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DOUBLET RESOLUTION ALGORITHM WITHOUT RECONSTRUCTION OF THE INITIAL SIGNAL

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Панебратцев Ю.А., Шахалиев Э.И., Сороко Л.М. E10-97-369
Алгоритм разрешения дублета
без восстановления исходного сигнала

Разработан алгоритм разрешения дублета без восстановления исходного сигнала. Этот алгоритм был применен для разрешения дублетной структуры, скрытой шумами в выходном сигнале от физического прибора, с симметричной или несимметричной функциями размытия. Проанализированы устойчивости к шуму разработанного алгоритма.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

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Panebratsev Yu.A., Shahaliev E.I., Soroko L.M. E10-97-369
Doublet Resolution Algorithm without Reconstruction
of the Initial Signal

The doublet resolution algorithm without reconstruction of the initial signal is developed. This algorithm has been applied to the resolution of the doublet structure hidden by the noise in the output signal from physical device with symmetrical or nonsymmetrical spread functions. The noise immunity of the developed algorithm is analyzed.

The investigation has been performed at the Laboratory of High Energies, JINR.

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1.Introduction

The resolving power of the physical device with symmetrical or nonsymmetrical spread function was considered in [1] on the basis of the Fourier Transformation approach. It was shown that the physical device with nonsymmetrical spread functions has higher resolving power than the physical device with symmetrical spread function with the same halfwidth. The simulating calculations show that the advantage of the physical device with nonsymmetrical spread function over the physical device with symmetrical spread function is getting pronounced for experimental quality $1/\epsilon \geq 3.5$.

The effective pulse resolution algorithms for detectors with Gaussian-like signals and for extremely high multiplicity of central ultrarelativistic nucleus-nucleus collisions have been proposed in [2]. The position and the amplitude of the initial signal have been calculated with parabolic approximation method (PAM). The developed procedure was generalized to the case of the correlated noise.

In this paper we consider the algorithm of the doublet resolution without reconstruction of the initial signal and without any approximation calculation methods. This algorithm has been applied to the resolution of the doublet structure hidden by the noise in the output signal from physical device with symmetrical or nonsymmetrical spread functions. The noise immunity of this algorithm is demonstrated on some examples.

2.Theory

For narrow doublets we may use the Fourier algorithm of the signal blurring described in [3,4]. According to this algorithm the initial signal f(x) is subjected by blurring in such a way that the output signal from the device g(x) can be considered as convolution of the initial signal f(x) with spread function of the device h(x):

$$f(x) \rightarrow g(x) = h(x) \otimes f(x)\,. \tag{1}$$

In the Fourier Transform space the convolution operation is going into the product operation:

$$G(x) = H(\omega) \cdot F(\omega) \tag{2}$$

where $F(\omega)$, $H(\omega)$ and $G(\omega)$ are the Fourier Transforms of the functions f(x), h(x) and g(x), respectively. For example,

$$F(\omega) = \int f(x)exp(-i\omega x)dx. \tag{3}$$

In general case the unknown function f(x) can be estimated from its Fourier Transform as

$$f'(x) = \int F'(\omega)exp(i\omega x)d\omega \tag{4}$$

where,

$$F'(\omega) = \frac{G(\omega)}{H(\omega)}. \tag{5}$$

However the very simple algorithm, Eqs.(4) and (5), involves many difficulties typical for all linear solution algorithms of the inverse problems [5,6]. Some of these difficulties have been removed by regularization procedure [7].

In this paper we use another approach, which do not involve the reconstruction stage, Eqs.(4) and (5), at all. So the information about the doublet structure of the initial signal can be deduced directly from the minima positions of the Fourier Transform of the detected signal $G(\omega)$.

For example, the initial signal with narrow maxima,

$$f_0(x) = \delta(x - \Delta) + \delta(x + \Delta), \tag{6}$$

with doublet splitting 2Δ is going into the Fourier Transform space as

$$F_0(\omega) = cos(\omega/2L), \tag{7}$$

where the modulation period L of the $F_0(\omega)$ function defines the doublet splitting 2Δ according to the simple relation

$$2\Delta = 1/L, \tag{8}$$

or

$$ln\Delta = -lnL + m \tag{9}$$

in the logarithmic scale.

3. Noise

In the real experiment there are noises from various sources. For the case of the additive noises we have

$$g(x) = h(x) \otimes f(x) + n(x). \tag{10}$$

To estimate the effect of the additive noise on the resolving power of our algorithm we have made several computer simulations. From the Eq.(9) we see that the relation between ln Δ and ln L in the logarithmitic scale has the form of the straight line with angle (-45^0) with respect to the (ln L) - axis.

If the doublet splitting Δ is small, the modulation period L is very large, and due to this the corresponding minimum is lost in the noise, the amplitude of which in the ω-axis is practically constant for all ω.

The additive noise used in our calculation was found according to the following scheme. At first we construct the function

$$N(x) = (k\%) \cdot max(A) \cdot (i) \tag{11}$$

in every x-point, where i-is the quasirandom positive definite number taken from the computer for $0 < i < 1$, $max(A)$ is the maximum value of the signal amplitude, and (k%) is the value of the accepted level of the noise.

We have repeated this operation m times and then the average value

$$\bar{n}(x) = \frac{1}{m} \cdot \sum_{k=1}^{n} N_k(x) \tag{12}$$

has been calculated. The accepted realization of the noise signal

$$n(x) = N(x) - \bar{n} \tag{13}$$

was a positive definite function.

The computer program, developed for computer simulation, hold all the values $|F(\omega)|$ in the special part, and then, another routine searches the minimum in the $|F(\omega)|$ values. The absolute value of $|F(\omega)|$ at the second point is with the previous and the subsequent ones. If for negative values of ω the following condition

$$|F(\omega_m - 1)| > |F(\omega_m)| < |F(\omega_m + 1)| \tag{14}$$

is fulfilled, then the ω_m valve is stored. The analogous condition has been used for positive values of ω. The corresponding ω_n value is stored. The distance between two minima is equal to

$$L = \omega_n - \omega_m = \omega_n + |\omega_m|. \tag{15}$$

4. Results

The simulation calculations have been performed according cited algorithm, for four typical spread functions: symmetrical gaussian, symmetrical lorentzian, nonsymmetrical gaussian and nonsymmetrical lorentzian.

The results for symmetrical gaussian spread function are shown in Fig.1. In Fig.1a the symmetrical gaussian spread function with $\sigma = 2.75$ is shown for $|x| < 65$. The Fourier Transform of this spread function is shown in Fig.1b for $|\omega| < 16.5$ in the logarithmic scale. In Fig.1c we see the initial signal in the form of the doublet with doublet splitting $\Delta = 3.0$. In Fig.1d we see its Fourier Transform with two sharp minima. The distance 2L between these minima is equal to 2L =19.0. The initial doublet signal with additive noise 5% is shown in Fig.1e, and its Fourier Transform in Fig.1f. The distance 2L between minima estimated according program described in §3, is equal to $(2L)^{5\%}_{est} = 19.0$. The initial doublet signal with additive noise 12% is shown in Fig.1g and its Fourier Transform in Fig.1h. The estimated distance 2L between minima is equal to $(2L)^{12\%}_{est} = 19.0$. The distance 2L estimated for different additive noise at various doublet splittings Δ is presented in Fig.2a and 2b. We see that at extremely small doublet splitting $\Delta, \Delta \sim 2 \div 4$, the estimated distance 2L between minima does not coincide with real of 2L in the absence of noise. For $\Delta = 3$ the error $\delta\Delta \approx 0.5$.

The results for symmetrical lorentzian spread function are given in Fig.3. The symmetrical lorentzian spread function with $\sigma = 2.75$ is shown for $|x| < 65$ in Fig.3a. The Fourier Transform of this spread function is shown in Fig.3b for $|\omega| < 50.$ in the logarithmitic scale. In Fig.3c we see the initial signal in the form of the doublet with doublet splittings $\Delta = 3.0$. In Fig.3d we see its Fourier Transform with two sharp minima. The distance 2L between these minima is equal to 2L = 19. The initial doublet signal with additive noise 5% is shown in Fig.3e, and its Fourier Transform in Fig.3f. The distance 2L between minima estimated according program described in §3, is equal to $(2L)^{5\%}_{est} = 19.0$. The initial doublet signal with additive noise 12% is shown in Fig.3g and its Fourier Transform in Fig.3h. The estimated distance 2L between minima is equal to $(2L)^{12\%}_{est} = 19.0$.

The distance 2L estimated for different additive noise at various doublet splittings Δ is given in Fig.4. We see that at extremely small doublet splittings $\Delta, \Delta \sim 2 \div 4$, the estimated distance 2L between minima does not coincide with real value of 2L in the absence of noise. For $\Delta = 2.5$ this error $\delta\Delta = 0.4$. To show the influence of the different noise realizations on the error $\delta\Delta$ we have calculated the relation $L = L(\Delta)$ for four different (10%) noise realization (Fig.5). We see that the weighted error $(\delta\Delta)_{AV} \leq 0.5$ for all doublet splittings Δ.

The results for non-symmetrical gaussian spread function are shown in Fig.6. In Fig.6a the non-symmetrical gaussian spread function with $\sigma_L/\sigma_R = 54$ is shown for $|x| < 65$. The Fourier Transform of this spread function is shown in Fig.6b for $|\omega| < 16.5$ in the logarithmic scale. In Fig.6c we see the initial signal in the form of the doublet with doublet splitting $\Delta = 3.0$. In Fig.6d we see its Fourier Transform with two sharp minima. The distance 2L between those minima is equal to 2L = 19. The initial doublet signal with additive noise 5% is shown in Fig.6e, and its Fourier Transform in Fig.6f. The distance 2L between minima estimated according program described in §3 is equal to $(2L)^{5\%}_{est} = 19.0$. The initial doublet signal with additive noise 12% is shown in Fig.6g and its Fourier Transform in Fig.6h. The distance 2L between minima estimated according our program is equal to $(2L)^{12\%}_{est} = 19.0$.

The relation between L and Δ, estimated in the presence of the additive noise for non-symmetrical gaussian spread function is given in Fig.7 for additive noises up to 12%. The weighted error $(\Delta\delta)_{AV} \leq 0.25$ for doublet splitting $\Delta > 3$.

The results for non-symmetrical lorentzian spread function are shown in Fig.8. In Fig.8a the non-symmetrical lorentzian spread function with $\sigma_L/\sigma_R = 54$ is shown for $|x| < 65$. The Fourier Transform of this spread function is shown in Fig.8b for $|\omega| < 16.5$ in the logarithmic scale. In Fig.8c we see the initial signal in the form of the doublet with doublet splitting $\Delta = 3.0$. In Fig.8d we see its Fourier Transform with two sharp minima. The distance 2L between those minima is equal to 2L = 19. The initial doublet signal with additive noise 5% is shown in Fig.8e, and its Fourier Transform in Fig.8f. The distance 2L between minima estimated according program described in §3 is equal to $(2L)^{5\%}_{est} = 19.0$. The initial doublet signal with additive noise 12% is shown in Fig.8g and its Fourier Transform in Fig.8h. The distance 2L between minima estimated according our program is equal to $(2L)^{12}_{est}\% = 19.0$.

The relation between L and Δ, estimated in the presence of the additive noise for non-symmetrical lorentzian spread function is given in Fig.9 for additive noises up to 15%. The weighted error $(\delta\Delta)_{AV} \leq 0.2$ for doublet splitting $\Delta > 3$.

To compare our results of the computer simulation we present two parameters: 1) The relative noise level of the Fourier Transform of the initial signal for different noises N(rel), and 2) The number of minima k in the Fourier Transform, up to which we may distinguish them in the ω - axis. This comparison is presented in Table 1.

Table 1

	%		Gaussian	Lorentzian	
	Noise	Symm	Non-symm	Symm	Non-symm
$N_{rel}x10^4$	5	8.8	10	2.8	-
	12	19	22	6.0	0.7
	15	-	-	-	1.4
k	5	1	1	2	-
	12	1	1	1.5	3
	15	-	-	-	3

From Table 1 we see that the best results give the noise immune spread function in the form of the non-symmetrical lorentzian. This behavior can be explain by the fact that this spread function gives the intense Fourier Transform components in the region of high frequency ω.

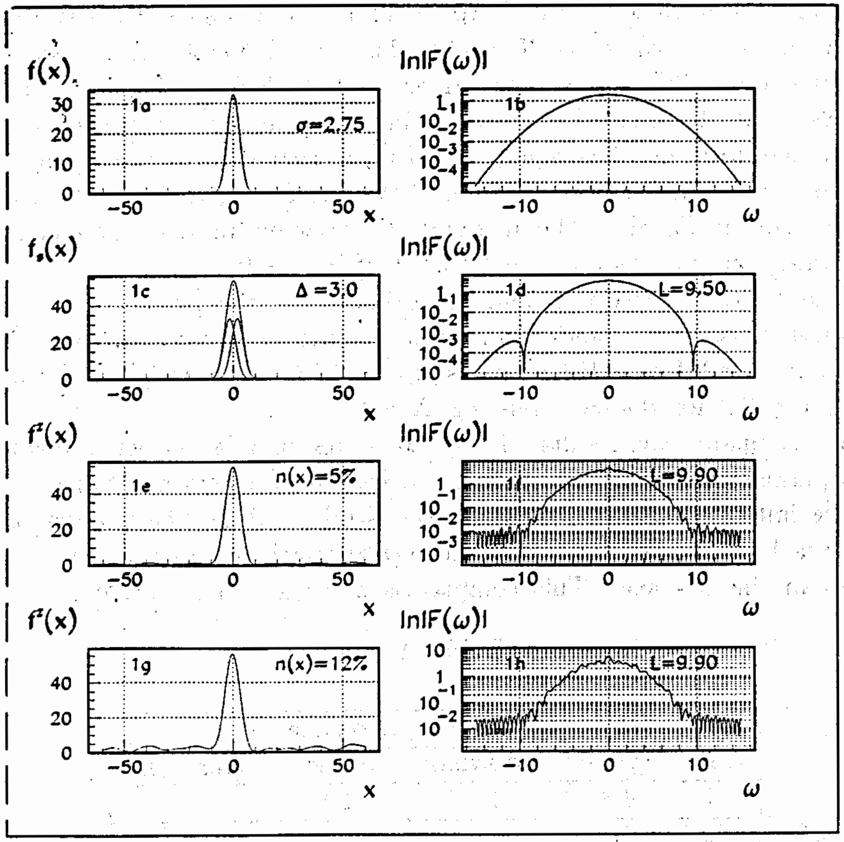

Fig. 1. The results of the simulation calculations for symmetrical gaussian spread function (see text).

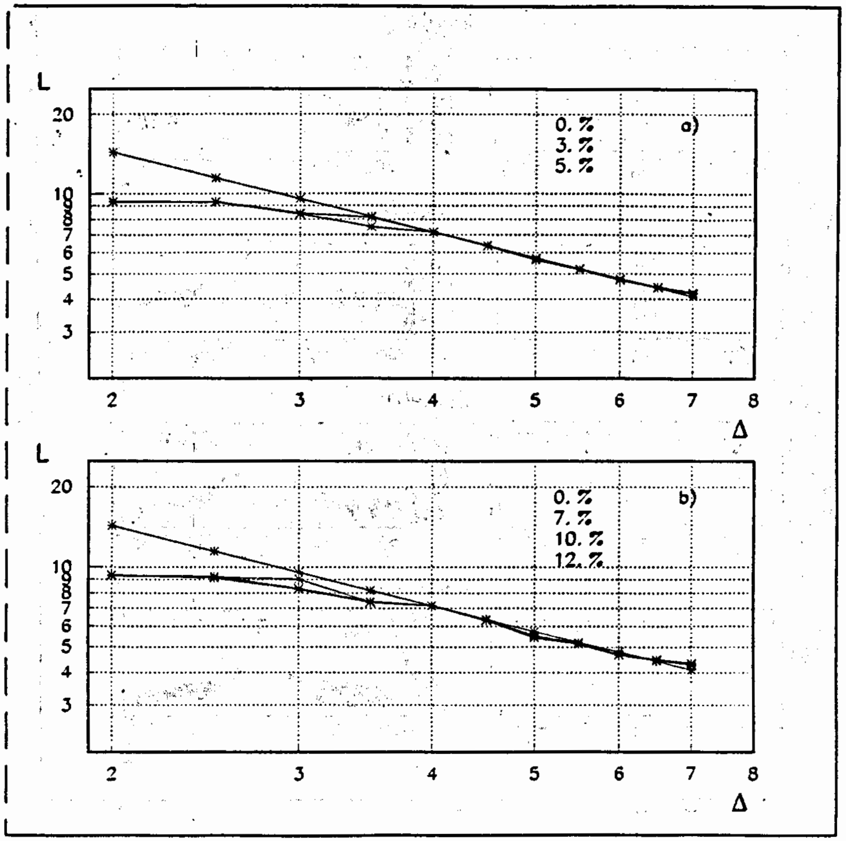

Fig. 2. The relation between distance L and the doublet splitting Δ for symmetrical gaussian spread function and various additive noise.

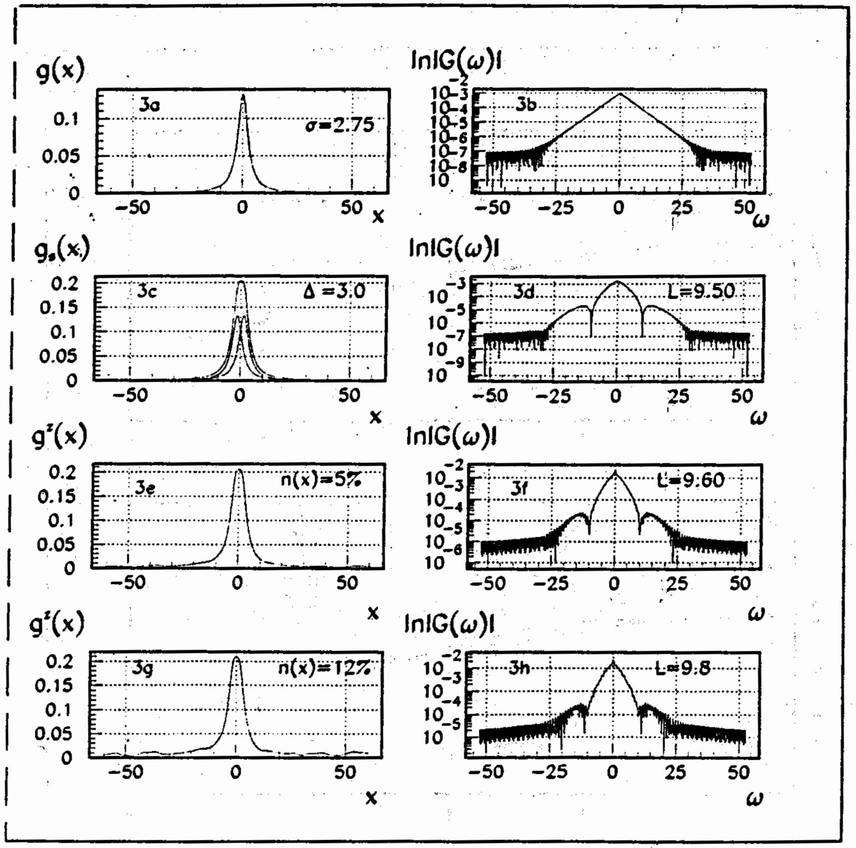

Fig. 3. The results of the simulation calculations for symmetrical lorentzian spread function (see text).

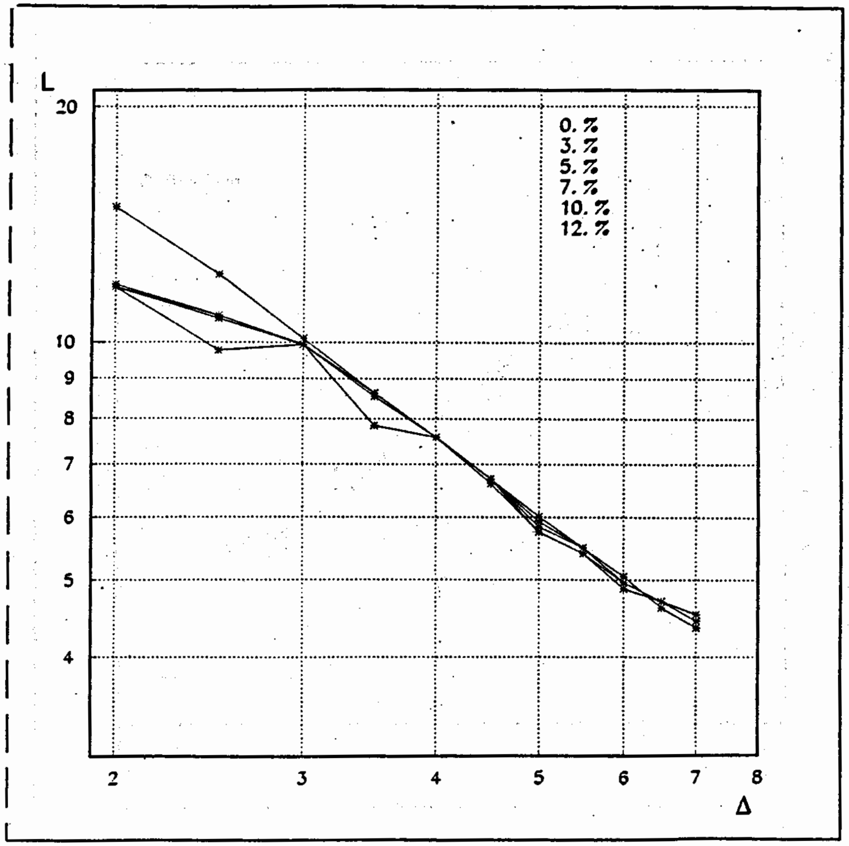

Fig. 4. The relation between distance L and the doublet splitting Δ for symmetrical lorentzian spread function and various additive noise.

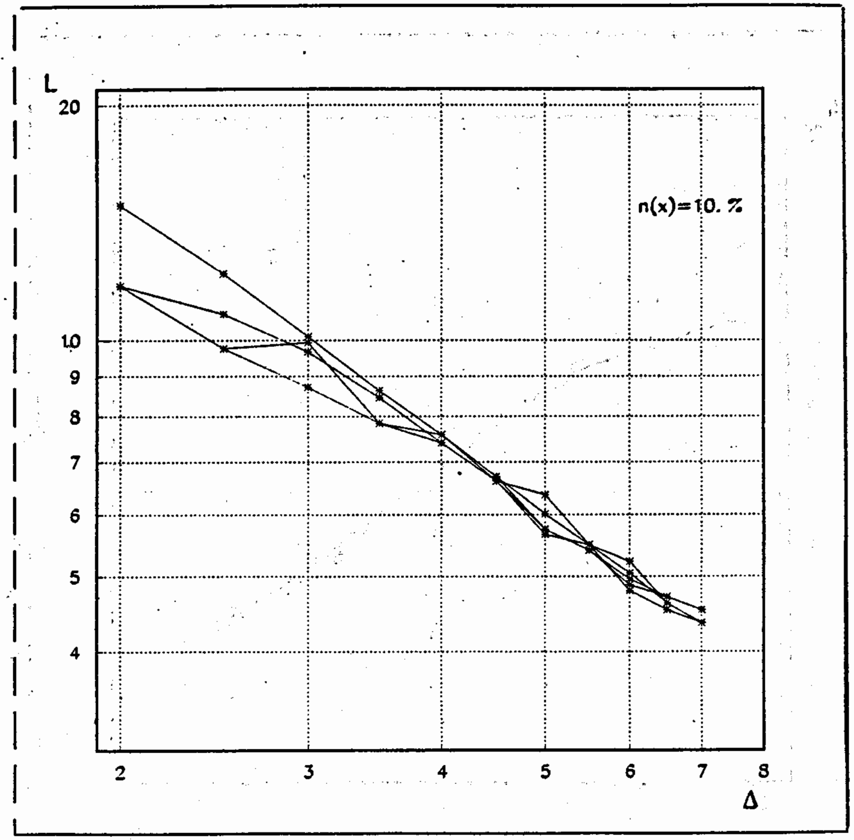

Fig. 5. The relation between L and Δ for four different noise (10%) realization.

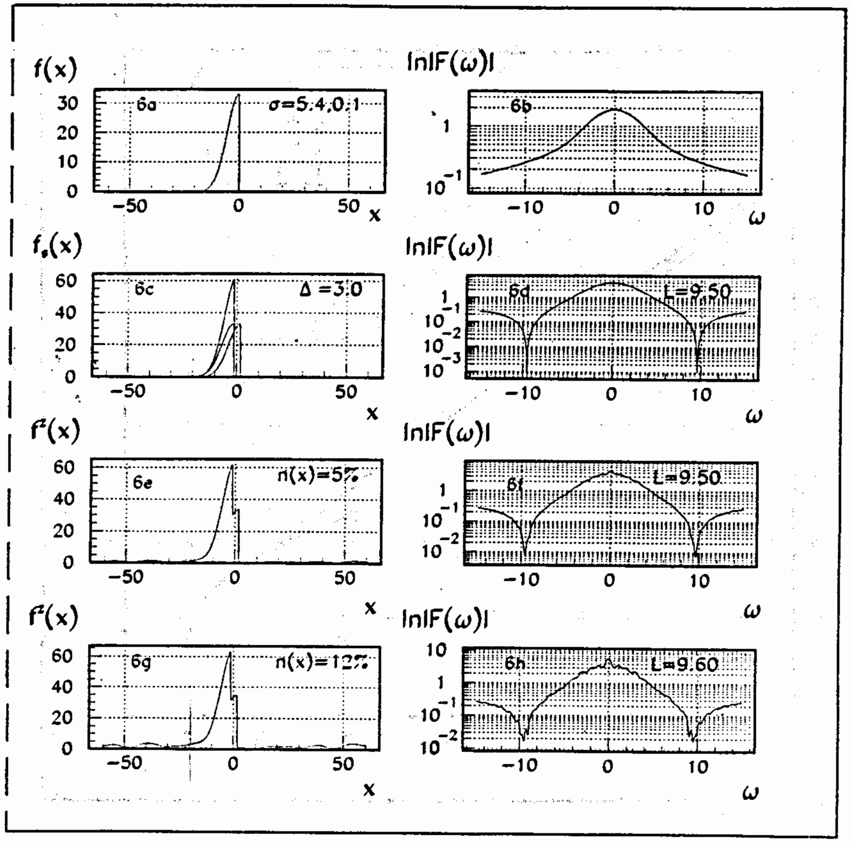

Fig. 6. The result of the simulation calculations for non - symmetrical gaussian spread function (see text).

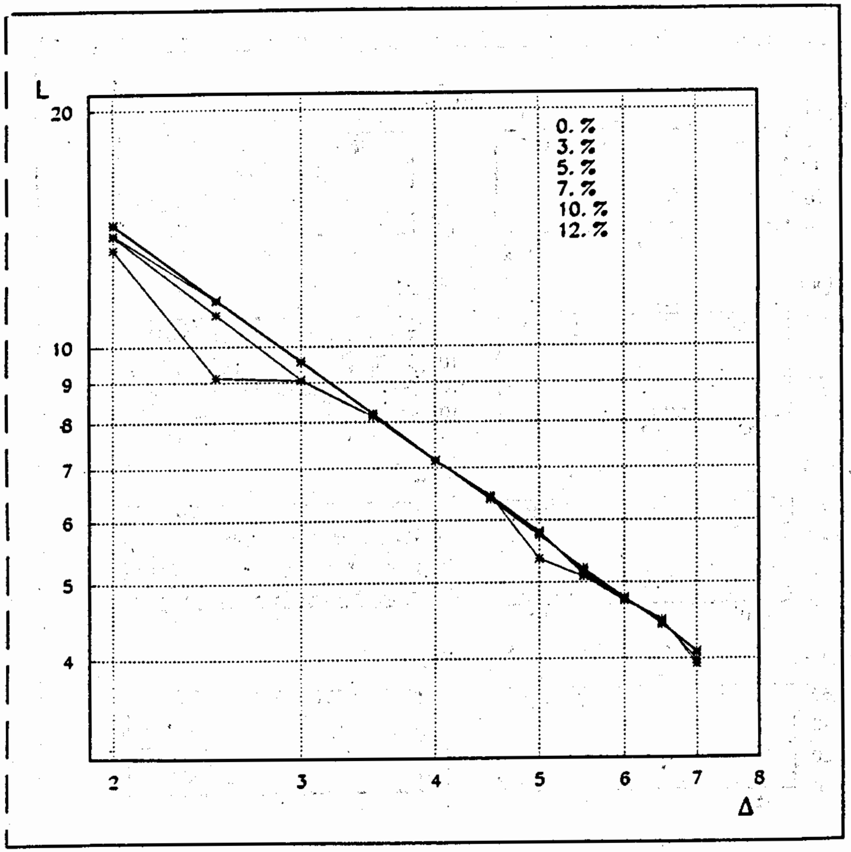

Fig. 7. The relation between L and Δ for non-symmetrical gaussian spread function and various additive noise.

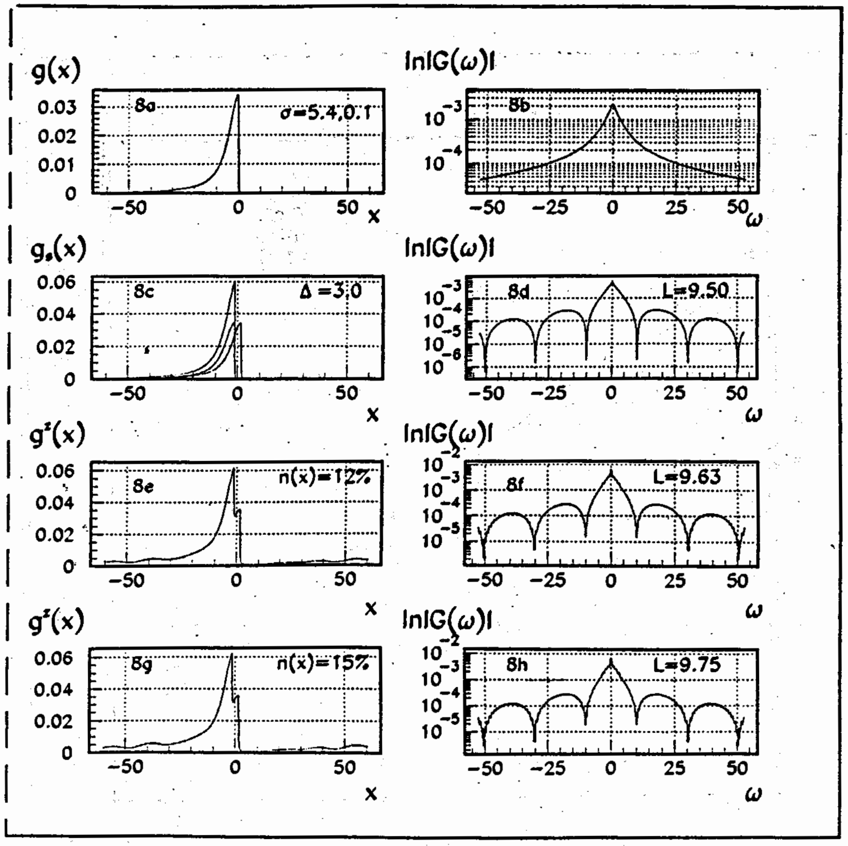

Fig. 8. The results of the simulation calculations for non - symmetrical lorentzian spread function (see text).

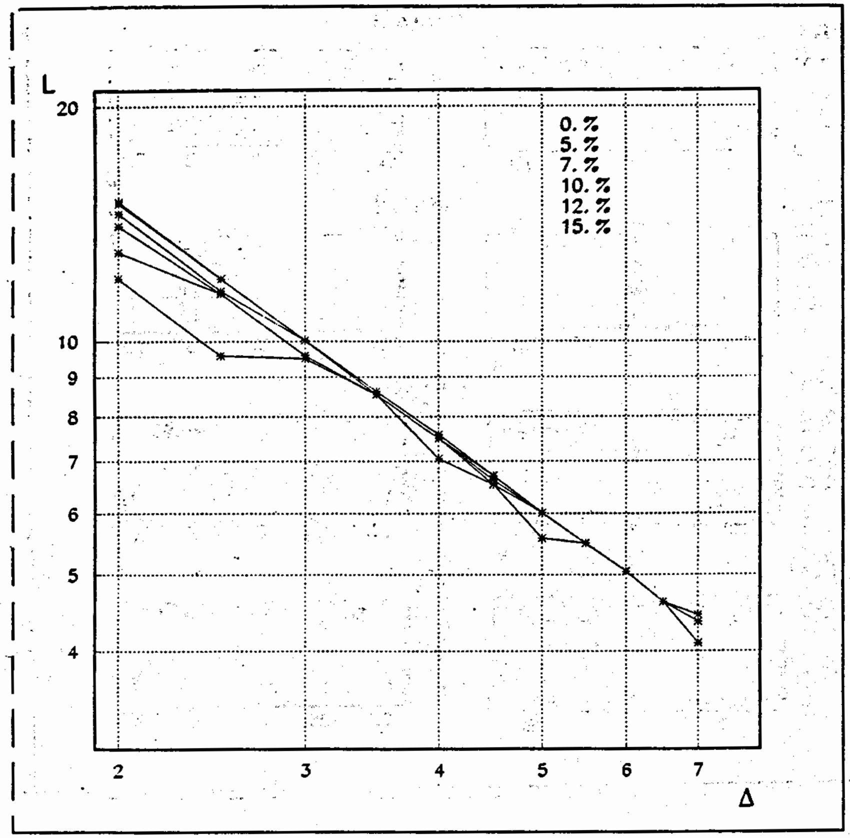

Fig. 9. The relation between L and Δ for non-symmetrical lorentzian spread function and various additive noise.

5. Conclusions

1. The doublet resolution algorithm without reconstruction of the initial signal has been simulated, for four different spread functions of the physical device.
2. It is shown that the best result and highest noise immunity give the physical device with non-symmetrical lorentzian function.
3. The results of this paper are in accordance with general recommendations which have been presented in paper [1].

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