объединенный
институт
ядерных
исследований
дубна

E11-84-451

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DYNAMICAL PROPERTIES OF MANY-DIMENSIONAL SOLITONS

Submitted to XXII International Conference on High Energy Physics (Leipzig, July, 1984)

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1984

1. INTRODUCTION

The inverse scattering transform (IST) method, having found numerous applications in investigations on nonlinear field theory models, turns out to be efficient for two-dimensional models. For many-dimensional partial differential equations successful results have been obtained with the aid of this method only for several models. Thus, besides the analytical methods of studying such models, it is of interest to investigate nonlinear field theory equations, admitting for soliton-like solutions, with the aid of numerical experiments [2].

Based on recent developments in numerical methods (fast Fourier transform [3], splitting methods [4], constructions of conservative and stable finite difference schemes for nonlinear equations [5]), it is possible to design numerical experiments for the study of dynamical properties of many-dimensional solitons. Simulations of interactions between soliton-like objects within the frame of various nonlinear field theory equations often provides us with information concerning the interaction dynamics which is not accessible using other ways.

At the same time, computer studies of qualitative properties of soliton-like solutions may suggest the manner of their subsequent analytical investigation.

In this paper the results concerning dynamical properties and stability of many-dimensional solitons are presented for classical fields within the framework of Klein-Gordon, Schrödinger and Dirac equations, respectively.

2. SIMULATION OF THE SOLITON-LIKE OBJECTS INTERACTIONS WITHIN THE FRAME OF KLEIN-GORDON EQUATION

Consider the Klein-Gordon equation

$$\phi_{tt} + \Delta_{1,2}\,\phi + \alpha\phi + \beta F(\phi)\phi = 0, \tag{1}$$

where Δ is the Laplace operator, $\Delta_1 u = \partial^2 u/\partial x^2$, $\Delta_2 u = \partial^2 u/\partial x^2 + \partial^2 u/\partial y^2$ and $F(\phi)$ is a nonlinear function. Computer studies of interactions of one-dimensional, (x,t), and two-dimensional, (x,y,t), quasisolitons have been performed [6] for nonlinearities of the following type:

$$F(\phi) = \begin{cases} |\phi|^2/(1+|\phi|^2), \\ \ln(|\phi|^2). \end{cases} \qquad (2)$$

Investigations have been performed using a symmetric second order difference scheme. The time step was $\Delta t = 0.1$; the spatial step $\Delta x = \Delta y$ was chosen from [0.1, 0.4].

In order to eliminate the influence of boundaries of computation area upon motion of solitons, an auxiliary condition in the form $\gamma\phi_t$ was added to (1). That term was taken into account merely in the neighbourhood of boundary having the width ≈5% of the dimension of the area (x,y). The optimum value of the coefficient γ was determined experimentally.

Let us deal in more detail with the way of choosing the initial configuration. Using the substitution

$$\phi(x,y,t) = \Psi(r)\,e^{i\omega t}, \qquad r = \sqrt{x^2+y^2}, \qquad (3)$$

we get from (1) the following boundary problem

$$\Psi_{rr} + \frac{1}{r}\Psi_r + a\omega^2\Psi - f(\Psi)\Psi = 0, \qquad \Psi_r(0) = 0, \qquad \Psi(\infty) = 0 \qquad (4)$$

and the problem leads to find a cylindrically-symmetric stationary one-soliton solution.

In order to solve the boundary problem (4) numerically the shooting method was used. The solution with prescribed accuracy obtained so far was then approximated using a class of Gaussian exponents

$$\Psi(r) \approx \sum_{i=1}^{n} a_i \exp\{\beta_i (r-\delta_i)^2\}. \qquad (5)$$

By choosing a_i, β_i, δ_i appropriately, it is possible to approximate $\Psi(r)$ in such a way that

$$\max|\Psi(r) - \sum_{i=1}^{3} a_i \exp\{\beta_i (r-\delta_i)^2\}| \le 0.005\,|\Psi(0)| \qquad (6)$$

holds true for all ω.

Using the Lorentz transformation

$$\phi(x,y,t) = \sum_{i=1}^{3} a_i \exp\{\beta_i (\sqrt{\gamma^2(x-vt)^2+y^2} - \delta_i)^2\} \times \exp\{-i\omega\gamma(t-vx)\} \qquad (7)$$

finally a moving soliton obtains. Subsequently, with the help of the above-mentioned difference scheme the soliton interactions were studied.

By carrying over numerical experiments for models possessing various types of nonlinearities, it was possible to single out four kinds of interactions:

1) resilient and quasiresilient interactions between quasisolitons;

2) decay (collapse) of quasisolitons after interaction;

3) decay (collapse) through a short-living bound state (resonance);

4) long-living bound state of two quasisolitons.

Two parameters have been changing in calculations, namely, the velocity of the relative motion v of quasisolitons and their charges:

$$Q = -i \int_{(x,y)} [\phi_t^*\phi - \phi_t\phi^*]\,dx\,dy. \qquad (8)$$

In both cases the four kinds of interactions have been observed. An analysis of numerical experiments suggested to formulate the conjecture, according to which the nature of soliton interactions is determined by the dispersion dependence $Q(\omega)$ and not by the type of the model.

A more detailed study of the nature of quasisoliton interactions resulted in the observation that it depends also on the angular momentum ℓ and on the initial phase difference $\Delta\theta$. Numerical experiments have shown:

1) There exists a certain resonance domain of the parameter ℓ, within which the resilience of interaction sharply decreases;

2) A purely antisymmetric angular configuration leads to a resilient interaction of quasisolitons.

In the series of numerical experiments /6/ there have been obtained seemingly paradoxical results. When placing unstable soliton-like objects sufficiently nearby each other (so that the kinematic time of their interaction becomes less than the decay time of each object), then under low enough relative motion velocities the appearance of a bound state - a two-dimensional bion (quasiperiodic solution) is observed. A possible explanation for stability of the observed bions with the help of a certain adibatic invariant may be found in ref./8/.

3. SIMULATION OF THE SOLITON-LIKE OBJECTS INTERACTIONS WITHIN THE FRAME OF NONLINEAR SCHRÖDINGER EQUATION

The nonlinear Schrödinger equation arises in many problems of solid state physics, nonlinear optics and plasma physics. The U(n) nonlinear Schrödinger equation with a repulsing-type potential may serve as an integrable variant of weakly nonideal Bose gas having n internal ("colour") degrees of freedom.

It arises also when describing many-chain antiferromagnetic spin systems with weak coupling between chains in the long-wave approximation [10].

A nonlinear Schrödinger equation with the noncompact isogroup U(1,1) describes a system consisting of two interacting bose-gases [9]. The nonlinear Schrödinger equation with cubic nonlinearity

$$i\Psi_t + \Psi_{xx} + a|\Psi|^2\Psi = 0 \tag{9}$$

has been investigated analytically using the IST method for various boundary conditions. The properties of the two-dimensional solutions (x,t) in case of U(1)-symmetry may be found in [11], in case of U(1,1)-isogroup in [12], for the case of U(2)-symmetry in [13], respectively. In [14], numerical experiments were used in the two-dimensional (x,t) case in order to study interactions between quasisolitons for the nonlinear Schrödinger equation possessing cubic nonlinearity and U(0,1)-, U(1,1)-symmetry groups, respectively. The numerical results are in favour of the conclusion that these models are fully integrable. Calculations have been performed using the semi-implicit finite difference scheme of approximation order $O(\tau + h^2)$, where τ and h designate the temporal and the spatial steps, respectively. Configurations of "soliton + antisoliton + vacuum" type, approximating the solution with exponential precision, have been employed as initial states.

The scattering of α-particles within the time-dependent Hartree-Fock theory was studied in [15]. The original three-dimensional problem is reduced to a two-dimensional one, the latter being described by a Schrödinger-type $\phi^4 - a\phi^6$ model

$$i\phi_t + \Delta\phi + a\phi + b|\phi|^2\phi - c|\phi|^4\phi = 0, \tag{10}$$

where a, b, c are constants and $\phi(x,y,t)$ is the one-particle wave function. Within this model interactions of two identical cylindrically-symmetric solitons having different energies and principal parameters have been studied. Numerical experiments suggest there arises a bound state at a certain value of energy $K \sim v^2$ of colliding solitons.

Interactions of cylindrically-symmetric gaussons within the nonlinear Schrödinger equation

$$i\phi_t + \{\frac{1}{2}\Delta + b\ln[a^D|\phi|^2]\}\phi = 0 \tag{11}$$

(a, b are constants, D is space dimensionality) have been studied in [16]. Fourier transformation was used for the purpose of numerical solution. Bound states of gaussons have not been observed in [16] for the model (11). However, only "uncharged"

gaussons interactions (once for which $\omega = 0$ at $v = 0$) have been considered there.

At present, we are carrying over a series of numerical experiments concerning interactions of three-dimensional gaussons (x,y,t) within the model (11). We use the splitting methods (which yield locally one-dimensional schemes) in order to solve the nonlinear Schrödinger equation (see [17]). The proposed method applies to Schrödinger equations possessing nonlinearities of the types

$$f(|u|)u = \begin{cases} a|u|^2u \\ a|u|^2u + b|u|^4u \\ \exp[-a|u|^2]u \\ |u|^2u|(1+|u|^2) \\ \ln(a|u|^2). \end{cases}$$

Within this approach a uniform temporal lattice $\omega_\tau = \{t_j = j\tau,\ j = 0, 1, \dots, N_t;\ N_t = t_{max}/\tau\}$ is employed. The multidimensional Schrödinger equation

$$i\phi_t + (\partial^2_{x_1x_1} + \partial^2_{x_2x_2} + f(|\phi|))\phi = 0 \tag{11'}$$

with the initial condition

$$\phi(x_1, x_2, 0) = \phi_0(x_1, x_2) \equiv \phi_0(\vec{x}) \tag{12}$$

is replaced by a chain of one-dimensional equations

$$\frac{1}{2} i \frac{\partial v_a}{\partial t} + A_a v_a = 0, \quad t_{j+(a-1)/2} < t \le t_{j+a/2}, \quad a = 1, 2, \tag{13}$$

where $A_a = \partial^2_{x_a x_a} + \frac{1}{2} f(|v_a|)$. Initial conditions of the type

$$v_1(\vec{x}, 0) = \phi_0(\vec{x}),$$

$$v_2(\vec{x}, t_{j+1/2}) = v_1(\vec{x}, t_{j+1/2}), \tag{14}$$

$$v_1(\vec{x}, t_{j+1}) = v_2(\vec{x}, t_{j+1})$$

are added. Each equation in (13) on the interval $t_{j+(a-1)/2} < t \le t_{j+a/2}$ is approximated by a semi-implicit finite-difference scheme. The total approximation order is $O(\tau + h_1^2 + h_2^2)$, where h_1, h_2 are the spatial steps along the coordinate axes. A configuration approximating the solution of (11) with exponential precision is chosen as the initial configuration.

4. INVESTIGATION OF STABILITY OF SOLITONS FOR THE NONLINEAR DIRAC EQUATION

The nonlinear Dirac equation

$$i\gamma^{\mu}\partial_{\mu}\Psi - m\Psi + 2\lambda(\bar{\Psi}\Psi)\Psi = 0, \quad \lambda > 0, \tag{15}$$

is employed in elementary particle physics for modelling extended quarks[18,19]. This model uses six Dirac fields which form two triplets Ψ_k and ϕ_k, $k = 1,2,3$. Here, ϕ_k is the charge conjugate to Ψ_k and ϕ_k, Ψ_k interact via 4-th order two-body forces. The particle is interpreted as a certain soliton solution to the field equations. By interpreting Ψ_k as a quark, and ϕ_k as an antiquark, the theory becomes amenable to comparison with experimental data. If the particle is the bound state of three Ψ_k (or of three ϕ_k), it corresponds to 3q (or $3\bar{q}$) state.

Curiously enough, the mechanism of confinement appears to be a particular case of the triviality so that it is not related to any quantum effects. In fact, the fields are created as c-numbers. This suggests the possibility that the confinement might be a property of the field equations at the classical level, which perhaps should be investigated before proceeding to their quantization.

In [19] it is shown there exist soliton solutions (↑ , ↓ - spin direction)

$$\Psi_{\uparrow} = e^{-i\Omega m t}\sqrt{\frac{m}{2\lambda}}\begin{pmatrix} G\begin{pmatrix}1\\0\end{pmatrix} \\ iF\begin{pmatrix}\cos\theta\\ \sin\theta\, e^{i\phi}\end{pmatrix}\end{pmatrix},$$

$$\Psi_{\downarrow} = e^{-i\Omega m t}\sqrt{\frac{m}{2\lambda}}\begin{pmatrix} G\begin{pmatrix}0\\1\end{pmatrix} \\ -iF\begin{pmatrix}\sin\theta\, e^{-i\phi}\\ -\cos\theta\end{pmatrix}\end{pmatrix}, \tag{16}$$

for all Ω with $\Omega^2 < 1$. Here, F and G are radial functions of spatial variables, satisfying the following system of equations:

$$F' + \frac{2}{\rho}F + (1 - \Omega + F^2 - G^2)G = 0, \quad G' + (1 + \Omega + F^2 - G^2)F = 0, \quad \rho = mr \tag{17}$$

The function F and G for soliton-like solutions are found in [20]. In [19] it is pointed out that it is a difficult task to investigate the stability of solutions within equations (15).

Also the stability of solutions with respect to deformations in two and three dimensions is pointed out in the quoted reference. It is noted that the scalar fields are highly unstable.

The stability of classical spinor field within the frame of Dirac equation with self-interaction $(\bar{\Psi}\Gamma\Psi)^2$ is investigated in [21] by the variational method in each two-dimensional and four-dimensional space-times. It is shown that three-dimensional spinor solitons generated by the scalar self-interaction $(\bar{\Psi}\Psi)^2_4$ are unstable at all frequencies. The instability region for three-dimensional solitons (coinciding with that one for one-dimensional solitons) of spinor fields under the selfinteraction $(\bar{\Psi}\Psi)^2_2$ is found to be

$$|\omega| < \frac{1}{\sqrt{2}}. \tag{18}$$

The dynamics of instability for the two-dimensional (x,t) Dirac equation with scalar selfinteraction $(\bar{\Psi}\Psi)^2_2$ is studied by numerical experiments (it has the form of energy contraction into narrow domains). The numerical calculations are carried over for (15) with the help of the second order symmetric finite difference scheme (i.e., of order $O(\tau^2 + h^2)$, where τ and h have the same meaning as before).

In [24] it is proved that for (15) there exist cylindrically-symmetric soliton-like solutions of the form

$$\phi(\rho, t) = e^{i\nu m t + in\phi}\left(i\frac{\not{x}}{\rho}\Psi_1(\rho) + \Psi_2(\rho)\right), \tag{19}$$

where Ψ_1, Ψ_2 are eigenfunctions of γ^0 corresponding to the same eigenvalue. Here

$$\not{x} = -\sum_{i=1}^{2}\gamma^i x_i, \tag{20}$$

ν - a positive real number, ϕ - the azimuthal angle around the x_3-axis, n - an integer and ρ - distance from the x_3-axis. Also the existence of spherically-symmetric soliton-like solutions to (15) is established. However, the proofs are based on a method introduced in [23], which is not mathematically rigorous. Rigorous proofs are given in [24] for the existence of soliton-like solutions for equations similar to those ones considered in [22]. The numerical experiments described in [22] show that the energy of cylindrically-symmetric solitons is always less than the energy of solutions in the form of planar waves. However, it is not possible to draw out of these experiments conclusions about stability of soliton-like solutions.

Consequently, it appears to be of great interest to design numerical experiments for the purpose of studying the stability properties of soliton-like solutions to nonstationary nonlinear and multidimensional Dirac equation. It is clear that any proposed algorithm should be unconditionally stable and efficient in the sense of minimizing the overall number of arithmetic operations $Q(\epsilon)$ needed to calculate the solution within a given precision $\epsilon > 0$. The latter requirement becomes extra-ordinarily important in multidimensional problems.

Let us describe one of efficient methods (the "method of total approximation"), applied to the nonlinear Dirac equation (15) in four-dimensional space-time. To this end rewrite (15) in the form

$$i(\gamma_0 \frac{\partial}{\partial t} + \gamma_1 \frac{\partial}{\partial x_1} + \gamma_2 \frac{\partial}{\partial x_2} + \gamma_3 \frac{\partial}{\partial x_3})\Psi - m\Psi + g^2(\bar{\Psi}\Psi)\Psi = 0, \qquad (21)$$

where (x_1, x_2, x_3) belongs to a certain spatial domain G and $0 < t \leq t_0$. By multiplying (21) by $-i\gamma_0$ form the left we obtain

$$\frac{\partial}{\partial t}\Psi + L\Psi + im\gamma_0 \Psi - ig^2(\bar{\Psi}\Psi)\gamma_0 \Psi = 0, \qquad (22)$$

where

$$L = \gamma_0\gamma_1 \frac{\partial}{\partial x_1} + \gamma_0\gamma_2 \frac{\partial}{\partial x_2} + \gamma_0\gamma_3 \frac{\partial}{\partial x_3}. \qquad (23)$$

We add the initial condition

$$\Psi(0,\vec{x}) = \Psi_0(\vec{x}). \qquad (24)$$

Let us construct a locally one-dimensional finite difference scheme for (22). We use the temporal lattice $\omega_\tau = \{t_j = j\tau$, $j = 0, 1, \dots, N_0$, $N_0 = t_0/\tau\}$. In place of (22) we introduce the following chain of one-dimensional systems

$$\frac{1}{3}\frac{\partial u_\alpha}{\partial t} + L_\alpha u_\alpha + im\gamma_0 u_\alpha - g^2(\bar{u}_\alpha u)\gamma_0 u_\alpha = 0,$$
$$t_{j+(\alpha-1)/3} < t \leq t_{j+\alpha/3}, \qquad (25)$$

where $L_\alpha = \gamma_0\gamma_\alpha \frac{\partial}{\partial x_\alpha}$, $\alpha = 1, 2, 3$. The value of the product $(\bar{U}_\alpha U)$ in (25) is taken from the preceding temporal fibre. Finally, the initial conditions are written in the form

$$\Psi_1(0,\vec{x}) = \Psi_0(\vec{x}),$$
$$\Psi_2(t_{j+1/3},\vec{x}) = \Psi_1(t_{j+1/3},\vec{x}),$$
$$\Psi_3(t_{j+2/3},\vec{x}) = \Psi_2(t_{j+2/3},\vec{x}), \qquad (26)$$
$$\Psi_1(t_{j+1},\vec{x}) = \Psi_3(t_{j+1},\vec{x}).$$

Each of the equations (25) with index α (which actually represents a system of four equations since Ψ is a 4-component spinor) is approximated by a finite difference scheme

$$\frac{u^{j+\alpha/3} - u^{j+(\alpha-1)/3}}{\tau} + \gamma_0\gamma_\alpha D_\alpha \bar{u}^{j+\alpha/3} + im\gamma_0 \bar{u}^{j+\alpha/3} -$$
$$- i\gamma_0(\bar{u}u)\bar{u}^{j+\alpha/3} = 0, \qquad (27)$$

where

$$D_\alpha = \frac{1}{2}(\nabla_{x_\alpha} + \nabla_{\bar{x}_\alpha}) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\nabla_x u_k = \frac{u_{k+1} - u_k}{h}, \qquad \nabla_{\bar{x}} u_k = \frac{u_k - u_{k-1}}{h}, \qquad (28)$$

$$\bar{u}^{j+\alpha/3} = \frac{1}{2}(u^{j+\alpha/3} + u^{j+(\alpha-1)/3}).$$

Let's introduce scalar product for the proof of absolute stability of one-dimensional finite difference scheme

$$(u,v) = \sum_{i_1=1}^{N_1-1}\sum_{i_2=1}^{N_2-1}\sum_{i_3=1}^{N_3-1} u^+_{i_1 i_2 i_3} v_{i_1 i_2 i_3} h_1 h_2 h_3, \quad ||u|| = \sqrt{(u,u)}, \qquad (29)$$

where

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}, \qquad u^+ = (u_1^*, u_2^*, u_3^*, u_4^*),$$

h_1, h_2, h_3 are the steps in directions of x_1, x_2 and x_3-axes, respectively. Let us multiply scalary the equation (27) by $\bar{u}^{j+\alpha/3}$ both from the right and from the left. By summing up the resulting equalities and by taking into account the fact that the matrices γ and D_α commute we get

$$\frac{1}{\tau}\{||u^{j+\alpha/3}||^2 - ||u^{j+(\alpha-1)/3}||^2\} + u^+\gamma_0\gamma_\alpha(D_\alpha + D_\alpha^+)\bar{u}^{j+\alpha/3} = 0, \qquad (30)$$

where the operator D_α^+ is the adjoint one to D_α. According to /26/,

$D^+_\alpha = -D_\alpha$. Consequently,

$$\| u^{j+\alpha/3} \| = \| u^{j+(\alpha-1)/3} \| . \qquad (31)$$

Let us take the sum of equations (31) for $\alpha = 1,2,3$. Then we get

$$\| u^{j+1} \| = \| u^j \| = \ldots = \| u^0 \| = \| \Psi_0 \| \qquad (32)$$

and this just means the stability relative to the initial conditions /25,26/ and the norm (30). The latter equations show that the proposed finite difference scheme is conservative with respect to the conservation low

$$I = \int \Psi^+ \Psi d^3x \qquad (33)$$

which is but the integral form of the continuity equation for the Dirac equation.

For the sake of simplicity suppose the solutions we seek for have to satisfy given periodic boundary conditions. In this way we get a system of three-point equations which is solvable using alternating-direct method /25/. The proposed algorithm is effective in the sense described above.

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Received by Publishing Department
on June 27, 1984.

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Динамические свойства неодномерных солитонов

Приведен краткий обзор работ по исследованию динамических свойств и структурной устойчивости неодномерных солитонов в рамках уравнения Клейна-Гордона с нелинейностями $\ln(|\phi|^2)\phi$ и $|\phi|^2\phi/(1+|\phi|^2)$, уравнения Шредингера с нелинейностью вида $f(|u|)u$ и уравнения Дирака с нелинейностью $(\bar{\psi}\psi)\psi$. Обсуждаются методы численного решения неодномерных нелинейных начально-краевых задач для дифференциальных уравнений теории поля. Для уравнения Клейна-Гордона рассмотрена симметричная конечно-разностная схема. Применительно к решению неодномерных нелинейных уравнений Шредингера и Дирака рассмотрен метод расщепления, построены локально-одномерные схемы с порядком аппроксимации $O(\tau+h^2)$, где τ - шаг по времени, h - максимальный шаг по пространственным переменным. Приведены оценки устойчивости численных алгоритмов.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1984

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Dynamical Properties of Many-Dimensional Solitons

The brief review of the dynamical properties and structural stability of many-dimensional solitons within the framework of the Klein-Gordon equation with nonlinearities of $\ln(|\phi|^2)\phi$ and $\frac{|\phi|^2\phi}{1+|\phi|^2}$ types, the Schrödinger equation with nonlinearity of $f(|u|)u$ type and Dirac equation with nonlinearity $(\bar{\psi}\psi)\psi$ is presented. Numerical methods for solving many-dimensional nonlinear differential equations of field theory are considered. The symmetric finite difference scheme is used for solving nonlinear Klein-Gordon equation. The splitting method for numerical solving of many-dimensional nonlinear Schrödinger and Dirac equations is developed. The local one-dimensional scheme with approximation of $O(\tau+h^2)$ order, where τ is temporal step and h is maximum dimensionless (x,y)-grid step is presented. The conditions of stability of numerical method are considered.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1984