объединенный
институт
ядерных
исследований
дубна

E11-87-110

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COMPUTER ALGEBRA IN PHYSICAL RESEARCH OF JINR

Submitted to "European Conference on Computer Algebra", Leipzig, 1987.

1987

I. Originally computers were designed for numeric computation. Computer structure and instruction system were directed just at this application field, though instructions from the very beginning include ones for logical operations, that allowed one to exceed the bounds of numeric information processing.

Against a breakdown of predominant numeric methods the separate papers /1-3/ on computer application to algebraic formula manipulations began to appear about 30 years ago. However, the two reasons delayed the development of computer algebra programs. Firstly, unsufficient training of experts and unpreparedness of their scientific fields for computer algebra usage. Secondly, difficulties of computer adaptation to formula manipulations, since a user was obliged by himself to train a computer to perform (i.e. to design a compiler, or an interpreter) algebraic transformations and only after that to use it for solving his problem. Therefore it was necessary to combine in one person two different professions system and applied programmer.

It is also necessary to note one more not the least of the factors connected with unsufficient core memory and slow performance of the first generation computers for algebraic manipulations.

Years went by, computer possibilities are rapidly extended from generation to generation, with increasing the efficiency of its usage in different applied fields.
All that furthers the appearance of needs in algebraic manipulations by computer which could already be effectively enough implemented in serial computers. It was getting more and more Soviet and foreign papers on computer algebra (see, for example, reviews /4-7/). Algorithmic and program methods were developed. Languages and compilers for symbolic information processing were created. Among them LISP was the most widely spreaded.

Many early and some present-day CAS had been written in a code or assembler language, sometimes including elements of such numerical languages as FORTRAN, ALGOL, PL/1 and others. It raised an immobility of such a system. An availability of high-level symbolic languages like LISP allowed one create mobile systems.

Just LISP underlies of the most developed and universal CAS, for example, widely-distributed systems REDUCE and MACSYMA. Such a system requires large computer resourses itself, i.e. typically 1 megabyte computer memory to say nothing of a problem to be solved.

However, the big universal systems could relatively easy be adopted at another computer. Moreover those CAS permit an extension by the use of addition to a program written in LISP or in "source" language for a given system.

Inspite of the appearance of those and other powerful universal CAS[7] (SMP, SCRATCHPAD-II, MuMATH-85, etc.) the process of creation, development and usage of special purpose CAS is in progress. In its own field such a system could in a sufficiently full measure satisfy the user's requirements. At the same time the special purpose systems gave, as a rule, the most effective algorithms and optimum internal representation for data, corresponding to the mathematical expressions and operations from the field of specialization for a given CAS. Therefore the specialized systems are usually much more effective in computer resourses then the powerful universal CAS.

As the stricking example of the specialized CAS it should be noted the system SCHOONSCHIP[8] which was developed more than 20 years ago (the first SCHOONSCHIP version has been created by Dutch physicist M.Veltman in 1965) and intended for computations in quantum field theory. Inspite of its "middle-aged" SCHOONSCHIP is till now beyond comparison in high energy physics. Just by means of its usage the record (with respect to amount of computation) results in quantum field theory were obtained. Below some of such computation will be briefly described.

In JINR up to now the most part of the problems connected with computer algebra application is solved with the help of SCHOONSCHIP. Among others such general purpose CAS as REDUCE and FORMAC are more widely used.

II. Comparing computer algebra development in the USSR and abroad one can note the same nature of its basic stages:

1. Appearence of separate papers which were pioneers in the field.

2. Creation of tens of specialized and general purpose CAS written in assembler languages or algorithmic languages FORTRAN, ALGOL, PL/1, LISP and others.

3. Intensive development of the algorithmic base of the present-day CAS and in first place of the general purpose systems. Appearance of new, considerably improved versions of such systems (REDUCE, ANALITIK, SCRATCHPAD). Creation of developed computer algebra software for mini- and microcomputers.

A shift should be noted in time of coming either stage in the USSR and abroad. If the first paper on computer algebra

appeared in our country and abroad approximately at the same time (first stage), then mass creation of CAS in the USSR (second stage) has begun in fact more later. It became possible only with appearance of the BESM-6 and then-serial ES computers. Now, several tens of CAS were created in the USSR (see review[6] and also proceedings of the conferences on computer algebra held in the Soviet Union[9-15]).

Among the Soviet special purpose CAS, ones intended to mechanical problems, are predominated[6]. While from general purpose systems: AVTO-ANALITIK[16], SIRIUS-SPUTNIK[17], AUM[18], ANALITIK[19] (see also[6]) the later is most widely used. In contrast to other Soviet CAS the language ANALITIK has been implemented by hardware, initially for MIR computers, then for the special processor SM-2410, which is a part of the two-processors complex SM-1410, and for the special processors ES-2680 destined for ES computers. Those special processors interprete by hardware the languages ANALITIK-79 (SM-2410) and ANALITIK-82 (ES-2680).

It is remarcable that such an approach to support by hardware of a high-level language started to be developed in the Institute of Cybernetics of the Ukrainian Academy of Sciences as early as the sixties. For some time past one can see a sharp rise of interest to the symbolic processors as abroad where the greatest attention was paid to the different LISP and PROLOG dialects, as also in the Soviet Union. Among home works it should be noted the investigations on creation of the symbolic processor which are carried on at the Institute of Applied Mathematics of the USSR Academy of Sciences[20].

REDUCE is the most widely-distributed foreign CAS in the USSR. It is used in different institutions for solving the scientific and applied problems. With the help of JINR REDUCE was adapted to ES computers in more than 50 Soviet institutions. Soviet papers based on REDUCE usage are sufficiently enough represented in references[10-15].

III. Let us go on to a brief description of basic works on computer algebra carried out at the Joint Institute for Nuclear Research.

The first investigations on computer realizations of symbolic mathematical operations were carried out at JINR as long ago as early 1960s.

Kim Ze Phen in 1963 has created the program for definite integration of some class of rational functions.

At the same time H.Kaizer has developed computer algorithms and program[21] for the algebra of Dirac γ - matrices.

In 1964 V.I.Sharonov has done extention of ALGOL-60 to make some formula manipulations and, in particular, to calculate the trace of the γ-matrices product[22].

The next step in computer algebra investigations at JINR began in the middle of 1970s after the first CAS SCHOONSCHIP had been obtained. It was in 1975 and created the favourable conditions for making investigations in computer algebra more active. In 1976 there were the first successful attempts[23,24] to use SCHOONSCHIP in quantum field theory.

Now JINR has 12 different CAS[25] for the ES-1060, ES-1061, CDC-6500 and BESM-6 computers (see the table):

ES-1060, ES-1061	CDC-6500	BESM-6
REDUCE 2, 3.2	REDUCE 2	AVTO-ANALITIK
SCHOONSCHIP	SCHOONSCHIP	UPP
FORMAC 73	CLAM	SAVAG
CAMAL	SYMBAL	GRATOS
ASHMEDAI		
AMP		

All systems for the BESM-6 computer are Soviet ones and are described in refs.[6,10-16].

An implementation of CAS has encountered great difficulties because of unsufficient computer memory, differences between operating systems, adaptation of computer dependent parts of CAS and so on.

IV. Another group of works is connected with the development of CAS to extend the field of its application in JINR. Some of such works are following:

1) Improvement for interface between SCHOONSCHIP and FORTRAN for symbolic-numeric computations[26],

2) Development of the algorithm for virtual memory control in case of compiled LISP functions in order to improve usage of REDUCE for the CDC-6500 computer[27],

3) Creation of general mathematical packages to extend possibilities of CAS for the following problems:

- solving by power series method of an ordinary differential equation of the form (REDUCE)[28]

$$y'' + p(x)y' + q(x)y = 0,$$

where $p(x)$ and $q(x)$ are rational function in x,

- construction of the determining equations for finding Lie-Bäcklund symmetries of differential equations (FORMAC, REDUCE)[29,30],

- determination of the Lie algebra of point and contact symmetries of differential equations (REDUCE)[31],

- classification of integrable scalar nonlinear evolution equations (FORMAC)[32]

$$u_t = F(u, u_1, \dots u_n), \quad u \equiv u(x,t), \quad u_i \equiv d^i u / dx^i,$$

- investigating the integrability of nonlinear evolution system of the form (FORMAC)[33]

$$\bar{u}_t = \Lambda \bar{u}_n + \bar{F}(x, \bar{u}, \bar{u}_1, \dots \bar{u}_{n-1}), \quad \bar{u} = (u^1, \dots u^m), \quad \bar{u}_i \equiv d^i \bar{u}/dx^i,$$

where $\Lambda = diag(\lambda_1, \dots \lambda_m)$, $\lambda_i \in \mathbb{C}$, $\lambda_i \neq 0$, $\lambda_i \neq \lambda_j$ $(i \neq j)$

- computation of symbolic determinants (SCHOONSCHIP)[34].

4) Creation of special packages for high energy physics

- calculation of the one- and two-loop Feynman integrals by the method of dimensional regularization (SCHOONSCHIP)[34],

- construction of renormalized coefficient functions of Feynman diagrams in scalar theories (SCHOONSCHIP)[35],

- realization of Feynman diagram technique for virton-quark model (Standard LISP)[36],

- simplification of polynomials in Pauli σ-matrices (SCHOONSCHIP)[34].

All the packages listed above form a core of general and special users libraries for REDUCE and SCHOONSCHIP[34] and also for FORMAC.

V. All the works on development of computer algebra systems and methods are closely connected with the scientific program of JINR. CAS are used in such fields of physics and mathematics as

- theoretical high energy physics,
- physics of atomic nucleus,
- statistical mechanics,
- quantum mechanics,
- electrodynamics of charges particles in accelerators,
- nonlinear problems of theoretical and mathematical physics,
- experimental high energy physics,

and others. Let us consider very briefly some of computer algebra applications in JINR.

The most traditional field of computer algebra application at the JINR is multiloop computation in quantum field theory based on CAS SCHOONSCHIP. As was shown by home and foreign practice it is the most suitable for such problems. Multiloop calculations in gauge and supersymmetric theories[23,37,38] were of great importance in analysis of their renormalization properties. As to computation of three-loop divergences in quantum chromodynamics[37] carried out in 1980 till now it is record. As a result of these computations, a number of universal programs[35,39,40] have been developed giving a possibility to automatize cumbersome algebraic manipulation at separate steps of Feynman diagram technique.

In other group of works (see ref.[41] and its bibliography) in the process of ten year usage of SCHOONSCHIP an effective method for solving a number of problems in theoretical high energy physics was developed. In these works a line has been successively realized, directed to the total algorithmization by means of SCHOONSCHIP of a computational procedure for elementary particles cross-sections, taking into consideration a contribution of a big number of high-order diagrams. By corresponding authors' efforts the general algorithm had been created for all computation of the chain "matrix-element" $\Longrightarrow$ "total cross-section" that is trace calculations, removal of ultraviolet and infra-red divergences, multiple integrations and orther complicated transformations. The algorithm was used for solving the physical problems, connected with an analysis of experimental data from the combined large-scale experiments of JINR and CERN (European Organization for Nuclear Research).

Development of applied software for CAS and in the first place for general-purpose systems REDUCE and FORMAC was greatly stimulated by its applications to nonlinear problems of theoretical and mathematical physics intensively studied at the Joint Institute. Among them are: - investigation of the nonlinear resonanses influence on charged particle motion in cyclic accelerators, using the asymptotic method by Krylov and Bogoliubov[42,43];

- construction of the general solution of the Chew-Low nonlinear dispersion equations for low-energy πN-scattering[44];
- investigation of nonlinear evolution equations, which at present give rise to a great interest owing to their soliton solutions[32,33,45];
- group analysis of differential equations; finding the system of determining equations for the Lie algebra of point and contact symmetries[31] and Lie-Bäclund symmetries[29,30].

To solve these problems a number of effective algorithms and universal program have been developed[28-34]. The programs[29,31,32] were included in a widely-spreaded CPC program library.

Computer algebra is used in the JINR not only for theoretical and mathematical problems but also in actual experimental investigations in high energy physics. For example, in papers[46] on the base of the new method of information registration from multiwire proportional chambers[47] a number SCHOONSCHIP and REDUCE programs are created. These programs are used for the development of the principal schemes of data compression devices, majority coincidence schemes and for devices realized the switch function. The latter is represented by the special polynomial, which is the element of Galua GF(2^m) field. Using the methods of algebraic coding theory a number of symbolic algorithm for coding and decoding of Compressed experimental data are developed. These algorithms and programs were used for designing special processors to registrate nuclear interactions.

VI. To conclude we note a close collaboration of JINR with many scientific and educational institutions in the USSR and socialist countries in the field of computer algebra. JINR passed a number of CAS and packages developed at the Joint Institute to scientific institutions of the Soviet Union, Bulgaria, Hungary, Czechoslovakia, the German Democratic Republic and Viet-Nam. Among the Soviet receivers are such large-scale centres as Moscow and Leningrad State Universities, Institute of Nuclear Physics (Novosibirsk), Institute of High Energy Physics (Moscow), Institute for Nuclear Research (Moscow), Steklov Institute of Mathematics (Leningrad Branch), Institute of Geophysics (Kiev), Institute of Physics (Minsk) and many others. Numerous common investigations in the field of computer algebra are well enough presented in Proceedings of International Conferences held at Dubna in 1979[10], 1982[12] and 1985[25].

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Received by Publishing Department
on February 19, 1987.

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Index	Subject
1.	High energy experimental physics
2.	High energy theoretical physics
3.	Low energy experimental physics
4.	Low energy theoretical physics
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8.	Cryogenics
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11.	Computing mathematics and technique
12.	Chemistry
13.	Experimental techniques and methods
14.	Solid state physics. Liquids
15.	Experimental physics of nuclear reactions at low energies
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17.	Theory of condenced matter
18.	Applied researches
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Федорова Р.Н. и др. E11-87-110

Компьютерная алгебра в физических исследованиях ОИЯИ

Дан обзор основных этапов исторического развития и современного состояния в ОИЯИ аналитических вычислений на ЭВМ. Отмечен ряд сходных черт и существенных различий процессов становления этого направления в СССР и за рубежом. Рассмотрены имеющиеся в ОИЯИ программные средства для аналитических вычислений. Описаны работы по развитию систем и методов аналитических вычислений, тесно связанные с научными исследованиями, проводимыми в ОИЯИ. Приведены примеры актуальных физических и математических исследований, существенно опирающихся на использование систем аналитических вычислений.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Fedorova R.N. et al. E11-87-110

Computer Algebra in Physical Research of JINR

A review is given of basic stages of computer algebra development and its present situation in the JINR. A number of similar and distinguishing features in the making this field in the USSR and abroad are noted. Computer algebra systems (CAS) which are available in the Joint Institute are listed. The investigations are described on development of CAS and creation of application packages for the scientific research of JINR. Examples of the actual physical and mathematical problems essentially based on usage of CAS are considered.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1987