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COMBINED ALGORITHMS IN NONLINEAR PROBLEMS OF MAGNETOSTATICS

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INTRODUCTION

Majority of problems in physics and engineering can only be solved by approximation methods. Quite common are the finite element method that uses discrete representation not only of the domain but also of its boundary, and the boundary element method that, on the other hand, uses only the discretization of the boundary. To solve the nonlinear boundary value problem of magnetostatics we have constructed combined algorithms that unite differential equations for a vector or a scalar potential and boundary integral equations /1,2/. Mathematical aspects in the theory of combined methods in the problems of magnetostatics without the preliminary discretization were studied in /3/. Here we give some fundamental results without proofs, attention is paid to three-dimentional problems. We also give the conditions of the convergence of iterative processes in the decomposition method of an unbounded domain with simple iteration as the transition operator and in the Newton method.

At the end we give some numerical results to illustrate the computations of spatial distribution of the magnetic field of a configuration with a dipole type magnet. The computations were performed using the packet of programs MOK31, especially written for this sort of problems.

GENERALIZED FORMULATION OF THE PROBLEM

Let $\Omega_1 \subset R^3$ be a bounded domain with Lipschitz boundary Γ_1 (it corresponds to a nonlinear medium); and $\Omega_3 \supset \Omega_1$, an auxiliary bounded domain with a smooth boundary Γ_3. Then the stationary Maxwell's equation with two scalar potentials can be transformed into a boundary value problem (with a nonlocal boundary condition in Ω_2)

$$E_\psi u = -\sum_{i=1}^{3} \frac{\partial}{\partial x_i} a_i(x, \omega) = 0, \tag{1.1}$$

$$\omega = \operatorname{grad} u, \quad u \in D(E_\psi),$$

with the domain of definition

$$D(E_\psi) = \{ u \mid u \in M(E_\psi), [u]_{\Gamma_1} = 0, [\tfrac{\partial u}{\partial n}]_{\Gamma_1} = \psi(x),$$
$$\alpha G_1 u + \beta \tfrac{\partial u}{\partial n} = \xi(x), x \in \Gamma_3, (u, g_0) = 0 \},$$

where $M(E_\psi) = C^2(\bar{\Omega}_1) \cup C^2(\overline{\Omega_2 \setminus \Omega_1})$. The functions $a_i(x, \omega)$, $i = 1, 2, 3$, $\omega = (y_1, y_2, y_3)$ are defined by

$$a_i(x, \omega) = \begin{cases} y_i, & x \in \Omega_2 \setminus \bar{\Omega}_1 \equiv \Omega_0, \\ \mu(x, |y|) y_i, & x \in \Omega_1, \end{cases}$$

where $|y| = (\sum_{i=1}^{3} y_i^2)^{1/2}$, the sign $[\cdot]$ defines the jump of the given function on the boundary Γ_1, α, $\beta \geq 0$, the operator $G_1 \in L_2(\Gamma_3)$ is symmetric and positively definite, g_0 is the density of the Robin potential, $K^* g_0 = g_0$, $\psi(x)$, $\xi(x)$ are given functions with the properties depending on the type of problem. The magnetostatic problem is represented by $\alpha = \beta = 1$, $\xi = 0$,

$$G_1 = L^{-1}(E + K), \tag{1.2}$$

where the integral operators K and L are defined by

$$Ku = \frac{1}{2\pi} \int_{\Gamma_3} \frac{\cos(r_{PM}, n_P)}{|r_{MP}|^2} u(P)\, d\sigma_P, \tag{1.3}$$

$$Lv = \frac{1}{2\pi} \int_{\Gamma_3} |r_{MP}|^{-1} v(P)\, d\sigma_P,$$

where $|r_{MP}|$ is the **Euclidean** norm of the vector r_{MP}, connecting the points $M, P \in \Gamma_3$, n_P is the inner normal vector at the point P, E is the identity operator. The $\alpha = 1$, $\beta = 0$, $\xi = 0$, $G_1 = E$ correspond to the homogeneous Dirichlet problem, and the $\alpha = 0$, $\beta = 1$, $\xi \in W_{2,1}^{-1/2}(\Gamma_3)$ to the Neumann problem.

Let V be reflexive Banach space, V^* its dual space, and, moreover, V can be continuously and densely embedded in a Hilbert space H, $D(E_\psi) \subset V$, $R(E_\psi) \subset H$, Y a Banach space. We construct an energetic extension of the operator E_ψ using the formula

$$A = T^* A_0 T, \quad A \in (V \to V^*), \tag{1.4}$$

where $T \in (V \to Y)$ is a continuous linear operator, such that $\|Tu\|_Y = \|u\|_V$, $A_0 \in (Y \to Y^*)$ is a nonlinear operator, $T^* \in (Y^* \to V^*)$. To achieve it, we use the space $L_2(\Omega)$, the Sobolev space $W_2^1(\Omega)$ and its subspace $\mathring{W}_2^1(\Omega)$. We also define the subspace $W_{2,g_0}^1(\Omega) \subset W_2^1(\Omega)$ of functions with the trace $\gamma_0 u$ orthogonal to g_0 on $\Gamma = \partial\Omega$, i.e. $(\gamma_0 u, g_0) = 0$. We use the Sobolev spaces of fractional powers, i.e. $W_2^{1/2}(\Gamma_3)$ /4/ and their dual spaces $W_2^{-1/2}(\Gamma_3)$. Similarly, we define the space $X \equiv W_{2,g_0}^{1/2}(\Gamma_3) \subset W_2^{1/2}(\Gamma_3)$ **of functions orthogonal to** g_0 on Γ_3, i.e. $(u, g_0) = 0$. Since $(g_0, 1) \neq 0$, we can consider that

$$W_{2,1}^{-1/2}(\Gamma_3) \equiv (W_{2,g_0}^{1/2}(\Gamma_3))^* \subset W_2^{-1/2}(\Gamma_3)$$

is a subspace of functions, **orthogonal** to unity, i.e. $(u, 1) = 0$. The duality mapping between V and V^* is denoted by $\langle \cdot , \cdot \rangle$. The trace operator $\gamma_{0,g_0} : W_{2,g_0}^1(\Omega_2) \longrightarrow W_{2,g_0}^1(\Gamma_3)$ is defined as the restriction of the linear continuous operator γ_0 on X /4/.

Let the linear operator $G_1 \in \mathcal{L}(X \to X^*)$ be selfadjoint and positively definite:

$$\langle G_1 u, u\rangle \geqq m_{G_1} \|u\|_X^2, \quad \forall u \in X, \quad m_{G_1} > 0. \tag{1.5}$$

We define the following spaces and operators

$$H = L_2(\Omega_2), \quad V = \{u \mid u \in W^1_{2,g_0}(\Omega_2)\}, \tag{1.61}$$

$$\|u\|^2 = \int_{\Omega_2} |\operatorname{grad} u|^2 dx + \int_{\Gamma_3} \bar{u}^2 d\sigma, \quad \bar{u} = \gamma_{0,g_0} u, \tag{1.62}$$

$$Y = L_2^3(\Omega_2) \times W^{1/2}_{2,g_0}(\Gamma_3), \quad Y^* = L_2^3(\Omega_2) \times X^*, \tag{1.63}$$

$$T: u \longrightarrow \left\{ \frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \frac{\partial u}{\partial x_3}, \gamma_{0,g_0} u \right\}, \tag{1.64}$$

$$A_{0K}: \{y; \varphi\} \longrightarrow \{a_1(x,y), a_2(x,y), a_3(x,y), G_1 y\} - \bar{g}, \ \varphi \in X, \tag{1.65}$$

where the linear functional $\bar{g} \in Y^*$ is, according to the Harn-Banach theorem, and extension of the linear continuous functional onto all $L_2^3(\Omega_2) \equiv L_2(\Omega_2) \times L_2(\Omega_2) \times L_2(\Omega_2)$,

$$g(Th) = \int_{\Gamma_1} \psi(s) \gamma_0 h(s) ds, \quad \forall h \in V, \ \psi \in W_2^{-1/2}(\Gamma_1). \tag{1.7}$$

Lemma /3/. Let $G_1 \in \mathcal{L}(X \to X^*)$ satisfy the condition (1.5). Then the operators and spaces defined by (1.61)-(1.65) give an energetic extension $A_K = T^* A_{0K} T$ of the operator (1.1).

We remind that the operator equation

$$A u = 0, \quad u \in V \tag{1.8}$$

is the functional-analytic (or generalized) formulation of the boundary value problem (1.1), if A is the energetic extension of the operator E_ψ.

The function $\mu(x, \xi) \in C(\Omega_1) \times C[0, \infty)$ can be subject to

any of the following conditions (for $x \in \Omega_1$, $t, r \in [0, \infty)$) :

$$\mu(x,t) \geqq m_1 > 0, \tag{1.91}$$

$$\mu(x,t)t - \mu(x,r)r \geqq m(t-r), \ t \geqq r, \ m > 0, \tag{1.92}$$

$$|\mu(x,t)t - \mu(x,r)r| \leqq M|t-r|, \tag{1.93}$$

$$\left|\frac{\partial}{\partial t}\mu(x,t)t\right| \leqq M, \quad \forall t \in [0,\infty), \tag{1.94}$$

From the results on solvability of nonlinear operator equations /5/ there follows :

Theorem 1. Suppose the operator $G_1 \in \mathcal{L}(X \to X^*)$ is symmetric and positively definite, and the function $\mu(x,t)$ satisfies the conditions (1.91)-(1.93). Then the operator A_K is radially continuous, coercitive and strictly monotonic and therefore, there exists unique generalized solution $u \in V$ of the equation (1.8).

Similar result holds for the problem (1.1) with the Neumann or Dirichlet conditions on Γ_3. Suppose we take the Neumann boundary value problem for (1.1) in generalized form : "Find a function $u \in W^1_{2,g_0}(\Omega_2)$ satisfying the integral identity

$$\int_{\Omega_2} \sum_{i=1}^{3} a_i(x,\omega) \frac{\partial \eta}{\partial x_i} d\Omega_2 - \int_{\Gamma_1} \psi \gamma_0 \eta \, ds = \langle g, \gamma_{0,g_0} \eta \rangle, \tag{1.10}$$

for any $\eta \in W^1_{2,g_0}(\Omega_2)$". Here the function $g \in X^*$. There exists a unique generalized solution of (1.10), with uniquely defined trace $\gamma_{0,g_0} u \in X$.

We define a nonlinear operator $S \in (X^* \to X)$ by the formula

$$\langle S(g), \eta \rangle = \langle \gamma_{0,g_0} u, \eta \rangle, \quad \forall \eta \in X^*. \tag{1.11}$$

The operator S is the "nonlinear Poincare-Steklov operator". The linear case was thoroughly studied in /6,7/.

Theorem 2. If the conditions (1.92)-(1.93) are fulfilled, then the operator S is strongly monotone, continuous and has a Lipschitz-continuous strongly monotone inverse $S^{-1}\in(X\longrightarrow X^*)$. If the condition (1.94) is fulfilled then the operator S^{-1} is Gateaux-differentiable and there holds the estimate

$$\langle (S^{-1})'(v)u,u\rangle \leqq M_1\|u\|_X^2 . \tag{1.12}$$

If the conditions (1.92), (1.93) are fulfilled and the function $t\longmapsto \frac{\partial}{\partial t}\mu(x,t)t$ is continuous for almost all $x\in\Omega_1$, then the operator $R=(S^{-1})'$ is positively definite :

$$\langle R(v)u,u\rangle \geqq M_0\|u\|_X^2 , \quad \forall u\in X, \tag{1.13}$$

and symmetric :

$$\langle R(v)\nu,\eta\rangle = \langle \nu, R(v)\eta\rangle \quad ,\forall \nu,\eta\in X. \tag{1.14}$$

Remark. Since A is a potential operator, it follows that the operator S^{-1} is also a potential operator.

Let S_Δ be a linear Poincare - Steklov operator for the problem (1.1), where we set $E_y=-\Delta$ in Ω_2 and $y=0$. Then the operator S_Δ is completely continuous on $L_2(\Gamma_3)$ and can be represented in the form

$$S_\Delta=(E-K)^{-1}L , \quad u\in L_2(\Gamma_3), \quad (u,1)=0 , \tag{1.15}$$

$(u,1)=0$ implying $(S_\Delta u, g_0)=0$. The inverse operator S_Δ^{-1} with the domain of definition $D(S_\Delta^{-1}\subset L_2(\Gamma_3)$ can be represented by the formula

$$S_\Delta^{-1}=L^{-1}(E-K) , \quad (u,g_0)=0. \tag{1.16}$$

That is why we turn our attention to the family of operators $G=G_1^{-1}$ (G_1 has been used in the definition of the operator E_y in (1.1)) satisfying the following conditions :

$$D(G^{-1}) = D(L^{-1}(E-K)); \tag{1.17}$$

G^{-1} is symmetric and positively definite in $L_2(\Gamma_3)$; (1.18)

$$(Gv,g_0)=0 \ \forall v\in L_2(\Gamma_3), \ (v,1)=0 , \ (G^{-1}u,1)=0 \ \forall u\in X. \tag{1.19}$$

Lemma 1. The operator $G=(E+K)^{-1}L$ defined on $L_2(\Gamma_3)$ satisfies the conditions (1.17)-(1.19).

Remark. The operator $G=(E+K)^{-1}L$ corresponds to the magnetostatic problem. In this case the generalized formulation (1.3) is equivalent to the operator equation

$$\Phi u_r \equiv S^{-1}u_r + G^{-1}u_r = 0 , \quad u_r\in X, \tag{1.20}$$

where $u_r=\gamma_{g_0}u$, u is a solution of (1.3). Since the operator Φ is strongly monotone and Lipschitz continuous in X, the equation $\Phi u=y$ has a unique solution $u\in X$ for all $y\in X^*$ /5/.

Suppose that $m_\Phi>0$ is the constant of strong monotonicity and M_Φ the Lipschitz constant of the operator Φ. To solve (1.20) we shall study the method of simple iterations :

$$J\left(\frac{u_{n+1}-u_n}{\tau}\right) = -\Phi(u_n) , \quad n=0,1,2,\dots \tag{1.21}$$

The above-mentioned properties of the operator Φ make it possible to use the results of /5/:

Theorem 3. The method (1.21) converges for $\tau\in\left(0,\frac{2m_\Phi}{M_\Phi^2}\right)$ to a unique solution of the Eq.(1.20) at the rate

$$\|u_n-u_r\|_X \leqq \frac{[K(\tau)]^n}{1-K(\tau)}\|\Phi u_0\|_{X^*} , \tag{1.22}$$

where $J=G^{-1}$, $K(\tau)=(1-2m_\Phi\tau+M_\Phi^2\tau^2)^{1/2}<1$.

The transition operator of (1.22) has the form :

$$u_{n+1}=T_\Phi u_n \equiv ((1-\tau)E-\tau G S^{-1})u_n , \quad u_n\in X.$$

Remark. Similar result holds for $J=S_\Delta^{-1}$, too.

To solve (1.20) we can also use the continuous Newton method:

Theorem 4. Suppose that the conditions of the Theorem 2 hold and that the function $t \longmapsto \frac{d}{dt}\mu(x,t)t$ is differentiable in t for almost all $x \in \Omega_1$, $t \in [0,\infty)$. Then the process

$$\frac{d\mu}{d\tau} = -\phi'(\mu(\tau))^{-1}\,\phi(\mu(\tau)), \quad \mu(0)=\mu_0 \tag{1.23}$$

converges to the unique solution of the equation (1.20) with an arbitrary initial approximation μ_0.

Remark. If the discretized equations approximate the operators S^{-1} and G^{-1} with sufficient accuracy, preserving the corresponding properties, then such finite-dimensional systems can be solved by the above-mentioned algorithms. The rate of convergence does not depend on the discretization step.

MATHEMATICAL FORMULATION OF THE MAGNETOSTATIC PROBLEM

Magnetic field is expressed via the two scalar potentials $\mu_1(x)$ and $\mu_2(x)$ through the following formulae:

$$H_1 = -\nabla \mu_1(x), \quad x \in \Omega_1, \tag{2.1}$$

$$H_2 = H^0 - \nabla \mu_2(x), \quad x \in \Omega_2', \tag{2.2}$$

Ω_2' is a vacuum region containing the point at infinity, $\Omega_1 \cup \Gamma_1 \cup \Omega_2' = R^3$. H^0 is taken to be known. The stationary Maxwell's equations get the form /8/ :

$$-\sum_{i=1}^{3} \frac{d}{dx_i}\left(\mu \frac{d}{dx_i} u_1\right) = 0, \quad x \in \Omega_1, \tag{2.3}$$

$$-\Delta \mu_2 = 0, \quad x \in \Omega_2' \tag{2.4}$$

the conditions on the boundary of the division of media Γ_1 are :

$$\frac{du_1}{dn_1} - \frac{du_2}{dn_2} = \psi(x), \qquad \mu_1 = \mu_2 - \phi(x), \quad x \in \Gamma_1, \tag{2.5}$$

and the condition at the infinity :

$$\mu_2(x) = O\left(\frac{1}{|x|^2}\right) \quad \text{for } |x| \longrightarrow \infty. \tag{2.6}$$

The functions $\psi(x)$ and $\phi(x)$ are defined on Γ_1 by the formula :

$$\psi(x) = -H^0 n_1, \qquad \phi(x) = \int_{X_0}^{X} (H^0 \times n_1)\, d\tau, \tag{2.7}$$

τ is the tangential vector to Γ_1, $X_0 \in \Gamma_1$ is an arbitrary point. To satisfy exactly the condition (2.6) we enclose the nonlinear medium into an auxiliary domain Ω_3 (see Fig.1). On the boundary of Ω_3 we impose a condition equivalent to (2.6) :

$$(E + K)\, u_2 = L \frac{du_2}{dn_3}, \tag{2.8}$$

where E is the identity operator, K and L are the operators already defined in (1.3) with $r_{MP} = X_P - X$, $n_P = n_3$, $u(P) = u_2(X_P)$.

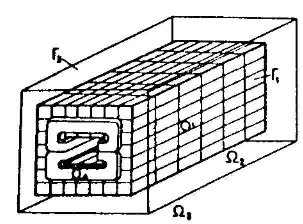

Figure 1

In the domain $\Omega_2 = \bar{\Omega}_3 \setminus \bar{\Omega}_1$ we retain the equation (2.4). To solve (2.3)-(2.5), (2.8) really means to solve the operator equation defined on the boundary of the auxiliary domain in relation to the function $y = \mu_2|_{\Gamma_3}$:

$$S_1^{-1} y + S_2^{-1} y = 0. \tag{2.9}$$

The operator S_1^{-1} is the inverse to the Poincare - Steklov operator, defined on the domain Ω_3 and the differential operator (3), (4), (5) for $\phi = 0$ /3/. $S_2^{-1} = L^{-1}(E + K)$ is the linear operator.

SOLVING THE PROBLEM

As we have already said in the introduction the computations were performed using MOK31. In making of MOK31 we have used one of the efficient methods of analysis of nonlinear spacial magnetic systems, based on the representation of the total field H as the sum of the field H^{o} from the sources at the vacuum and of the field H^{m} of the magnetized medium. Moreover, only one scalar function is needed to describe the field H^{m} /8/. In this case the problem of computation of the field H^{o} of a coil and the problem of computation of the function u are separated and can be optimized independently from each other. When using this approach it is very important to approximate with necessary accuracy the behaviour of the scalar function u at infinity. An efficient approach to solving this problem, as is shown in /7,10/, seems to be transforming it into an equivalent boundary value problem for scalar potential containing inside an auxiliary domain the core and with a special condition of integral type on the boundary of this domain. This special boundary condition takes into account the behaviour of the potential u at infinity exactly. The questions of solvability and iterational methods of solving such boundary value problems were studied in /3,11/. The use of special domains as the auxiliary ones gives the possibility of solving the integral equations that arise on the boundary of these domains numerically.

As the auxiliary domain in MOK31 is being used a parallelepiped with a square as its base. This region is denoted as Ω_3 on Fig.1, its boundary as Γ_3 and its inner normal is n_3. The region Ω_3 contains the nonlinear medium, region Ω_1 contains the ferromagnet with magnetic permeability μ depending on the modulus of the magnetic induction B. The region Ω_1 is bounded by the surface Γ_1 with the inner normal n_1, the region Ω_A correspods to the conducting coil. The direction of the coordinate axes and one eighth of the domain, where the computations were performed are depicted on Fig.2.

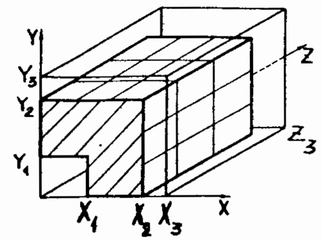

Figure 2

The method of simple iterations to solve the problem (2.9) used in MOK31 has the following form :

$$y_{n+1} = (1-\tau)y_n - \tau S_2 S_1^{-1} y_n . \qquad (2.10)$$

It consists of four stages :

1. The computation of the vector $v = S_1^{-1} y_n$. This is equivalent to solving the nonlinear problem (2.3)-(2.5) in the domain Ω_3 and the Dirichlet boundary condition on Γ_3 : $u_2(x) = y_n(x)$, $x \in \Gamma_3$. Then we find

$$v(x) = \frac{du_2(x)}{dn_3}, \quad x \in \Gamma_3 .$$

2. The computation of the vector $y_{n+1/2} = S_2 v$.
At this stage we solve the boundary integral equation $(E+K)\, y_{n+1/2} = Lv$ on the surface Γ_3 of the auxiliary domain and we find the vector $y_{n+1/2} = (E+K)^{-1} Lv$. All the used algorithms were described in /2,3, 9-13/.

3. Relaxation $y_{n+1} = (1-\tau) y_{n+1/2} + \tau y_n$. The iterational process can be speeded up essentially when the diagonal matrix is used as τ.

4. Return to the stage 1.

RESULTS OF MODELLING

The computations of the spatial distribution of the field corresponding to the dipole magnetic field[13] were performed with the MOK31, using the method described in the previous section. The computations were performed on the sequence of grids (12,12,7) - (23,23,13) - (45,45,25) using the results obtained on a coarser grid as the initial approximation on a finer grid. Organizing the computations in this way economizes the processing time. An idea about the behaviour of convergence of the solutions on the three grids is presented in the Table. The magnetic properties of the iron core are described in[14].

Numerical experiments with MOK31 have shown:

a) weak dependence of the rate of convergence of the iteration process on the characteristics of magnetic permeability of the ferromagnet placed inside the auxiliary domain,

b) independence of the rate of convergence on the discretization step. The number of iterations needed for the relative accuracy of 10^{-3} is 7,8 and 8 for the grids /12,12,7/, /23,23,13/ and /45,45,25/, respectively. The number of points in the grids has been doubled in each direction on consecutive grids, while the magnetic permeability has been a constant function.

Intensity of the current in the coil is 500 A.

Table

Grid	H(0,0,0)T	H(0,0,14.85)T	H(0,0,19.8)T	H(0,0,24.70)T
I	0.5485	0.5500	0.5536	0.0873
II	0.5483	0.5477	0.5162	0.0734
III	0.5482	0.5470	0.5706	0.0687

Intensity of the current in the coil is 2000A.

Grid	H(0,0,0)T	H(0,0,14.85)T	H(0,0,19.8)T	H(0,0,24.75)T
I	2.1454	2.1457	2.1583	0.3348
II	2.1489	2.1404	2.0109	0.2818
III	2.1500	2.1388	1.8304	0.2632

The results of computation of the distribution of magnetic field[13] can be used in programs modelling the trajectories of a beam of charged particles in accelerating systems.

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Комбинированные алгоритмы в нелинейных проблемах магнитостатики

Для решения краевых задач магнитостатики в неограниченной двух- или трехмерной области построены комбинированные алгоритмы основанные на комбинировании метода граничных интегральных уравнений с сеточными методами. Рассматривается обоснование комбинированных методов в нелинейных задачах магнитостатики без предварительной дискретизации уравнений. Приводятся результаты по сходимости итерационных процессов решения возникающих краевых задач в нелинейном случае. Рассмотрены экономичные итерационные процессы решения граничных интегральных уравнений на некоторых поверхностях и алгоритмы их использования. Приводятся примеры численных расчетов задач магнитостатики возникающих при моделировании полей в электрофизических установках.

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Combined Algorithms in Nonlinear Problems of Magnetostatics

To solve boundary problems of magnetostatics in unbounded two- or three-dimensional regions, we construct combined algorithms based on a combination of the method of boundary integral equations with the grid methods. We study the question of substantiation of the combined method in nonlinear magnetostatic problems without the preliminary discretization of equations and give some results on the convergence of iterative processes that arise in nonlinear cases. We also discuss economical iterative processes and algorithms that solve boundary integral equations on certain surfaces. Finally, examples of numerical solutions of magnetostatic problems that arose when modelling the fields of electrophysical installations are given, too.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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