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SOME QUESTIONS OF USING CODING THEORY AND ANALYTICAL CALCULATION METHODS ON COMPUTERS

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1. FORMULATION OF THE PROBLEM

Increasing information from multichannel detectors of nuclear particles has generated a need for studying the questions of optimal coding and of data readout and processing methods. Under real conditions a small number of particles is commonly registered in multichannel detectors. This number is small as compared to the total number of registration channels. The problem lies in the creation of a very fast device of data compression or a digital filter without a memory. Priority encoders usually operate using synchronization pulses. When there is a large number of channels, very much time is needed for encoding the coordinates of registered perticles.

In the last ten years studies have been carried out of the possibility of usung coding theory and practice, particularly algebraic coding theory, for the construction of efficient devices of data compression. To solve this problem, analytical calculation methods on computers [1,2] are applied, in perticular using the SCOONSHIP program. Below we consider the results of studies that are of interest from the practical and theoretical points of view.

2. METHOD OF SYNDROME CODING

This method is described in detail in papers [3,4]. The essence of the method is the following (fig. 1). There are n sources at the transmitting side. A small part (10-15%) of their total number can only operate simultaneously. The number of sources simultaneously operated is denoted by t. If no source operates, we get a zero code word, and ones occuring when the sources operate are considered as an error vector to the code word. This

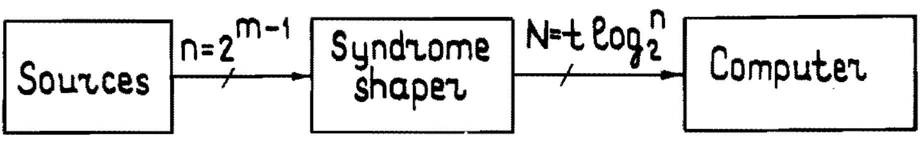

Fig. 1. Block-diagram of the data transfer system using the method of syndrome coding.

word arrives at the input of a syndrome shaper (encoder). The number of transmission channels at the output of the encoder decreases to $N = t \log_2 n$. For $n = 63$ and $t = 3$, $N = 18$. The efficiency of the method increases with increasing n.

The use of correcting code theory and practice helps to answer the following question: "How is a parallel encoder constructed for $t > 1$?" To make the best use of coding theory, the author has suggested a system of analogies of coding theory and the theory of multichannel hodoscopic systems /5/. For example, the error vector e corresponds to an event in hodoscopic system theory which generates pulses from the sources (a scintillator, MWPC wires and others). A cluster of errors in the communication channel corresponds to the cluster arising from the operation a group of neighbouring sources even from one particle. Therefore correcting code theory can be used for the construction of cluster counters /6,7/.

Further the parameter t is the number of information symbols which can be corrected by a given code. This parameter corresponds to a maximum number of the operated sources of a hodoscopic system. The number of check-parity symbols, N (syndrome), is an important parameter of the code. This value depends on code block length and on t. For the well-known Hamming code having $t = 1$ and $N = m$, the code block lengt $n = 2^m - 1$ and $N = m$. For the codes having $t > 1$, $N = mt$. As noted above, in hodoscopic systems the value of t corresponds to the number of outputs or registration channels. Finally the code efficiency is determined by the ratio n/N (transmission speed). As applied to our problems, this parameter is called compression coefficient C_c. The most important result of studying coding theory is the suggestion to use encoders as efficient digital filters in multichannel hodoscopic systems. For example, as shown in paper /5/, for $t = 1$ an encoder of the Hamming code represents a parallel coder. A unitary position code is converted to an ordinary binary code by means of this coder. Complicate the problem and assume that $t = 2$ and $n = 31$. A parallel coder (without memory elements) should be constructed with the aid of which the coordinates of two positions can be simultaneously encoded. This problem is successfully solved using the algebraic theory of BCH codes /7,8/. To draw a principal diagram of the encoder, it is necessary to construct the matrix H^T. The syndrome is calculated with the aid of this matrix. Such matrices are called connection ones /5/ as coupling between the sources and the circuit inputs of syndrome calculation can be determined using their structure. Figure 2 presents one of syndrome calculation of the BCH code correcting two errors. It consists of two parts. The elements of the Galois field $GF(2^5)$ generated by irreducible polynom

$$X^5 + X^2 + 1 \qquad (1)$$

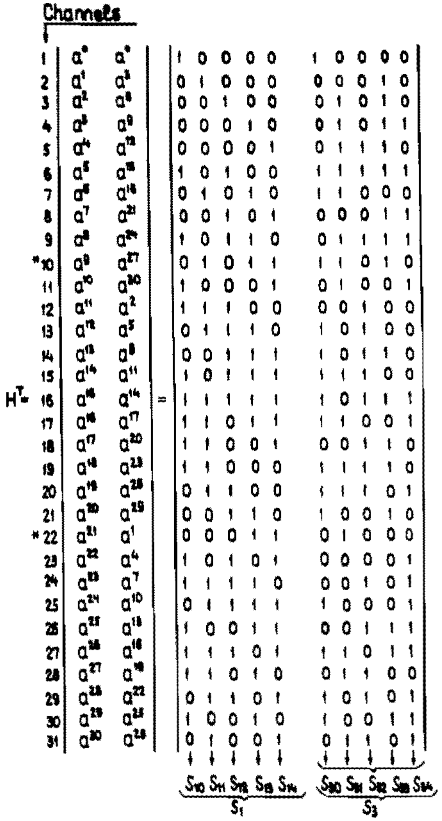

Fig. 2. BCH-code parity check matrix for the correction of two mistakes.

are presented on the right. Here $a^0 = 10000$, $a^1 = 01000$, $a^2 = 00100$, $a^3 = 00010$ and $a^4 = 00001$ are basis elements of the field and a^1 is the polynom root (1). The remaining 26 elements can be calculated from the equality $a^5 = a^2 + a^0$. As the field is finite, $a^{31} = a^0 = 1$. The cubes of the elements are given in the second column of the matrix H^T. The elements as binary codes are presented on the right (fig. 3). The numbers of channels (sources), which logic pulses are sent from, are designated by numerals. Parity checkers and modulo-2 adders are used as microcircuits. Assume for definiteness that the pulses are supplied from the 10th and 22nd sources. From the theory it follows that $S_1 = X_1 + X_2$ and $S_3 = X_1^3 + X_2^3$ with X_1 and X_2 the coordinates of

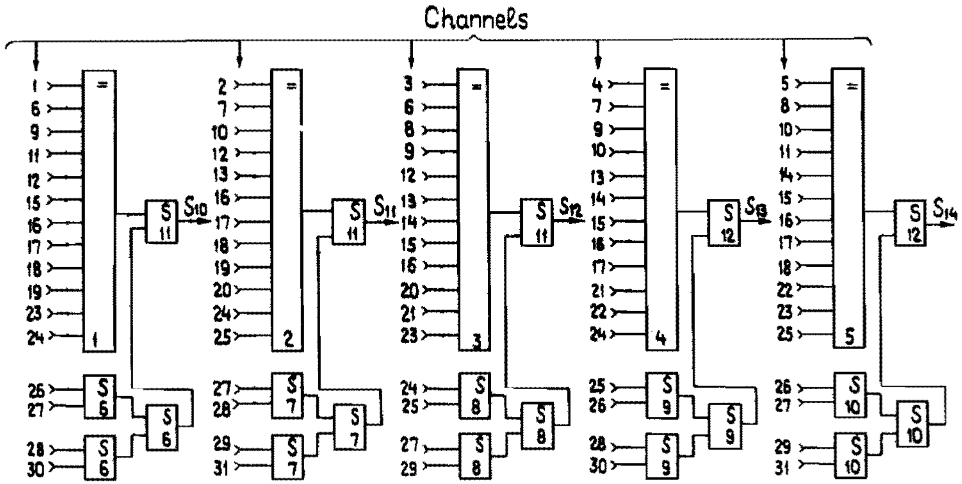

Fig. 3. Principal scheme for syndrome calculation.

the sources operated ($X_1 = a^9$ and $X_2 = a^{21}$). Then the X_1 and X_2 coordinates are the roots of the equation

$$X^2 + \sigma_1 X + \sigma_2 = 0, \qquad (2)$$

where $\sigma_1 = S_1$ and $\sigma_2 = \frac{S_1^3 + S_3}{S_1}$. Then $S_1 = a^9 + a^{21} = a^1$ and $S_3 = a^{27} + a^1 = a^{29}$. In this case $\sigma_2 = \frac{a^3 + a^{29}}{a} = a^{30}$. One can verify that the elements a^9 and a^{21} are the coordinates of the equation

$$X^2 + aX = a^{30} = 0. \qquad (3)$$

The rules of calculation in the Galois field $GF(2^m)$ described in detail in [4,7,8] are not taken into account. Note that the modulo-2 sum is denoted by the sign +. A 10-bit syndrome is obtained at the encoder output for $t = 2$. According to coding theory, the syndrome contains information on the number of operated sources and on their coordinates. The coordinates can be easier found not by solving eq. (3) but by means of PROM as shown in fig. 4, where X_1^1 and X_2^1 are the coordinates of the sources having operated in the binary code.

To fine the number of operated sources, one should solve the following determinants: det 1 = S_1 and det 2 = $S_1^3 + S_3$ [9]. In addition, let us introduce two other parameters "even" and "odd". These parameters can be easily calculated if all the inputs of the encoder are connected to a parallel parity-check circuit. If $S_1 \neq 0$, there is at least one pulse at the inputs of the encoder, i.e., $t \geq 1$. If $S_1 \neq 0$, det 2 = 0 and there is an odd pulse, $t = 1$. Further $t = 2$ if $S_1 \neq 0$, det $\neq 2$ and there is an even pulse. Finally if det $\neq 0$ and there is an odd pulse, $t = 3$. In our case $S_1 = a^1$ and $S_3 = a^{27}$ for $t = 1$. Then $S_1^3 + S_3 \neq 0$.

As the elements of the Galois field represent a cyclic group, this means that rather complicated algebraic expressions can be calculated with the aid of PROM tables. Figure 5 presents a diagram used to calculate the expression $S_1^3 + S_3$. More complicated expressions for $t > 2$ can be calculated in a similar way. Thus, the use of correcting code theory makes it possible to construct qualitatively new, very fast devices such as majority units and parallel counters. To create an encoder for other n values, it is necessary to choose irreducible polynoms of the corresponding degree from the tables of paper [7] and to construct matrices of the H^T type at given t. For example, for $n = 15$ and 63, $X^4 + X + 1$ and $X^6 + X + 1$ are irreducible polynoms.

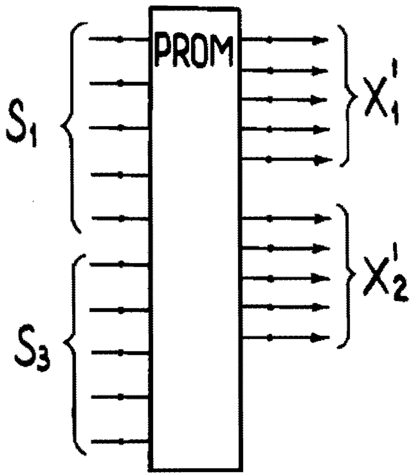

Fig. 4. Scheme for syndrome code transformation to the binary code. X_1' and X_2' are binary coordinate sources. Microcircuits MC 10149.

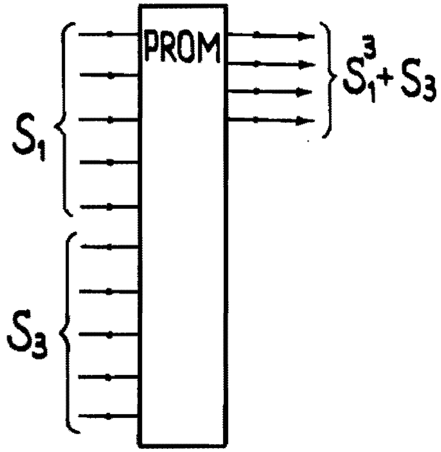

Fig. 5. Scheme for calculation of the expression $S_1^3 + S_3$.

3. SUPERIMPOSED CODES

To create parallel encoders, modulo-2 adders are mainly used. Logic signals must be supplied to the inputs of the adders for their correct operation. However, the question arises: "Won't an encoder be constructed that can operate when weak analog signals are supplied to its inputs? Such correcting codes exist. They are referred to as superimposed codes [10]. To form syndromes of these codes, amplifiers-mixers, e.g., photomultipliers or electric amplifiers-mixers, can be used. However, at other equal parameters the value of C_c of these codes is smaller than that of conventional optimal codes. We have $1 + 1 = 0$, $1 + 0 = 1$, $0 + 1 = 1$ and $0 + 0 = 0$ by modulo-2 adding and $1 + 1 = 1$, $1 + 0 = 1$, $0 + 1 = 1$, $0 + 0 = 0$ by Boolean adding, i.e., the number of differeft combinations is smaller. Nevertheless, for small t and large n values superimposed codes can be applicable, e.g., to scintillation hodoscopes and MWPCs for the purpose of decreasing the number of amplifiers.

Figure 6a presents a matrix H. An encoder for the scintillation hodoscope [10] can be constructed with the aid of this matrix. In this hodoscope $H = 2\sqrt{n}$ photomultipliers are required, and it is possible to register a signal or a triple cluster from one particle. Figure 6b shows a more economical matrix for the construction of an encoder having $n = 15$ and $N = 6$. As is shown in paper [10], the efficiency of coding significantly increases with n. For $n = 28$ the number of combinations, C_c, is $8 \cdot 7/2 = 28$. From this it follows that an encoder having 28 inputs and 8 outputs can be constructed. It should be noted that a similar code is used to construct majority coincidence circuits having a large number of inputs [12].

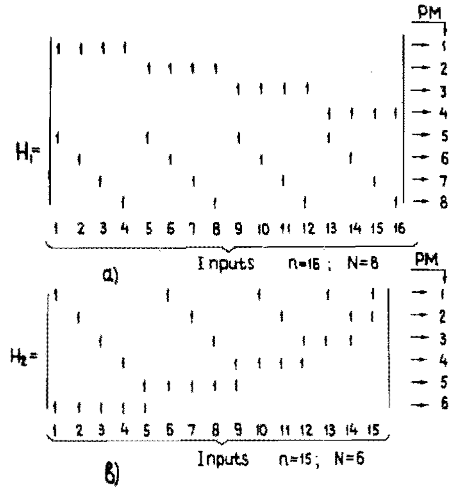

Fig. 6. Parity check matrix for: a) the known coding scheme ($n = 2N = 16$). *b) the coding scheme* ($n = C_N^2 = C_6^2 = 15$) *suggested by the author.*

4. CALCULATION OF DIGITAL FUNCTIONS

The theory of Calois field $CF(2^m)$ is a natural continuation of the theory of Boolean field. Representation of the Boolean functions as Galois field elements has a number of advantages. In particular, one can describe digital functions (DF) as a polynom in which coefficients and variables are the Galois field elements. For example, for a large number of variables ($m > 3$) analytical programming systems and present-day computers can be used to calculate digital logic devices with given properties.

The point is that any DF $f(X_o, X_1, \ldots, X_{m-1})$ of m arguments can be presented as a polynom [13]:

$$f(X_o, X_1, \ldots X_{m-1}) = B(0) + A(1)X + A(2)X^2 + A(3)X^3 + \ldots A(2^m - 1)X^{2^m - 1}$$

and the coefficients A(k) are calculated from the expression:

$$A(k) = \sum_{i=1}^{2^m - 1} a_i^{-k} [B(0) + B(a_j)]$$

with $B(a_j)$ are substitution elements taken from a truth table of inputs and outputs; and $B(0)$, the function at a zero point. A computer practically presents this result in a minimized mode.

Using the calculations obtained in paper [14], a new method of construction of universal, dynamically programmed modules has been suggested. The use of a set of similar modules opens up possibilities for fast programming,

using a program written on a microcomputer, of the operation of trigger systems without varying mechanical connections.

In conclusion the author expresses his gratitude to V.P.Shirikov and R.I.Gaidamaka for useful cooperation, A.V.Selikov for his help in programming PROM and D.V.Shirkov for his attention and support of this work.

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Никитюк Н.М. E11-87-10
Некоторые вопросы применения теории кодирования и аналитических методов расчета на ЭВМ

Приводятся основные результаты работ по применению теории и практики кодов, исправляющих ошибки, для создания быстродействующих устройств отбора физических событий, зарегистрированных в многоканальных детекторах ядерных частиц. На основе использования этой теории и аналитических вычислений на ЭВМ созданы принципиально новые устройства комбинационного типа, например, параллельные декодеры. Обсуждаются также вопросы создания нового алгоритма для расчета переключательных функций с помощью ЭВМ и проблемы построения универсальных динамически перепрограммируемых логических модулей.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1987

Nikityuk N.M. E11-87-10
Some Questions of Using Coding Theory and Analytical Calculation Methods on Computers

Main results of papers devoted to the application of theory and practice correcting codes are presented. These results are used to create very fast units for the selection of events registered in multichannal detectors of nuclear particles. Using this theory and analytical computing calculations, practically new combination devices, for example, parallel encoders, have been developed. Questions concerning the creation of a new algoritm for the calculation of digital functions by computers and problems of devising universal, dynamically reprogrammable logic modules are discussed.

The investigation has been performed at the Laboratory of High Energies, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1987