/8 part of the sample.After solving eq.(19) the vector σ was used for recount magnetic field at an arbitrary point.The mathematical expectation of the demagnetization factors calculated from (9) and also the dispersion of N^{zz} distribution are shown in the Table.The dispersion is obtained from(9)by means of statistical analysis.For simplicity it is assumed that $|\vec{H}_{ext}|=1$ and $a=1$ .The third moment of N^{zz} distribution is less than 0.001 for all sample sizes.The values of H^X and H^Y components of magnetic field in the sample make up one tenth as many as the value H^Z component. It is straightforward to see that the mathematical expectation of N^{zz} obtained numerically are in good agreement with the analytical value of N for the infinite plate in magnetic field for the cases when $c \ll a$ and $c \ll b$ (N=0) or $a \ll b$ and $a \ll c$ (N=1).Figure 2. illustrates the spectral behaviour of N^{zz} in the probe for b=5,c=1÷10.

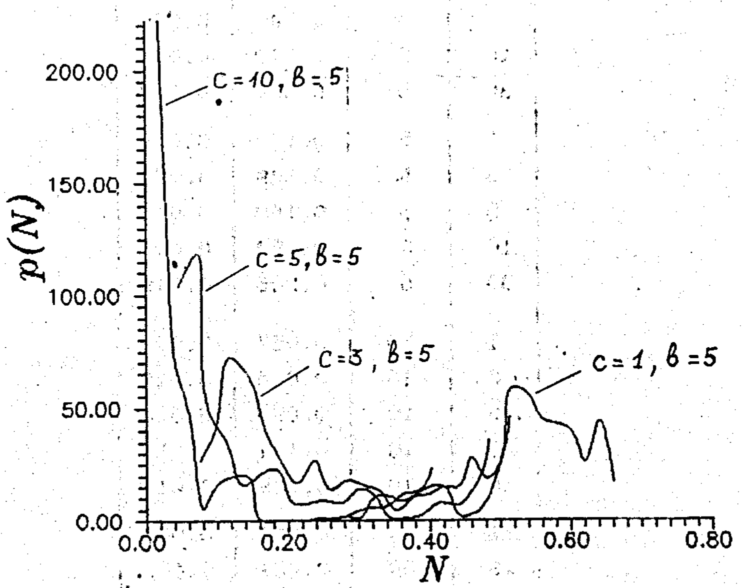

Figure 2.The probability distribution plot of N^{zz} for rectangular samples.

Table. The mathematical expectation and the dispersion of N^{zz} for rectangular samples

b	c	$\langle N \rangle$	$\langle \Delta N^2 \rangle$
1	0.2	0.718	0.007
3	0.2	0.782	0.006
7	0.2	0.804	0.008
10	0.2	0.806	0.005
30	0.2	0.814	0.004
1	1	0.365	0.015
3	1	0.462	0.011
5	1	0.488	0.010
10	1	0.509	0.008
30	1	0.525	0.007
1	3	0.158	0.020
3	3	0.226	0.021
5	3	0.249	0.020
10	3	0.272	0.019
30	3	0.290	0.017
1	5	0.105	0.016
3	5	0.149	0.020
5	5	0.169	0.019
10	5	0.190	0.019
30	5	0.206	0.019
1	10	0.057	0.009
3	10	0.085	0.013
5	10	0.099	0.013
10	10	0.111	0.014
30	10	0.129	0.013
1	30	0.017	0.001
3	30	0.031	0.002
5	30	0.038	0.004
10	30	0.045	0.004
30	30	0.056	0.005

where $\langle \Delta N^2 \rangle = \langle N^2 \rangle - \langle N \rangle^2$

From the Table one can see that the second moment of N does not exceed 0.03 over the sample sizes. For high T_c superconductors this means that the penetration depth λ, which relates to the frequency line width and the variance of the field intensity as $\lambda^4 \simeq 1/\langle \Delta B^2 \rangle$, must be corrected. The demagnetization broadening $\langle \Delta B^2 \rangle_{Dem}$ is related with the observed broadening $\langle \Delta B^2 \rangle_{exp}$ as:

$$\langle \Delta B^2 \rangle_{Dem} = \frac{\langle \Delta N^2 \rangle}{(1-\langle N^{zz} \rangle)^2} \langle \Delta B^2 \rangle_{exp} \qquad (20)$$

So, the maximum value of $\langle \Delta B^2 \rangle_{Dem}$ is attained then $\langle N^{zz} \rangle \simeq 1$ (a=1, b=7, c=0.2 in our sizes range). In this case $\langle \Delta B^2 \rangle_{Dem}$ is: $\langle \Delta B^2 \rangle_{Dem} = 0.2 \langle \Delta B^2 \rangle_{exp}$. The corresponding correction of the λ is defined from the relation :

$$\frac{d\lambda}{\lambda} = \frac{1}{4} \frac{d(\langle \Delta B^2 \rangle)}{\langle \Delta B^2 \rangle} \leq 0.05 \qquad (21)$$

For instance, in compounds ($La_{1.8}Sr_{0.15}CuO_4$) /4/ the value λ=2420±60 A may be corrected approximately by 60 A. Therefore the penetration depth λ in high - T_c ceramics is underestimated by not more than 5% .

4. Conclusion

We have dealt with the effect leading to additional macro-inhomogeneity of the internal magnetic field for rectangular probes. The space distribution of the magnetic field is computed for the ideal diamagnetic approximation. On the basis of this calculations, the mathematical expectation and dispersion of the demagnetizing tensor component are evaluated for a real field geometry. These estimations are valid for various substances, which have the same form. The demagnetization fields can have a very marked effect in compounds which have the demagnetization factor $\langle N \rangle \simeq 1$.

Acknowledgments.

The authors wish to thank Dr.V.Y Yushankhai and V.Y.Pomjakushin for helpful discussions.

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Received by Publishing Department
on December 29,1990.