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A.B.Shvachka

SYMMETRY PROPERTIES OF SOME NONLINEAR FIELD THEORY MODELS

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1. In this paper various approaches towards the study of symmetry properties of nonlinear equations as well as possible ways of their computer inplementation using the language of systems of analytical computations are discussed. A detailed investigation on the problems met in this paper may be found in [1-3]. In each section we deal with the possibilities of automation of routine calculations, connected with the study of symmetry properties of nonlinear field theory equations. Special attention is paid to the method of pseudopotential investigation of formal integrability and isovector method for the equations of balance.

2. The method of quasipotentials has been proposed by Wahlquist and Estabrook[4]. It allows one to write down for a nonlinear evolution equation a finite or infinite set of conservation laws as well as to find the Bäcklund transformation and the equations of associated linear scattering problem, by using the tools of outer differential forms. In order to illustrate the approach, consider the following[5] nonlinear 2nd order equation

$$H(\xi, \eta, u_{\xi\xi}, u_{\eta\eta}, u_{\xi\eta}, u_{\eta}, u_{\xi}, u) = 0. \qquad (2.1)$$

A column vector is said to be a quasipotential if it satisfies the following overdetermined system of the 1st order differential equations

$$y^{\mu}_{\xi}(\xi, \eta) = F^{\mu}(\xi, \eta, u, u_{\xi}, u_{\eta}, \dots, u^*, u^*_{\xi}, u^*_{\eta}, \dots, y^{\mu}, y^{\mu *}),$$
$$y^{\mu}_{\eta}(\xi, \eta) = G^{\mu}(\xi, \eta, u, u_{\xi}, u_{\eta}, \dots, u^*, u^*_{\xi}, u^*_{\eta}, \dots, y^{\mu}, y^{\mu *}), \quad \mu = 1,2,\dots,\eta. \qquad (2.2)$$

In this case the consistency conditions for the system (2.2), i.e.,

$$F^{\mu}_{\eta} = G^{\mu}_{\xi}, \qquad (2.3)$$

are equivalent with the original equation (2.1).

Hence, the problem is reduced to that one of writing down equations (2.2) for a given equation (2.1) (i.e., to find F^{μ} and G^{μ}) and, subsequently, of solving them relative to $y^{\mu}(\xi,\eta)$. The essence of the method consists of replacement of (2.1) by a closed Pfaffian system of differential forms which together with prolongation forms θ^{μ} form a differential ideal.

The condition

$$\Theta^{\mu} = dy^{\mu} - F^{\mu} d\xi - G^{\mu} d\eta \qquad (2.4)$$

leads to a family of commutation relations between vector-fields. On the next step, it is necessary to find the vector-fields y^{μ} in the explicit form of their commutation relations for components of vector-fields, and thereby also the functions F^{μ} and G^{μ}. The algorithm of finding quasipotentials requires cumbersome calculations. In order to automatize such calculations one can employ a system of analytical computations (say, REDUCE /7/). In ref. /7/ the algorithm is described and the text of program is given on the language of REDUCE for performing the basic operations with outer differential forms. Our experience shown that the program works rather slowly. Therefore the author jointly with I.G.Resnikov and V.L.Topunov created a program on the language of a system of analytical calculations, which carries over the following steps on a computer: 1) replaces the original equation by a system of outer differential forms, 2) tests the Frobenius condition for a given equation, and 3) writes down the system of equations for the purpose of finding isovectors /7/ and simplifies that system when possible. Consequently, the most part of routine work is carried out by a computer. In ref. /8/ a description is given to the programs in REDUCE which construct prolongation structures for nonlinear equations. Unfortunately, we have no possibility of becoming familiar with that paper. In /6,9/ REDUCE-programs are described for the study of Lie symmetries of differential equations.

3. A somewhat different, nevertheless conceptually close to WE /4/ approach towards the study of symmetry properties for nonlinear equations is given in /10,11/ In particular, in P.Kepsten's paper /11/ the formalism of local jets /12/ is employed. Similarly to the WE approach, the original equation is replaced by a family of outer differential forms. For the nonlinear Dirac equation

$$\sum_{k=1}^{3} h \frac{\partial}{\partial x_k}(\gamma_k \psi) - ih\frac{\partial}{\partial x_4}(\gamma_4 \psi) + m_0 c\psi + n_0 \psi(\bar{\psi}\psi) = 0 \qquad (3.1)$$

the class of generators of the local symmetry group is constructed in /11/. Using contact forms the ideal I of differential 1-forms, connected with (3.1), is constructed. The infinitesimal symmetries of the closed ideal I in the n-dimensional space formed by the class of differential forms $\alpha(I),\ldots,\alpha(m)$ are determined by vector-fields

$$V = V^i \frac{\partial}{\partial x_i} \qquad (3.2)$$

such that $\mathcal{L}_V I \subset I$, where $\mathcal{L}_V$ stands for the Lie derivative relative to the vector-field V. As is seen from (3.2), the vector-field V necessarily satisfies the conditions

$$\mathcal{L}_V\, \alpha(i) + \gamma(i,j) \wedge \alpha(j) = 0, \quad (i = 1,\ldots, m), \qquad (3.3)$$

where $\gamma(i,j)$ are the corresponding differential forms. Beside the definition of the generators for the local symmetry group of the nonlinear Dirac equation, using the program package on the language of REDUCE in /2/ there are determined also the preserved flows. The author jointly with I.G.Resnikov and V.L.Topunov carried over calculations and determined the generators of local symmetry groups for a number of the nonlinear equations. This required development of the corresponding program packet on the language of a system of analytical computations. In this way all routine calculations connected with the determination of generators and the construction of preserved flows are carried over with the aid of a computer /36/.

4. When investigating formal integrability of nonlinear evolution equations (NEE) one may successfully employ Computer Algebra Systems (CAS). In /13/ there are described algorithms and programs on the language REDUCE designed for the study of formal integrability of NEE having the form

$$u_t = F(u, u_1,\ldots,u_n),\ n \geq 2, \qquad u \equiv u(x,t),\ u_i = \frac{d^i u}{dx^i}. \qquad (4.1)$$

A systematic account of questions related to formal integrability, existence of nontrivial Lie-Bäcklund algebra, and infinite series of conservation laws may be found in /2,12/. Here we restrict ourselves to merely presenting basic concepts and necessary formulae /13/.

An equation of the form (4.1) is said to be formally integrable if there exists a formal series

$$L = \sum_{i=-\infty}^{\infty} a_i D^i,\quad a_i = a_i(u, u_1, \ldots, u_{k_i}),\quad D \stackrel{\text{def}}{=} \frac{d}{dx},$$

satisfying on solutions to (4.1) the following operator relation

$$L_t - [F*, L] = 0, \qquad (4.2)$$

where $F* \stackrel{\text{def}}{=} \sum_{i=0}^{n} \frac{\partial F}{\partial u_i} D^i,\quad L_t \stackrel{\text{def}}{=} [\frac{d}{dt}, L]$.

The set of all functions $H(u, u_1, \ldots, u_m)$ such that

$$H_*(F) - F_*(H) = 0 \tag{4.3}$$

is said to be the Lie-Bäcklund algebra, A(F), of the equation (4.1). A Lie-Bäcklund algebra is called nontrivial, if it contains elements different from u_1 and F. A conservation law for (4.1) is the relation

$$\frac{d}{dt}\, p(u, u_1, \ldots, u_k) = \frac{d}{dx}\, q(u, u_1, \ldots, u_{k+n}) \tag{4.4}$$

valid on the solutions to (4.1). The order of conservation laws is, by definition, the number $\frac{1}{2}\,\mathrm{ord}\frac{\delta p}{\delta u}$, where ordf=n is defined as such an n for which $\frac{df}{\partial u_n} \neq 0$, $\frac{df}{\partial u_m} = 0$, m>n; and $\frac{\delta}{\delta u} \overset{\mathrm{def}}{=} \sum_{i=0}^{k} (-1)^i D^i \frac{\partial}{\partial u_i}$ is the variational derivative. If $p \in \mathrm{Im}(D)$ (i.e., p coinsides with the total derivative with respect to x), then $\delta p/\delta u = 0$ and the conservation law becomes trivial. Let us summarize basic results on formally integrable equations [14-16].

Theorem 1. If the equation (4.1) admits for an infinite Lie-Bäcklund algebra, then it is formally integrable [14].

Theorem 2. If the equation (4.1) admits for an infinite series of conservation laws of form (4.4), then it is formally integrable [15].

Theorem 3. The equation (4.1) is formally integrable if and only if there exists an infinite series of conservation laws [16] of the following type:

$$(\mathrm{Res}L^{-1})_t \in \mathrm{Im}D, \tag{4.5a}$$

$$(\mathrm{Res}L^{-1}L_t) \in \mathrm{Im}D, \tag{4.5b}$$

$$(\mathrm{Res}L^{m})_t \in \mathrm{Im}D, \quad m = 1,2,\ldots, \tag{4.5c}$$

where $\mathrm{Res}\sum b_i D^i \overset{\mathrm{def}}{=} b_{-1}$.

Conditions (4.5a) and (4.5b) may be rewritten in terms of the right-hand side F of (4.1)

$$\left[\left(\frac{\partial F}{\partial u_n}\right)^{-1/n}\right]_t \in \mathrm{Im}D, \tag{4.6a}$$

$$\left(\frac{\partial F/\partial u_{n-1}}{\partial F/\partial u_n}\right) \in \mathrm{Im}D. \tag{4.6b}$$

In order to find explicit formulae for the coefficients of the series L and of densities $\mathrm{Res}L^m$ the following algorithm was proposed in [13]. It follows from (4.2) that the coefficients of the series $L^m = \sum_{i=-\infty}^{m} n_i D^i$ should satisfy the relations

$$b_i = \frac{F_n^{i/m}}{n} \int dx \left\{ F_n^{-\frac{(i+n)}{n}} [\, c_{n+i-1}|_{b_i=0} + \partial_t b_{n+i-1} \,]\right\}, \tag{4.7}$$

$$i = m-1,\ m-2, \ldots;\quad m = 1,2,\ldots,\quad F_n = \frac{\partial F}{\partial u_n},$$

where c_j are coefficients of D^j in the commutator $[L^m, F_*]$, $b_j = 0$ for $j > m$. The integration constants in (4.2) are taken to be zeroes.

Since the expressions c_{n+i-1} depend on b_j, $j \le i$, the relation (4.7) makes it possible to define an arbitrary number of coefficients b_i by recurrence. In particular, when $i = -1$, we may determine the explicit form of $\mathrm{Res}\,L^m$.

The algorithm described so far enables us to solve the following problems [13].

a) Testing the conditions for formal integrability. For any concrete equation of the form (4.1) one first has to check the conditions (4.6a) and (4.6b). If the result is positive, then the recurrent procedure (4.7) is employed and the quantities $\mathrm{Res}L^m$, $m = 1,2,3,\ldots$, are calculated. If the original equation is formally integrable, then the integrands in (4.7) must be total derivatives with respect to x. Hence, an implicit checking of conditions (4.5c) is contained in the recurrent procedure: if for $m = 1$ and on the i-th step a "nonintegrable" expression appears under the integral sign in (4.7), then the condition (4.5c) for $m = 1-n-i$ is not satisfied.

b) Inclusion of the requirement of formal integrability into the equations. When the proposed algorithm is applied to equations containing arbitrary constants, then these constants in general will appear in the left-hand sides of (4.6a), (4.6b), as well as in the integrand of (4.7), i.e., in expressions which for formally integrable equations should coincide with total derivatives. Hence, by taking the variational derivatives of these expressions equal to zero, we can resolve the resulting equations and in this way adjust the constants so that the original equation becomes formally integrable. The same results may be also obtained in a more simple manner, namely, by singling out the "nonintegrable" part of (4.7) and taking this equation to zero.

c) Finging the densities of conservation laws. Here it suffices to observe that for concrete equations of the form (4.1) the algorithm yields explicit formulae for $\mathrm{Res}L^m$, $m = 1,2,\ldots$, these being the densities we seek for.

d) Finding the elements of Lie-Bäcklund algebra. Calculations of $b_m, b_{m-1}, \dots, b_0$ according to (4.7) give us the preliminary information for the determination of the m-th order element of the Lie-Bäcklund algebra for any given equation of the form (4.1). The corresponding algorithm is described in [16]. It allows either to find the element of a prescribed order or, to show that such an element does not exist.

A detailed description of programs developed by the author jointly with A.Yu.Zharkov is given in [13]. The programs are applied to S.I.Svinolupov-V.V.Sokolov equations

$$u_t = u_5 + 5(u_2 - u_1^2 + \alpha e^{2u} + \beta e^{-4u}) u_3 - 5u_1 u_2^2 + 15(\alpha e^{2u} - 4\beta e^{-4u})^2 u_1 u_2 + u_1^5 + 90\beta e^{-4u} u_1^3 + 5(\alpha e^{2u} + \beta e^{-4u}) u_1, \quad \alpha, \beta \in C, \tag{4.8}$$

and result in a nontrivial element of the Lie-Bäcklund algebra in the form [13]

$$\begin{aligned} H = {} & u_7 + 7(u_2 - u_1^2) u_5 + 14(u_3 - 2u_1 u_2) u_4 - 21 u_1 u_3^2 - \\ & - 14(2u_2^2 + u_1^2 u_2 - u_1^4) u_3 - \frac{28}{3} u_1 u_2^3 + 28 u_1^3 u_2^2 - \frac{4}{3} u_1^7 + \\ & + 7(\alpha e^{2u} + \beta e^{-4u}) u_5 + 7(5\alpha e^{2u} - 16\beta e^{-4u}) u_1 u_4 + \\ & + 14(5\alpha e^{2u} - 13\beta e^{-4u}) u_2 u_3 + 28(2\alpha e^{2u} + 29\beta e^{-4u}) u_1^2 u_3 + \\ & + 42(\alpha e^{2u} - 76\beta e^{-4u}) u_1^3 u_2 + 14(7\alpha^2 e^{4u} - 13\alpha\beta e^{-2u} - 20\beta^2 e^{-8u}) u_1 u_2 + \\ & + 1260\beta e^{-4u} u_1^5 + 70(\alpha^2 e^{4u} + 2\alpha\beta e^{-2u} + 10\beta^2 e^{-8u}) u_1^3 + \frac{28}{3}(\alpha e^{2u} + \beta e^{-4u})^3 u_1. \end{aligned} \tag{4.9}$$

5. In [17,18] various concepts of group symmetry for differential equations are discussed. Since information concerning the group symmetry in many cases is equivalent to information concerning the solutions of a given equation, it appears reasonable to employ an as general concept of group symmetry as possible. We present the definition of the symmetry group proposed in [19,20]. Suppose the functions $\phi(x_0, \dots, x_{n-1})$ on the n-dimensional space $R^n(x)$ satisfy

$$A\phi = 0, \tag{5.1}$$

where A is a differential operator.

Definition 1. Let a family of operators g_L form the Lie algebra for some group G. The group G is said to be the symmetry group of the equation (5.1) if g_L and A commute on the set of solutions to (5.1):

$$[A, g_a]\phi(x) = 0. \tag{5.2}$$

In other words, the generators of G map one solution, $\phi(x)$, into another one, $\psi_a(x) = g_a \phi(x)$.

A more general definition of symmetry groups for equations is given in [17].

Definition 2. Let a family of operators $g_a (1 \le a < \infty)$ form an algebra closed under multiplication (·) and generate some group G. The group G is said to be a symmetry group for $A\phi = 0$, if

$$A \cdot A \cdot \ldots \cdot A . g_a \phi(x) = A^p g_a \cdot \phi(x) = 0 \tag{5.3}$$

(i.e., the multiplication p - yields an operator transforming a non-zero solution $\phi(x)$ into the zero solution) or, if

$$\psi_a(x) = A^{p-1} \cdot g_a \phi(x) \tag{5.4}$$

(i.e., the operator g_a considered on the set of all solutions to $A\phi = 0$, yields an operator transforming one non-zero solution into another one (in general, non-zero as well)).

The algebra of generators $g_a (1 \le a < \infty)$ of the symmetry group G is said to be the invariance algebra of the equation $A\phi = 0$ [17]. It is clear that the second definition contains the first one as a particular case, namely, one for p = 1 and the multiplication (.) corresponds to commutation, i.e., when G is a Lie group.

6. The symmetry properties of multidimensional nonlinear differential equations make it possible, in some cases, to construct the solutions explicitly. In [21,22] classes of exact solutions are found using the symmetry properties for the following multidimensional nonlinear equations:

$$\Box u + \lambda u^k = 0, \quad u = u(x), \quad x = (x_0, x_1, \dots, x_{n-1}), \tag{6.1}$$

$$\Box u + \lambda \exp(u) = 0, \quad \Box = -p_\mu p^\mu, \tag{6.2}$$

$$p^\mu = i g_{\mu\nu} \frac{\partial}{\partial x_\nu}; \tag{6.3}$$

$$\frac{\partial u}{\partial x_\mu} \cdot \frac{\partial u}{\partial x^\mu} = m^2, \quad [\gamma_\mu p^\mu - m - \lambda(\bar\psi\psi)^k]\psi = 0, \tag{6.4}$$

where γ_μ is the Dirac matrix, ψ is a spinor and m, k, λ are arbitrary constants.

In order to find explicit solutions to equations (6.1) through (6.4) the following approach is used [21]:

$$u(x) = \phi(\omega) f(x) + g(x), \tag{6.5}$$

where $\phi(\omega)$ is a certain unknown function (or a vector-function in case of a system of differential equations) depending on new invariant variables $\omega = \omega(x) = \{\omega_1, \omega_2, \dots, \omega_{n-1}\}$, the number of these being n-1, where n is the number of variables in the original equation. New variables $\omega(x)$ and explicit expressions for $f(x)$ and $g(x)$ are determined from the system of Euler-Lagrange equations

$$\frac{dx_0}{\xi^0} = \frac{dx_1}{\xi^1} = \dots = \frac{dx_{n-1}}{\xi^{n-1}} = \frac{du}{\eta}, \tag{6.6}$$

where ξ^μ and η are functions which determine the infinitesimal invariance group for (6.1)-(6.3), that is,

$$x'_\mu = x_\mu + \epsilon \xi^\mu(x, u) + O(\epsilon^2), \quad u'(x') = u(x) + \epsilon\eta(x, u) + O(\epsilon^2). \tag{6.7}$$

Of course, the explicit form of ξ^μ and η depends upon the concrete symmetry.

In order to solve the nonlinear Dirac equation (6.4) one uses [21]

$$\psi(x) = A(x)\phi(\omega), \tag{6.8}$$

where $A(x)$ is a 4x4 nonsingular matrix, $\phi(\omega)$ is an unknown 4-component spinor depending merely on invariant variables. The explicit form of the matrix A is determined from the equation [21]

$$QA(x) = (\xi^\mu(x)\partial\mu + \eta_x)A(x) = 0, \tag{6.9}$$

(i.e., Q is infinitesimal generator of the invariance group for (6.4)). Explicit forms of solutions to (6.4) are given in [23]. Besides the equations (6.1)-(6.4) the above explained method was employed by Fuschich and coauthors in order to find classes of exact solutions for multidimensional nonlinear Schrödinger equation [24] Hamilton-Jacobi equation [21] and Born-Infeld equation [25], respectively. It was shown that:

1. If a nonlinear equation is in possession of nontrivial symmetry, there is a hope to determine multi-parameter families of its exact solutions.
2. The exact solutions may serve the purpose of "etalons" when developing constructive approximative methods for solution of nonlinear equations.

The method proposed in [21-25] can be implemented on the language of systems of analytic computations. At present, the author jointly with I.G.Resnikov and V.L.Topunov are working on its computer realization.

7. When studying systems of differential equations in different areas of mathematics the necessity arises in checking their consistency [26]. This is usually connected with cumbersome symbolic calculations. One of the methods designed for this purpose is Cartan's algorithm [27].

The possibility of using a computer for the above-mentioned problem is pointed out in [28]. In [29-37] the algorithm is described for the investigation on consistency properties of various systems of equations.

Suppose we are given a system of m-th order differential equations

$$(s)\ \Phi_\mu(x, u, p) = 0, \quad \mu = 1, 2, \dots, s, \tag{7.1}$$

where $x = (x_1, \dots, x_n)$, $u = (u_1, \dots, u_n)$, p are the derivatives up to some specified order m of the functions u relative to the variables x, i.e., $p^\alpha_\mu = \frac{\partial^{(\alpha)} u_\mu}{\partial x_\alpha}$, $\alpha = (\alpha_1, \dots, \alpha_n)$. In order to solve the consistency problem for (7.1) it is necessary to clarify whether the given system is in involution, otherwise, it is necessary to construct its prolongation with the aid of adding differential consequences $\frac{d\Phi_\mu}{dx_i} = 0$, where d/dx_i is the total derivative with repsect to x_i. The system (7.1) is in involution, if the following conditions are satisfied [35]:

1) $c_{x_0}(s)$ is an involutory space;
2) there exist a neignbourhood $\mathcal{G} \subset J^m$ of the point $x_0 \in S$ such that the triple $(p(s) \cap (p^{m+1}_m)^{-1}(y), s\ \mathcal{G} p^{m+1}_m)$ is a fibred manifold,

where s is the manifold determined by $\Phi_\mu = 0$, $\frac{d\Phi_\mu}{dx_i} = 0$, J^m is the set of all m-jets j^m_x, and $p^{m+1}_m(j^{m+1}_x(f)) = j^m_m(f)$ respectively.

In [27] the application of analytical computations is illustrated on the Navier-Stokes equation and the existence of a partially invariant solution is exhibited. Furthermore, the corresponding system of equations determining that solution is reduced to a system in involution, and it is pointed out that there is a certain freedom in doing so.

When partial solutions are determined via the method of differential connections [35] then the original system of differential equations is completed using different differential connections.

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Исследование групповых свойств некоторых нелинейных моделей теории поля

Обсуждаются различные подходы к исследованию групповых свойств некоторых нелинейных эволюционных уравнений, а также возможные способы реализации этих подходов в виде программ на языке систем аналитических вычислений. Наиболее детально обсуждаются метод псевдопотенциалов и метод изовекторов для уравнений баланса в связи с исследованием интегрируемости нелинейных уравнений.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1984

Shvachka A.B. E11-84-495
Symmetry Properties of Some Nonlinear Field Theory Models

Verious approaches towards the study of symmetry properties of some nonlinear evolution equations as well as possible ways of their computer implementation using the computer algebra systems language are discussed. Special attention is paid to the method of pseudopotential investigation of formal integrability and isovector method for the equations of balance.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR

Preprint of the Joint Institute for Nuclear Research. Dubna 1984