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дубна

E11-84-512

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USE OF SOME TYPES OF POLYNOMIALS IN CALIBRATION PROBLEMS FOR MEASURING DEVICES IN HIGH ENERGY PHYSICS

Submitted to VI International Symposium
on Computational Statistics (Praha, 1984)

1984

Direct and indirect (inverted) calibration transforms are formally defined by

$$u = Av, \qquad (1)$$

$$v = A^{-1}u, \qquad (2)$$

where A is an operator, corresponding to the transformation considered and A^{-1} is its inverse. The problem is to determine A and A^{-1}. Each problem belongs to the class of problems which often lead to incorrect, (ill-posed)[1] for the normal equations, matrices, corresponding to the approximation problems. An operator A is a response function (also called a transfer function or an apparatus function).

A is usually nonlinear and approximately defined and its simple inversion is not always attainable. We seek an approximate form of A and A^{-1} satisfying certain extremum conditions[2]. This is a characteristic of very complex devices in high energy physics: spiral reader data, filmless data acquisition (TV) system for streamer chamber[3], monitoring computing scanner[4]. These operators require the restoration of spatial coorsinates of events which are either optically or electronically (or both) nonlinearly distorted[3].

In the present paper we give 1) effective applications of specially built orthogonal polynomials; 2) comparative data from some calibration procedures of another types of polynomials.

Let $\{v_i\}$, $i = 1,2,\dots,m$ be the values of the quantity v, measured at m reference points. Let the corresponding values of the observable u be $\{u_i\}$, $i = 1,2,\dots,m$ and the respective accuracies $\{\Delta u_i\}$, $i = 1,2\dots,m$. Assuming a normal error distribution we can associate to each point v_i a positive weight

$$W_i = \frac{1}{(\Delta u_i)^2}. \qquad (3)$$

At the first calibration step the coordinates of fiducial cross centres are calculated[3,5]. At the second one the operators A and A^{-1} may be constructed in various ways. The choice of their types depends on the adapted mathematical model. Once the problem is given in a mathematically precise form, there appear several aspects in its solution: 1) the choice of approximation function and of the distance function; 2) the existence of a solution; 3) the uniqueness of a solution; 4) special properties of the solution; 5) the computation of the solution.

The polynomial models are flexible enough to describe any sort of transfer functions. More often the complexity of devices used allows us the employment of polynomial ones. We shall be looking for the following representation of (1) and (2):

$$u = \sum_k a_k P_k(v), \qquad (4)$$

$$v = \sum_\ell a_\ell Q_\ell(u), \qquad (5)$$

where $P_k(v)$ and $Q_\ell(u)$ are suitable sets of mutually independent polynimials. More frequently the polynimial set $\psi_1 = (1, x, y, xy, \dots)$ is used[4]. The direct computation of such ordinary polynomial series is handicapped by the high condition numbers of the matrices involved. In[6] ψ_2 and ψ_3 polynomial sets are used; ψ_2 are bivariate polynomials orthogonal over continuous set $[-1,1]\times[-1,1]$, that leads to quasidiagonal matrices and to the condition numbers ≈ 1. ψ_3 are bivariate polynomials, orthogonal over concrete discrete point set. If $\{P_k\}$ and $\{Q_\ell\}$ are orthonormal over point sets as follows:

$$\sum_{i=1}^{m} P_k(v_i)\, w_i\, P_\ell(v_i) = \delta_{k\ell}, \qquad \sum_{i=1}^{m} Q_k(u_i)\, w_i\, Q_\ell(u_i) = \delta_{k\ell},$$

then the series coefficients are easily computed via:

$$a_k = \sum_{i=1}^{m} u_i w_i P_k(v_i), \qquad (6)$$

$$b_\ell = \sum_{i=1}^{m} v_i w_i Q_\ell(u_i), \qquad (7)$$

In our case, we propose polynomials, numerically built by method of Forsythe-Weisfeld[7,8,9]. $P_L = C_L[(u_k - a_{L-1})P_k(u) - \sum_{\substack{r=1 \\ r\neq k}}^{L-1} \beta_L^r P_r(u)]$

Now we can report on the (x,y) calibration data for filmless data acquisition system of RISK-track chamber (Fig.1). In this case the chamber is scanned using TV-cameras[3]. Both the direct and inverted calibration problems consist of establishing one-to-one correspondence between image coordinates (in steps) and actual ones (in linear units), and vice versa. There are problems for two dimensional case. We formulate the calibration problem in the form of two discrete approximate problems

$$L_2^{u_1} = \sum_{i=0}^{m} [u_{1,i} - \sum_{k=0}^{n} a_{1,k} P_k(v_{1i}, v_{2i})]^2,$$

$$L_2^{u_2} = \sum_{j=0}^{m} [u_{2,j} - \sum_{\ell=0}^{n} a_{2,\ell} P_\ell(v_{1j}, v_{2j})]^2. \qquad (8)$$

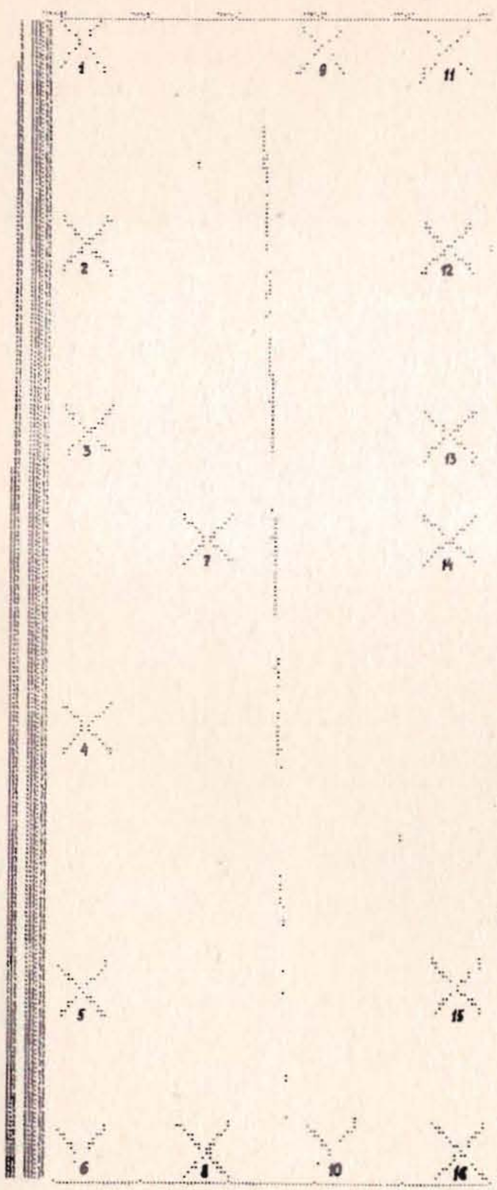

Fig.1

Now we can express u_1 and u_2 as functions of (v_1, v_2):

$$u_1 = \sum_k a_{1,k} \; P_k(v_1, v_2), \qquad (9)$$

$$u_2 = \sum_k a_{2,k} \; P_k(v_1, v_2), \qquad (10)$$

$$v_1 = \sum_\ell b_{1,\ell} \; Q_\ell(u_1, u_2), \qquad (11)$$

$$v_2 = \sum_\ell b_{2,\ell} \; Q_\ell(u_1, u_2), \qquad (12)$$

where $\{a_1\}$, $\{a_2\}$, $\{b_1\}$, $\{b_2\}$ are computed according to (6), (7). The approximate calibration transforms have been found. The criteria by which the lengths of the series (4) and (5) are selected are the following: 1) the min χ^2 per degree of freedom; 2) smoothness considerations; 3) measurement accuracy. The choice of 28 members (degree 5) is suitable by (2) and (3) criteria.

This Ortho-Normal-Expansion Method is adapted to the requirements of calibration problems [10]. On Fig.2 the fiducial mark positions are plotted, and the distortion of a rectangle grid is represented. It was computer by means of (11), (12). Figure 2 shows a very good quality of the numerically found calibration transforms despite of the poor data of 13 points only and of uninformity of a grid.

A package of special Fortran programs for two calibration steps is composed in LCTA, Dubna, where the above-mentioned four systems of polynomials are employed. The processing was carried on a CDC-6500 at JINR, Dubna [3,5].

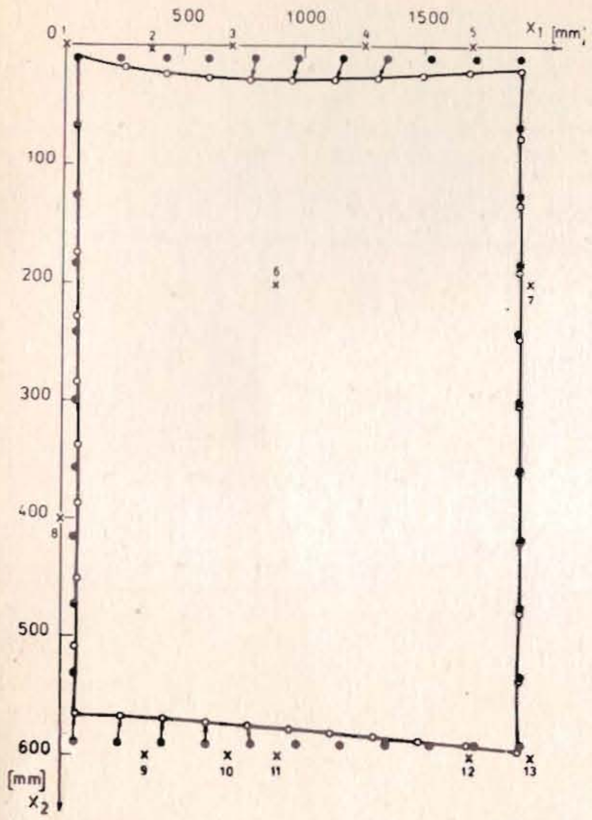

Fig.2

Here we present some new results (see the Table) on calibration with ONEM (ψ_4) and another polynimial model (ψ_2) for filmless-acquisition systems (by Gramm-Schmidt recursion formulae [5]).

Results and figures show that the orthogonal polynomial system by Forsythe-Weisfeld can be more successfully used comparing with another polynomial models.

Table

	Nonlinear Model (ψ_2)	ONEM (ϕ_4)
Maximum noncompensed distortion	0.019 mm	0.016 mm
Mean value of noncompensed distortion	0.009	0.001
Matrix inversion	yes	no
Iterating	yes	no
Stability troubles	yes	no
Max number of parameters limited by	storage	precisement of measurement
Machine time (relative units)	1	0.5

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Received by Publishing Department
on July 13, 1984.

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E11-84-512

Применение некоторых типов полиномов в калибровочных задачах для сложных измерительных устройств в физике высоких энергий

Рассматривается задача калибровки для измерительных приборов с прямоугольной системой координат. Прямое и обратное калибровочные преобразования определяются с помощью двумерных полиномов, наилучшим образом аппроксимирующих некоторый набор заданных координат. Предложены полиномы, построенные методом Форсайта-Вайсфельда. Сравнительное изучение их свойств показало высокое качество и применимость к калибровке бесфильмовой системы съема РИСК.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1984

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E11-84-512

Use of Some Types of Polynomials in Calibration Problems for Measuring Devices in High Energy Physics

The calibration problem for measuring devices with rectangular coordinate systems is considered. The direct and inverted calibration transformations are defined as the best least squares approximations of some coordinate set by polynomials in two variables. The proposed polynomials are numerically built by Forsythe-Weisfeld method. Some comparative study of their properties shows a good quality and applicability to the calibration of the filmless data read-out RISK system.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1984