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CALCULATIONS OF THE DEMAGNETIZATION FACTORS FOR THE CYLINDRICAL SAMPLES

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1. Introduction

In some physical experiments, e.g. for superconductivity investigations, the samples to be tested are placed in the magnetic field. The sample is magnetized and the external field is distorted near the surface of the sample, depending on the sample's shape and orientation of the external field. In this case the demagnetization field arises in every point of the sample and it is related to the magnetization as

$$\vec{H}_{dem} = N\,\vec{M}, \qquad (1.1)$$

where $\vec{M}$ is the magnetization, N is the proportionality coefficient, also called the demagnetization factor.
The quantity N is determined by the shape of the magnetic. Generically, for the anisotropic magnetic N is a tensor. Exact analytical calculation of N is possible for magnetics of the ellipsoid form only. In this case the internal field is constant. For arbitrary shapes of samples the magnitude and direction of the vector $\vec{M}$ are functions of the coordinates. In this article the cylindrical specimens are considered in a homogeneous magnetic field. A sample in the magnetic field is shown schematically in figure 1.

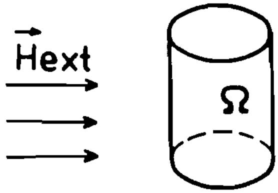


Figure 1.

We define the component of the magnetic field in the direction of $\vec{H}_{ext}$ in every point of the specimen Ω as the difference of the projection of the external field and the demagnetization fields:

$$(\vec{H}(\xi_i),\vec{e})=(\vec{H}_{ext},\vec{e})-N(\xi_i)(\vec{M}(\xi_i),\vec{e}), \qquad (1.2)$$

where $\xi_i \subset \Omega$, $\vec{e} = \dfrac{\vec{H}_{ext}}{|\vec{H}_{ext}|}$.

It is known that, in the magnitostatics the magnitude of the magnetic induction is defined in the following way:

$$\vec{B}(\xi_i)=\vec{H}(\xi_i)+4\pi\vec{M}(\xi_i). \qquad (1.3)$$

Multiplying (1.3) by $\vec{e}$ and using (1.2) we obtain for $\vec{B}(\xi_i)$:

$$(\vec{B}(\xi_i),\vec{e})=(\vec{H}_{ext},\vec{e}) +(4\pi-N(\xi_i))(\vec{M}(\xi_i),\vec{e}). \qquad (1.4)$$

Using (1.4) the demagnetization factor may be defined as

$$N(\xi_i)=4\pi\left[\ 1+\frac{(\vec{H}_{ext},\vec{e})\ -(\vec{B}(\xi_i),\vec{e})}{4\pi(\vec{M}(\xi_i),\vec{e})}\right]. \qquad (1.5)$$

To calculate N we used the ideal diamagnetic approximation ($\vec{B} \equiv 0$ in the sample).In this case (1.5) becomes:

$$N(\xi_i)=4\pi\left[\ 1-\frac{|\vec{H}_{ext}|}{(\vec{H}(\xi_i),\vec{e})}\right]. \qquad (1.6)$$

Evaluating the sum $(\vec{H}(\xi_i),\vec{e})$ over the whole specimen or over

its thin surface layer,we shall obtain the quantity $\int_{\Omega'}(\vec{H}(\xi_i),\vec{e})dV$, where Ω' is the area of the whole sample or its surface layer,respectively. Owing to the symmetry of the specimen the direction of the total magnetic field $\vec{H}_{sum}=\int_{\Omega'}\vec{H}(\xi_i)dV$ will coincide with the direction of $\vec{H}_{ext}$.Thus we may define the average value of $(\vec{H}(\xi_i),\vec{e})$ in the sample as

$$S_m = \frac{\int_{\Omega'}(\vec{H}(\xi_i),\vec{e})dV}{\int_{\Omega'} dV} = \frac{|\vec{H}_{sum}|}{\int_{\Omega'} dV}. \qquad (1.7)$$

Now using eqs. (1.6) and (1.7) we define the mean value of the demagnetization factor for the sample in the direction of $\vec{H}_{ext}$ as:

$$N = 4\pi\left[\ 1 - \frac{|\vec{H}_{ext}|}{S_m}\ \right]. \qquad (1.8)$$

The demagnetization factor for the direction transverse to $\vec{H}_{ext}$ is zero.This fact ensues from eq.(1.2) and the following observation:

$$(\vec{H}_{ext},\vec{n})=0,\quad \sum_{\xi_i\subset\Omega'}(\vec{H}(\xi_i),\vec{n})=0,$$

where the symmetry of the sample has been taken into account and $\vec{n}$ satisfies the conditions $(\vec{e},\vec{n})=0$.However,in order to find N from (1.8),it is necessary to compute $\vec{H}_{sum}$. For calculating the magnetic fields in the specimen the boundary integral equations method was used.

2.The method of boundary integral equations

In order to calculate the strength of the magnetic field, it is necessary to solve the three-dimensional Maxwell equations for a sample placed in a homogeneous magnetic field:

$$\begin{cases} \operatorname{div} \vec{B} = 0 \\ \operatorname{rot} \vec{H} = 0 \\ \vec{B} = \mu \vec{H} \end{cases} \quad (2.1)$$

Besides, at the phase boundary the vectors $\vec{B}$ and $\vec{H}$ should satisfy the following conditions: $B_{1n}=B_{2n}, H_{1t}=H_{2t}$.

The integral analog of (2.1) is

$$\vec{H}(\vec{a})=\vec{H}_{ext}+\frac{\nabla_{\vec{a}}}{4\pi}\left\{\int_{\Omega}(\vec{B}(\vec{x})-(\vec{H}(\vec{x})),\nabla_{\vec{a}}(\frac{1}{|\vec{x}-\vec{a}|})dv\right\}, \quad (2.2)$$

where $\vec{H}_{ext}$ is the external field, a is the observation point.

Assuming that the magnetic permeability μ in Ω is constant, the eq. (2.2) is reduced to the boundary integral equation:

$$\vec{H}(\vec{a})=\vec{H}_{ext}-\frac{\nabla_{\vec{a}}}{4\pi}\int_{d\Omega}\left[\frac{(\vec{B}(\vec{x})-\vec{H}(\vec{x}),d\vec{S}_{\vec{x}})}{|\vec{x}-\vec{a}|}\right]. \quad (2.3)$$

For the ideal diamagnetic $\vec{B}(\vec{a})=0$ holds in Ω. Therefore $\mu=0$ in this case, and (2.3) becomes:

$$\vec{H}(\vec{a})=\vec{H}_{ext}+\frac{\nabla_{\vec{a}}}{4\pi}\int_{d\Omega}\frac{(\vec{H}(\vec{x}),d\vec{S}_{x})}{|\vec{x}-\vec{a}|}. \quad (2.4)$$

Introducing the new variable $\sigma(x)=(\vec{H}(x),\vec{n}(x))$, where $\vec{n}(x)$ is the external normal to the boundary $d\Omega$ at point $\vec{x}$ and multiplying (2.4) by $\vec{n}(\vec{a})$ we obtain :

$$\sigma(\vec{a})=(\vec{H}(\vec{a}),\vec{n}(\vec{a}))+\frac{\partial}{\partial n_{\vec{a}}}\left[\frac{1}{4\pi}\int_{d\Omega}\frac{\sigma(\vec{x})dS_{\vec{x}}}{|\vec{x}-\vec{a}|}\right]. \quad (2.5)$$

For the discretization of (2.5) the boundary $d\Omega$ is divided into boundary elements $\{\Omega_i\}$ so that $d\Omega = \bigcup_{i=1}^{L}\Omega_i$. Adopting that $\sigma(a)$ is constant within the element Ω_i: $\sigma = \sigma_i$, and choosing the observation points a_i in the center of Ω_i, we obtain the system of equations

$$\sigma_i = (\vec{H}_{ext}(\vec{a}_i),\vec{n}_i) + \sum_{j=1}^{L}\frac{\partial}{\partial n_{\vec{a}}}\cdot\frac{1}{4\pi}\int_{d\Omega}\frac{\sigma_j\, dS}{|\vec{x}-\vec{a}|}\Bigg|_{\vec{a}=\vec{a}_i}, \quad (2.6)$$

where $\vec{n}_i$ is the external normal to Ω_i, $i=1,\ldots,L$.

Symbolically, expression (2.6) may be rewritten as :

$$\hat{\sigma} = H + [\ A\ ]\ \hat{\sigma}, \quad (2.7)$$

were $\hat{\sigma}=(\sigma_1,\ldots,\sigma_L)^T$, $\hat{H}=\left[(\vec{H}_{ext}(\vec{a}_1),\vec{n}_1),\ldots,(\vec{H}_{ext}(\vec{a}_L),\vec{n}_L)\right]^T$,

and $[\ A\ ]$ is an $L \times L$ matrix.

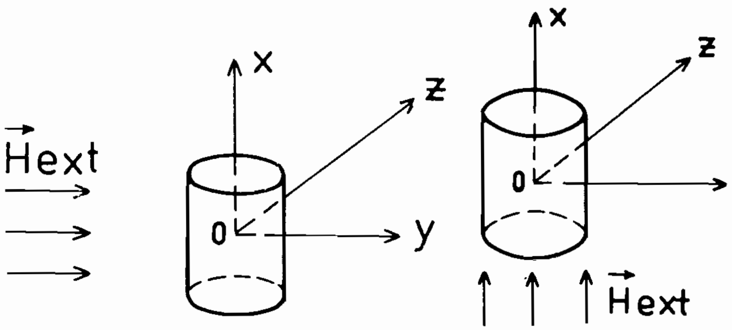
For solving (2.7) the following iterative process was used:

$$\hat{\sigma}_{k+1} = \hat{H} + [\, A \,]\, \hat{\sigma}_{k}\,, \qquad \hat{\sigma}_0 = 0.,\ k=0,1,\ldots,L. \qquad (2.8)$$

Since $\vec{H}(\xi_i)$ in the sample does not have an axial symmetry in general case,the special method proposed in (Akishin.1988) was used to discretize (2.6) and to solve the arising equations.This method takes into account the axial symmetry of the sample.

3. Results of the numerical computation

The numerical simulations were performed with the EC-1061 computer (2 million operations per sec.) using a package of FORTRAN programs.For the specimen placed in a homogeneous magnetic field the values of $\vec{H}$ and $\vec{B}$ were calculated inside and outside of the sample.The simulation was carried out under the assumption that $\vec{H}_{ext}=1$.The specimens with the transversal and longitudinal magnetic field applied, are shown in figure 2 .



(a) (b)

Figure 2. The sample placed in (a) transversal magnetic field. (b) longitudinal magnetic field.

Figure 3 shows the decomposition of the sample surface for simulation by the method of boundary integral equations. Besides, the symmetry or antisymmetry of the magnetic field w.r.t. the (*OZY*) plane was taken into account.

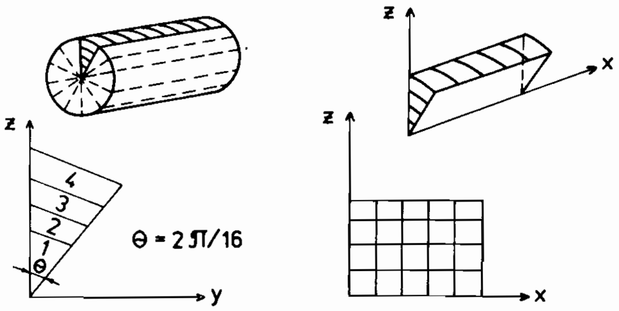


Figure 3. The decomposition of the sample surface.

The number of the decomposition elements along the *X*-axis used in the simulation varies from 40 to 60. The dimension of system equations changed from 768 to 1088 variables, depending on the length of the sample.The vector σ calculated from (2.7) was used to recount the field strength at an arbitrary point.To find the average value $(\vec{H},\vec{e})$ in the sample the integral was replaced by the sum over the subspaces obtained by decomposition Ω into 6 equal parts w.r.t the radius,40 parts w.r.t. the angle and into 20 to 40 parts w.r.t the axis of the sample. In computations the value of $\vec{H}$ was considered constant in each subspace and equal to the value in the center of the subspace.Taking into account the symmetry of the sample,the computation time was reduced by the factor of 8. The full time required for the central processor to solve the system of equations and calculate the average field,was changed from 18′ to 46′ depending on the geometry of the sample .The verification was made for the cylindrical specimen in the longitudinal magnetic field. The calculated values of the demagnetization factors *N* were compared with the experimental data obtained in (Bozorth and Chapin 1942).The calculated and experimental results are shown in table 1.

Table 1. The demagnetization factors for the cylindrical samples in the longitudinal field.

k	0.1	0.2	0.3	0.5	0.7	1.0	2.0	5.0
N	0.827	0.711	0.625	0.522	0.431	0.352	0.221	0.182
N^*	0.758	0.603	0.526	0.404	0.345	0.271	0.165	0.070
N_{exp}	—	—	—	—	—	0.27	0.14	0.04

k is the ratio of the length of the sample and its diameter, N_{exp} is the data of measurements, N and N^* are the numerical results.

The numerical values of the demagnetization factors N and N^* are calculated by (1.8). To compute N (resp. N^*) the field values were chosen from the whole specimen (resp. from the thin surface layer of the specimen). The better agreement of N^* and N_{exp} is probably due to the fact that the measurements were done by a winding applied to the specimen. As a result, only the surface values of the magnetic induction were registered .

For the samples in the transversal magnetic field the demagnetization factors were estimated for different k. The behavior of the magnetic field in the sample and beyond it illustrated in fig.4. All calculations were performed for the ideal diamagnetic. Fig.4, shows the $|H_y|$ as a function of the coordinates for k=0.6 in the (ZOY) plane.

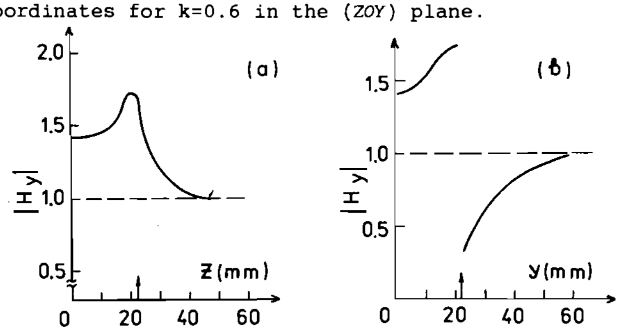


Figure 4. The distribution $|H_y|$ along the OZ axis (a) and along OY axis (b) in the sample and beyond it. ↑ – denotes the edge of the sample.

The calculated values of the demagnetization factors are given in table 2.

Table 2. The values of the demagnetization factors for cylinders in the transversal field, obtained numerically

k	S_m	S_m^L	$N/4\pi$	$N^*/4\pi$
0.1	1.230	1.377	0.187	0.274
0.2	1.280	1.526	0.219	0.344
0.3	1.344	1.600	0.256	0.375
0.4	1.410	1.636	0.291	0.389
0.5	1.466	1.667	0.318	0.400
0.6	1.510	1.693	0.338	0.409
0.7	1.540	1.713	0.354	0.416
0.8	1.583	1.733	0.368	0.423
0.9	1.613	1.750	0.380	0.428
1.0	1.640	1.766	0.390	0.434
3.0	1.882	1.930	0.469	0.482
5.0	1.956	1.983	0.488	0.496

Here S_m and S_m^L are the average values of the Y component of the magnetic field in the full volume of the specimen and in a thin (1.5 mm) surface layer, respectively; N and N^* are the demagnetization factors calculated by (1.8) for these values.

It is straightforward to see that the demagnetization factors obtained numerically for $k\to\infty$ are in a good agreement with the analytical value of the demagnetization factor for the infinite cylinder (Osborn.1945): $N/4\pi=0.5$.

The maximum value $|H_y|$ in the space is also interesting. The maximum $|H_y|$ is attained near the surface of the specimen. In table 3 the dependence of the maximum $|H_y|$ on k is given. The dynamics of the numerical values of $|H_{y,max}|$ is in a good agreement with the asymptotic value of $|H_{y,max}|$: $|H_{y,max}|=2$, when $k=k_\infty$.

Table 3.The maximum values $|H_{y,max}|$ near the surface of samples.These values are shown at the point A(0,0,R+δ),where R is the radius of the sample,δ = 1mm

k	$\|H_{y,max}\|$
0.1	1.214
0.2	1.302
0.3	1.380
0.4	1.441
0.5	1.496
0.6	1.543
0.7	1.584
0.8	1.621
0.9	1.653
1.0	1.682
1.5	1.783
2.5	1.893

The accuracy of the discretized model used for the computation of the magnetic field is 0.5% for the decomposition of the sample surface shown in figure 3.The accuracy of the calculations depends on the decomposition of the sample surface used in the simulations.When the number of elements of decomposition increases,the accuracy grows as well and as a result, the full time of the central processor needed to solve the system of equations and calculate the average field also increases.

4.Conclusion

The proposed numerical method allows one to find the demagnetization factors,which could not be found from the direct measurements for the cylindrical samples in the transversal magnetic field .For the samples in a longitudinal magnetic field a comparison of the theoretical and experimental results has been made.Besides, this case yields a number of the missing data for k=0÷1.

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