

# СО0БЩЕНИЯ ОБЪЕДИНЕННОГО ИНСТИТУТА Я्रДЕРНЫХ ИССЛЕДОВАНИЙ 

## Дубна

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## EFFECTIVE PULSE RESOLUTION ALGORITHMS

 FOR DETECTORS WITH GAUSSIAN-LIKE SIGNAL SHAPE**[^0]
## 1 Introduction

Because of the excellent spatial resolution and relatively small pixel occupancy Silicon Drift Detectors [1] are used in many different experiments in relativistic nuclear physics (NA45, WA98, STAR et al.). The STAR Collaboration ( Solenoidal Tracker at RHIC) is designing a microvertex detector, the Silicon Vertex Tracker (SVT), which uses SDD to measure charge particle lit position near the beam vertex [2]. The CERES/NA45 experiment [3] at SPS CERN measures low-mass pair production in nuclear collisions and to use a doublet of 3 inches cylindrical silicon drift detectors [4] as a stand-alone vertex detector. The dublet of silicon drift detectors provide external part tracking and increase the ring finding efficiency pointing a ring centre candidates as well as background rejection of conversion and close Dalitz pairs.

Due to an extremely high multiplicity of central $\mathrm{Pb}-\mathrm{Pb}$ collisions (up to 1500 tracks) the crucial point of this SiDD tracking system is the demand for the highest accuracy and speed of any algorithm intended to determine the centre and the energy of each electron cloud detected by SiDD in conditions when over $30 \%$ of signals overlap each other in case of geometry of the SiDD doublet in the CERES set-up.

This note will cover some of the basic approaches in new algorithms of overlapping peak resolution, which finally enables us to cope with the high charged multiplicity environment in $\mathrm{Pb}-\mathrm{Pb}$ collisions taking into account both such factors as SiDD signal asymmetry and the noise correlation.

## 2 Problem formulation

The SiDD detector has a disk shape with the active inner and outer radii of $R_{\text {in }}=6$ and $R_{\text {out }}=32 \mathrm{~mm}$, respectively. The 360 radially oriented anodes register signals, i.e. arrived charges of
electron clouds appeared when particles hit the detector. While drifting of an electron cloud its size increases due to diffusion and electrostatic interaction between electrons. As it was shown in [6] the signal has a form of two-dimensional Gaussian with the maximum amplitude $A$ :

$$
\begin{equation*}
N\left(x, r ; A, x_{0}, R_{0}\right)=A e^{-\left(\frac{\left(x-x_{0}\right)^{2}}{2 \sigma_{x}^{2}}+\frac{\left(r-R_{0}\right)^{2}}{2 \sigma_{r}^{2}}\right)} \tag{1}
\end{equation*}
$$

where $x=r \varphi$ is perpendicular to the radius direction, $R_{0}$, $x_{0}=R_{0} \varphi_{0}$ are the initial coordinates of the electron cloud, $\sigma_{r}^{2}=2 D t, \sigma_{x}^{2}=2 D t R_{o u t} / R_{0}, D$ is the diffusion constant of electrons in silicon, $t=\left(R_{\text {out }}-R_{0}\right) / V_{\text {drift }}$, the radial velocity $V_{d r i f t}$ supposed to be constant.

The single particle signal (electron cloud charge) could be considered as being registered by several adjacent cells of 2D grid formed by 360 anodes in azimuthal direction and time-bins in radius direction. So the charge distribution to each cell can be calculated by 2D integration of (1) over this cell. If two signals are ovelapped, their contributions to the same cell are superposed.

Recent measurements [5] show that the integrated front-end electronics [7] did slightly disturb the assumption about the Gaussian-like symmetrical shape of the registered signals. Thus for each fixed radius $R$ the values of $\sigma_{r}$ and $\sigma_{x}$ could be tabulated during a special statistical processing of $\mathrm{Pb}-\mathrm{Pb}$ experimental data. In our further consideration we assume these parameters $\sigma_{r}$ and $\sigma_{x}$ to be known.

Besides, the front-end electronics produces an additional contribution to every part of signal registered in each cell, which can be considered as a correlated Gaussian noise with rms up to $10 \%$ of the average signal amplitude. The correlation exists in the radial direction only and the covariance function vanishes after 3-4 time bins. By taking into account the information about this
correlation one would improve estimations of signal parameters. However, in our first aproach we simplify the situation assuming noise in every cell to be independent. It should give us the upper bound of these estimations. Further in section 6 we extend our formalizm to the case of real signals with their asymmetry and correlation.

Our aim is to estimate the center and the amplitucle of a digitized signal, which can be considered as two-dimentional histogram $\left\{a_{i j}\right\}$ formed by extracting cluster of ajacent cells with amplitudes exceeding a given threshold. The straightforward approach to our estimation is to fit to this historgam 2D Gaussian (1). However due to its factorized riew we can split this task in two steps:

1. According to the number of anodes covering by the given cluster (let it be $k$ ) split 2D array $\left\{a_{i j}\right\}$ into $k$ one-dimentional histograms. Then for each fixed $\varphi_{j}$ fit to the corresponding partial histogram one-dimentional Gaussian

$$
\begin{equation*}
g(r ; A, R)=A e^{-\frac{(r-R)^{2}}{2 \sigma_{r}^{2}}} \tag{2}
\end{equation*}
$$

with $\sigma_{r}$ supposed to be a known constant inside of the given cluster. That gives you $k$ estimations $\left(\hat{A}_{j}, \hat{R}_{j}\right)$ for cach $\hat{x}_{j}=R_{j} \varphi_{j}$.
2. The wanted position of the signal $\left(R_{0}, x_{0}\right)$ can be calculated as the center of gravity of these $k$ estimations:

$$
\begin{equation*}
R_{0}=\frac{\sum_{j} \hat{A}_{j} \hat{R}_{j}}{\sum_{j} \hat{A}_{j}} ; \quad x_{0}=\frac{\sum_{j} \hat{A}_{j} \hat{x}_{j}}{\sum_{j} \hat{A}_{j}} \tag{3}
\end{equation*}
$$

To obtain its amplitude $A$ we can minimize the functional

$$
\begin{equation*}
L(A)=\sum_{j}\left(\hat{A}_{j}-N\left(\hat{x}_{j}, \hat{R}_{j} ; A, x_{0}, R_{0}\right)\right)^{2} \tag{4}
\end{equation*}
$$

where the function $N\left(x, r ; A, x_{0}, R_{0}\right)$ is taken from (1). That
gives

$$
\begin{array}{r}
\hat{A}=\left(\sum_{j} \hat{A}_{i, j} e^{-\left(\frac{\left(\hat{x}_{j}-x_{0}\right)^{2}}{2 \sigma_{x}{ }^{2}}+\frac{\left(\hat{R}_{j}-R_{0}\right)^{2}}{2 \sigma_{r}{ }^{2}}\right)}\right) /  \tag{5}\\
\left(\sum_{j} e^{-\left(\frac{\left(\hat{x}_{j}-x_{0}\right)^{2}}{2 \sigma_{x}{ }^{2}}+\frac{\left(\hat{R}_{j}-R_{0}\right)^{2}}{2 \sigma_{r}{ }^{2}}\right)}\right) .
\end{array}
$$

Unfortunately the main snag in this nice scheme is the very probable case when the extracted cluster are created by two (or even more) signals. For such cases we have to develop the following:

- a fast computational algorithm for the single signal case, which would implement the fit of the Gaussian (2) to onedimentional histogram;
- a reliable criterion for making a decision whether we have a single signal case or multi-signal one;
- generalize the above methods and algorithms to multi-signal cases.

These items are considered in the further sections.

## 3 Fast algorithm for fitting one-dimentional histogram

Let us have the histogram $\left\{a_{i}\right\}, i=\overline{1, n}$ with the unit binsize, i.e. $\Delta r=r_{i+1}-r_{i}=1$. We have to fit to this histogram the function $g\left(r ; A, r_{0}\right)$ given by (2). The corresponding least square functional

$$
\begin{equation*}
\mathcal{L}\left(A, r_{0}\right)=\sum_{i}\left(a_{i}-\int_{r_{i}}^{r_{i+1}} g\left(r ; A, r_{0}\right) d r\right)^{2} \tag{6}
\end{equation*}
$$

contains the unknown parameter $r_{0}$ under the integral sign. This obstacle is avoidable by replacing each integral in (6) by its approximate mean value: $g\left(\bar{r}_{i} ; A, r_{0}\right)$, where $\bar{r}_{i}=\left(r_{i+1}+r_{i}\right) / 2$, so (6) is simplified to

$$
\begin{equation*}
\tilde{\mathcal{L}}\left(A, r_{0}\right)=\sum_{i}\left(a_{i}-g\left(\bar{r}_{i} ; A, r_{0}\right)\right)^{2} \tag{7}
\end{equation*}
$$

Searching for its minimum one should solve the corresponding system of normal equations obtained by equating to zero the $\tilde{\mathcal{L}}\left(A, r_{0}\right)$ partial derivatives. However, this system is, unfortunately, transcendental. That requests to develop a special iterative procedure to solve it. As initial values $A^{(0)}, r_{0}^{(0)}$ of unknown parametes for this procedure in a single signal case we use $A^{(0)}=\max _{\bar{r}_{i}} a_{i}$ and its position (the center of gravity

$$
\begin{equation*}
A_{c o g}=\left(\sum_{i} a_{i} \bar{r}_{i}\right) /\left(\sum_{i} a_{i}\right) \tag{8}
\end{equation*}
$$

is also admissible, since it is more accurate). Then considering (7) as a function in 3D space of two parameters we approximate it in the vicinity of $A^{(0)}, r_{0}^{(0)}$ by an elliptic paraboloid

$$
\begin{equation*}
z=a x^{2}+b y^{2}+c x y+d x+e y+f \tag{9}
\end{equation*}
$$

where $x, y$ are current values of parameters $r, A$. To find six coefficients of (9) it is necessary to calculate the values of $\tilde{\mathcal{L}}\left(A, r_{0}\right)$ in the point template, i.e. in six specialy selected points surrounding $x^{0}=r_{0}^{(0)}, y^{0}=A^{(0)}$ choosen as the base point of this template. We use the simplest template design: the base itself, step left, step right from it in each dimension and the last point by step right in both dimensions. After solving the corresponding system of six linear equations to find our paraboloid coefficients its minimum coordinates are easily calculated

$$
\begin{equation*}
x_{\min }=x^{0}+\frac{-2 b d+c e}{4 a b-c^{2}}, \quad y_{\text {min }}=y^{0}+\frac{-2 a e+c d}{4 a b-c^{2}} . \tag{10}
\end{equation*}
$$

The obtained coordinates are used as $A^{(1)}, r_{0}^{(1)}$, i.e. as the base point for the second iteration. Although a few iterations. is enough, as a rule, the iteration process is controlled by testing of achieving either the maximum admissible accuracy of the minimum position or the fixed maximal number of iterations.

Further we refer to this method as to Paraboloidal Approximation Method - PAM.

## 4 Criterion: one or more signals

It is not a hard problem to check if in the given cluster sample $\left\{a_{i j}\right\}$ we have more then one explicit and distinguished local maxima. The problem arises when we have two (or more), but so close each other that they are indistinguishable producing the only one maximum.

Let us restrict ourself by one-dimensional case and assume unprobable to have more then two peaks when a histogram has the only one maximum. Then we could have two possibilities for two close peaks:

1. their amplitudes are so different that the smallest is "hidden under the wing" of bigger one;
2. their amplitudes are equal (very rare situation, but could happened).

Having the only one maximum in both such cases we should apply PAM to fit one Gaussian. However, since it is not a case we have to obtain too big value of $\tilde{\mathcal{L}}\left(\hat{A}, \hat{r}_{0}\right)$ (see (7)). According to the theory of mathematical statustics this functional in the point of its minimum should obey to the $\chi^{2}$-distribution with the degree of freedom equal to the number of bins minus the number of parameters. It could give us, in principle, a threshold for testing hypotheses about peak numbers. Unfortunately, due to
the violation of theory assumptions for real and even simulated data it was found that the distributions of $\tilde{\mathcal{L}}\left(\hat{A}, \hat{r}_{0}\right)$ for a single and double peaks ovelap each other too much to use any treshold to distiguish them confidently.

So we have to look for a more comprehensive criterion analysing both cases enumerated above. In the first case we should have a histogram asymmetry. The easiest test of it is the difference between the positions of maximum and the center of gravity (8)

$$
\begin{equation*}
\Delta_{a s y m m}=\max _{\bar{r}_{i}} a_{i}-\left(\sum_{i} a_{i} \bar{r}_{i}\right) /\left(\sum_{i} a_{i}\right) \tag{11}
\end{equation*}
$$

More cumbersom is the known expression for the third central momentum $\mathcal{M}_{3}=\left(\sum_{i} a_{i}\left(\bar{r}_{i}-A_{\text {cog }}\right)^{3}\right) /\left(\sum_{i} a_{i}\right)$.

In the second case of two close peaks with equal amplitudes they produce a symmetric, but much wider histogram. Its width can be tested by calculating of the second central momentum $\mathcal{M}_{2}=\left(\sum_{i} a_{i}\left(\bar{r}_{i}-A_{\text {cog }}\right)^{2}\right) /\left(\sum_{i} a_{i}\right)$, which must considerably differ from the given $\sigma_{r}^{2}$.

## 5 Double peak resolution method

The shape of a histogram produced by two superposing peaks can be described like

$$
\begin{equation*}
f\left(x_{i} ; A_{1}, x_{1}^{(0)}, A_{2}, x_{2}^{(0)}\right)=A_{1} e^{-\frac{\left(x_{i}-x_{1}^{(0)}\right)^{2}}{2 \sigma^{2}}}+A_{2} e^{-\frac{\left(x_{i}-x_{2}^{(0)}\right)^{2}}{2 \sigma^{2}}} . \tag{12}
\end{equation*}
$$

This expression depends on four parameters. To find them we have to minimize a functional generalizing the view of (6)

$$
\begin{equation*}
\mathcal{L}_{4}=\sum_{i}\left(a_{i}-f\left(x_{i} ; A_{1}, x_{1}^{(0)}, A_{2}, x_{2}^{(0)}\right)\right)^{2} \tag{13}
\end{equation*}
$$

A direct generalization of the PAM procedure on four-parameter functional (13) would lead to treating 5D elliptic paraboloid. To avoid that we take into account that the partial derivatives of $\mathcal{L}_{4}$ in respect to $A_{1}, A_{2}$ are linear, so we can easily calculate both amplitudes by solving the system of two linear equations

$$
\begin{equation*}
\frac{\partial \mathcal{L}_{4}}{\partial A_{1 ; 2}}=0 \tag{14}
\end{equation*}
$$

Then we can apply the above iterative PAM-procedure for minimizing $\mathcal{L}_{4}$ in respect to two resting parameters $x_{1}^{(1)}, x_{2}^{(1)}$.

The cardinal problem of this way of solution is the most accurate choice of initial values of parameters, since it determines the convergence and the speed of the iteration process.

In the easiest case, when our histogram has two explicit and distinguished local maxima, we can take as initial values the amplitudes and positions of both of them. However, when we have the only one maximum, but it does not satisfy the single peak criteria, we need a special procedure for determine the starting values for the "invisible" peak.

The first approach (next called as sequential elimination method - SEM) consist in applying the PAM procedure with the functional (6) for obtaining $A_{1}, x_{1}$ and then use them to generate an artificial theoretical histogram imitating the first peak: $\tilde{a}_{i}=\int_{x_{i}}^{x_{i+1}} g\left(x ; A_{1}, x_{1}^{(0)}\right) d x$ with $g\left(r ; A_{1}, x_{1}^{(0)}\right)$ from (2). After substracting $\tilde{a}_{i}$ from original histogram we determine from the residual histogram the starting values $A_{2}^{(0)}, x_{2}^{(0)}$ as the maximum amplitude and its position. Then we apply PAM again to the residual histogram. This process can be continued iteratively for getting better accuracy, but for our purpose the first iteration is enough.

After inserting the pair of initial values $x_{1}^{(0)}, x_{2}^{(0)}$ to the linear system (14) we can solve it to obtain $A_{1}^{(0)}, A_{2}^{(0)}$ and then, as it was noted above, apply PAM-procedure for minimizing $\mathcal{L}_{4}$ to calculate next iterative values of $x_{1}^{(1)}, x_{2}^{(1)}$.

The second approach for calculating more accurate approximations of parameters further called as Linearization Method LM is based on the iterative solution of the linearized system of trascendental equations:

$$
\begin{equation*}
\frac{\partial \mathcal{L}_{4}}{\partial x_{1 ; 2}^{(0)}}=0 \tag{15}
\end{equation*}
$$

The linearization of (15) is done as follows. Taking the initial value of $x_{1}^{(0)}$ as $\max _{\bar{r}_{i}} a_{i}$ we calculate $x_{2}^{(0)}$ as $x_{1}^{(0)}+3$ signn $\Delta_{\text {asymm }}$ (see (11)). Both differencies $\Delta x_{1}=x_{1}-x_{1}^{(0)}, \Delta x_{2}=x_{2}-x_{2}^{(0)}$ are assumed to be small enough to have any of their products to be the higher order of smallness. Thus, we can substitute in (15) each parameter by the sum of its initial value and corresponding delta-correction and omit all members of the second and higher order of smallness obtaining a linear equation system

$$
\left\{\begin{array}{c}
\Delta x_{1} C+\Delta x_{2} D=G  \tag{16}\\
\Delta x_{1} E+\Delta x_{2} F=H .
\end{array}\right.
$$

Here we denote
$C=\sum_{i}\left(d_{i}^{(0)}\left(1+\frac{\left(x_{i}-x_{i}^{(0)}\right)^{2}}{\sigma^{2}}\right)+A_{1} \frac{\left(x_{i}-x_{i}^{(0)}\right)^{2}}{\sigma^{2}} e^{-\frac{\left(x_{i}-x_{i}^{(0)}\right)^{2}}{2 \sigma^{2}}}\right) e^{-\frac{\left(x_{i}-x_{i}^{(0)}\right)^{2}}{2 \sigma^{2}}}$
$D=\frac{A_{2}}{\sigma^{2}} \sum_{i}\left(x_{i}-x_{1}^{(0)}\right)\left(x_{i}-x_{2}^{(0)}\right) e^{-\frac{\left(x_{i}-x_{1}^{(0)}\right)^{2}}{2 \sigma^{2}}} e^{-\frac{\left(x_{i}-x_{2}^{(0)}\right)^{2}}{2 \sigma^{2}}}$
$E=\frac{A_{1}}{\sigma^{2}} \sum_{i}\left(x_{i}-x_{1}^{(0)}\right)\left(x_{i}-x_{2}^{(0)}\right) e^{-\frac{\left(x_{i}-x_{1}^{(0)}\right)^{2}}{2 \sigma^{2}}} e^{-\frac{\left(x_{i}-x_{i}^{(0)}\right)^{2}}{2 \sigma^{2}}}$
$F=\sum_{i}\left(d_{i}^{(0)}\left(1+\frac{\left(x_{i}-x_{2}^{(0)}\right)^{2}}{\sigma^{2}}\right)+A_{2} \frac{\left(x_{i}-x_{i}^{(0)}\right)^{2}}{\sigma^{2}} e^{-\frac{\left(x_{i}-x_{i}^{(0)}\right)^{2}}{2 \sigma^{2}}}\right) e^{-\frac{\left(x_{i}-x_{2}^{(0)}\right)^{2}}{2 \sigma^{2}}}$
$G=\sum_{i} d_{i}^{(0)}\left(x_{i}-x_{1}^{(0)}\right) e^{-\frac{\left(x_{i}-x_{i}^{(0)}\right)^{2}}{2 \sigma^{2}}}$
$H=\sum_{i} d_{i}^{(0)}\left(x_{i}-x_{2}^{(0)}\right) e^{-\frac{\left(x_{i}-x_{2}^{(0)}\right)^{2}}{2 \sigma^{2}}}$
where
$d_{i}^{(0)}=a_{i}-A_{1} e^{-\frac{\left(x_{i}-x_{1}^{(0)}\right)^{2}}{2 \sigma^{2}}}-A_{2} e^{-\frac{\left(x_{i}-x_{2}^{(0)}\right)^{2}}{2 \sigma^{2}}}$.

After solution this linear system we add obtained corrections to the initial values of parameters receiving $x_{1 ; 2}^{(1)}$. Inserting them to the linear system (14) we obtain $A_{1 ; 2}^{(1)}$ and repeat the whole procedure iteratively until the corrections become less than a prescribed value or the number of iterations attains its limits.

## 6 Asymmetrical signals and correlated noise modifications

On the representative sample of central $\mathrm{Pb}-\mathrm{Pb}$ collisions the pulses were fitted using a composition of two gaussians with widths $\sigma_{0}-\Delta$ below the maximum, and $\sigma_{0}+\Delta$ above the maximum:

$$
f(x)=\frac{A}{\sqrt{2 \pi} \sigma_{0}} e^{\frac{-\left(x-x_{0}\right)^{2}}{2 \sigma^{2}}}
$$

where $x \equiv x_{i}$ are time bins, $A$ is amplitucle, $x_{0}$ is a position of the gaussian maximum, and

$$
\sigma= \begin{cases}\sigma_{0}-\Delta & \text { for } x \leq x_{0}  \tag{17}\\ \sigma_{0}+\Delta & \text { for } x>x_{0}\end{cases}
$$

The results from fitting a selection of pulses which have only small admixture of double pulses yields $\sigma_{0}=1.73$ time bins at $x_{0}=50$ time bins and $\sigma_{0}=2.09$ time bins at $x_{0}=220$ time bins The dependence is rather linear, namelly $\sigma_{0}=1.626+0.2097 \cdot 10^{-2} \cdot x_{0}$. The delta parameter clrops, $\Delta=$ 0.18 at $x_{0}=50$, and $\Delta=0.02$ at $x_{0}=220$.

To take this asymmetry into account in our calculation scheme (see eqs.(2) and (12)), it is necessary to use $\sigma$ from (17) in corresponding ranges of $x$.

To handle a problem of the noise correlation it is necessary to include in our model the following formalism.

In order to simulate a normally distributed vector $\mathbf{Y}\left(y_{1}, \ldots y_{n}\right)$ with the given covariance matrix $C\left(c_{i j}\right)$ we should start from
modelling a normally distributed vector $\mathbf{X}\left(x_{1}, x_{2}, \ldots, x_{n}\right)$ with independent components. Then one can convert it into the sought vector $\mathrm{Y}\left(y_{1}, \ldots y_{n}\right)$ by using a triangular transformation

$$
y_{k}=b_{k 1} \cdot x_{1}+b_{k 2} \cdot x_{2}+\ldots+b_{k k} \cdot x_{k}, k=\overline{1, n},
$$

where the coefficients $b_{i j}$ can be obtained by the recurrent formula:

$$
b_{i j}=\frac{c_{i j}-\sum_{k=1}^{j-1} b_{i k} b_{k j}}{\sqrt{c_{j j}-\sum_{k=1}^{j-1} b_{j k}^{2}}} .
$$

Now if we have mẹasured points $\left(x_{i}, y_{i}\right), i=1, n$ with the known covariance matrix $c_{i j}, i, j=\overline{1, n}$, then in order to fit a curve $y=f(x ; b)$ to these data we have to minimize a functional

$$
L(b)=\sum_{i j}^{n} w_{i j}\left(y_{i}-f\left(x_{i} ; b\right)\right)\left(y_{j}-f\left(x_{j} ; b\right)\right)
$$

where $w_{i j}$ is the elements of the matrix which is inverted to $C=$ $\left\|c_{i j}\right\|$.

Equating to zero the first derivative of $L(b)$ by $b$ one obtains

$$
\begin{array}{r}
\frac{d L(b)}{d b}=\sum_{i j}^{n} w_{i j}\left(\left(y_{i}-f\left(x_{i} ; b\right)\right) \frac{\partial f\left(x_{j} ; b\right)}{\partial b}+\right.  \tag{18}\\
\left.\left(y_{j}-f\left(x_{j} ; b\right)\right) \frac{\partial f\left(x_{i} ; b\right)}{\partial b}\right)=0 .
\end{array}
$$

If $b$ is a vector of parameters we have a system of such equations.
In our case, when $f\left(x ; A_{1}, x_{1}^{(0)}, A_{2} . x_{2}^{(0)}\right)$ given by formula (12), we should obtain after the linearization of this system the linear system, which is exactly the same as (16), but with the more complicate coefficients

$$
\begin{aligned}
C=\sum_{i j} w_{i j} & \left(d_{j}^{(0)}\left(1+\frac{\left(x_{i}-x_{1}^{(0)}\right)^{2}}{\sigma^{2}}\right)+\right. \\
& \left.+\frac{A_{1}}{\sigma^{2}}\left(x_{i}-x_{1}^{(0)}\right)\left(x_{j}-x_{1}^{(0)}\right) e^{-\frac{\left(x_{j}-r^{(0)}\right)^{2}}{2 \sigma^{2}}}\right) e^{-\frac{\left(x_{i}-f^{(0)}\right)^{2}}{2 \sigma^{2}}}
\end{aligned}
$$

$$
\begin{aligned}
& \begin{aligned}
& D= \frac{A_{2}}{\sigma^{2}} \sum_{i j} w_{i j}\left(x_{i}-x_{1}^{(0)}\right)\left(x_{j}-x_{2}^{(0)}\right) e^{-\frac{\left(x_{i}-x_{1}^{(0)}\right)^{2}}{2 \sigma^{2}}} e^{-\frac{\left(x_{j}-x_{2}^{(0)}\right)^{2}}{2 \sigma^{2}}} \\
& E= \frac{A_{1}}{\sigma^{2}} \sum_{i j} w_{i j}\left(x_{j}-x_{1}^{(0)}\right)\left(x_{i}-x_{2}^{(0)}\right) e^{-\frac{\left(x_{j}-x_{j}^{(0)}\right)^{2}}{2 \sigma^{2}}} e^{-\frac{\left(x_{i}-x_{i}^{(0)}\right)^{2}}{2 \sigma^{2}}} \\
& F= \sum_{i j} w_{i j}\left(d_{j}^{(0)}\left(1+\frac{\left(x_{i}-x_{2}^{(0)}\right)^{2}}{\sigma^{2}}\right)+\right. \\
&\left.\quad+\frac{A_{2}}{\sigma^{2}}\left(x_{i}-x_{2}^{(0)}\right)\left(x_{j}-x_{2}^{(0)}\right) e^{-\frac{\left(x_{j}-x_{0}^{(0)}\right)^{2}}{2 \sigma^{2}}}\right) e^{-\frac{\left(x_{i}-x_{2}^{(0)}\right)^{2}}{2 \sigma^{2}}} \\
& \quad
\end{aligned} \\
& G=\sum_{i j} w_{i j} d_{j}^{(0)}\left(x_{i}-x_{1}^{(0)}\right) e^{-\frac{\left(x_{i}-x_{1}^{(0)}\right)^{2}}{2 \sigma^{2}}} \\
& H=\sum_{i j} w_{i j} d_{j}^{(0)}\left(x_{i}-x_{2}^{(0)}\right) e^{-\frac{\left(x_{i}-x_{2}^{(0)}\right)^{2}}{2 \sigma^{2}}}
\end{aligned}
$$

In the next section results of comparative analysis SEM-PAM, LM-PAM and MINUIT applications for simulated data are given.

## 7 Results

The method was tested using both Monte-Carlo simulated data and data from $160 \mathrm{GeV} / \mathrm{u} \mathrm{Pb}-\mathrm{Au}$ collision fron 1995 CERES run.

For the sake of brevity we present here only results of compared methods testing on double overlapping signals, since they are most critical for the SiDD data handling. To provide a data set for testing a simple routine was written, which simulates histograms according to (12)-(17) for two overlapped "asymmetrical" Gaussians with $\sigma_{0}=1.626+0.2097 \cdot 10^{-2} \cdot x_{0}$ and corresponding $\Delta$. Contributions of every Gaussian into each bin are superposed (summed).

Then to each bin a random noise is added with the normal distribution with $\sigma=10 \%, A_{\text {noise }}$ is $10 \%$ of the mean amplitude value of pulses. Two versions of the noise distribution were simulated: first, the independent gaussian noise; second, when the
noise counts in adjacent bins were correlated according to the given covariance matrix. However, it should be noted that data processing in the accordance with the formulae of the previous section of the model simulated by the second version showed a very weak influence of the noise correlation due to the covariance matrix obtained by special measurements. Simulating two overlapping signals we chose the different set of both distances between two gaussian maxima and their amplitude ratios.

In this note we present results of comparison of only MINUIT and LM-PAM performances, since LM-PAM procedure shows results obviously better then SEM-PAM or some other of being tested.

Our calculations show that in comparison with MINUIT LMPAM algorithm has the same or even better accuracy for a double peak parameter reconstruction being 5-7 times faster in the range of distances from 7 timebins up to $\sigma_{0}$.


Fig 1. Distributions of $\sigma$.

We studied on a simulated data sample the dependence of the algorithm efficiency upon the gap between pulses and found that efficiency of double pulse reconstruction is on the level of $93 \%$ for gaps greater than $1.5 \sigma$ and decreases to the level $8 \%$ for gaps smaller than $1.5 \sigma$ (one half of the mid-pulse-width). The dependence of the accuracy of the pulse reconstruction algorithm upon the distance between pulses is shown in fig. 1. Besides, we would like to stress here, that while PAM-LM can proceed events with gaps between pulses equal even to 2 , the MINUIT method could not proceed any case when this distance is smaller or equal
to 2.5 .
The small systematic shift in pulse positioning by both methods within of the calculation errors should be mentioned.

Fig. 2 shows distributions of mean value of $\chi^{2}$ as a function of the distance between pulses (normalised to $\sigma$ ) in MINUIT and ROBUST methods for $\mathrm{Pb}-\mathrm{Au}$ ' 95 data. As one can see, the $\chi^{2}$ - values of LM-PAM are much lower than such values for the MINUIT method.


Figure 1: $\chi^{2}$ distributions.

## 8 Conclusion

Various approaches to the effective and precise determination of characteristics of signals detected by the SiDD tracking system of the CERES spectrometer were studied in conditions of the high multiplicity of $\mathrm{Pb}-\mathrm{Pb}$ collision experiment. The study was focused on the most delicate problem of the fast and most accurate resolution of overlapping siguals. In the assumption of the symmetrical gaussian-like signal shape the problem was reduced to the case of handling of one-dimentional histograms. For the majority of cases when digitized signals are overlapped, but still distinguished the parabolic approximation method (PAM) is proposed for the effective calculation of suclı signal parameters as its position and amplitude. The linearization method (LM) was developed for providing the initial parameter values for the PAM iteration scheme. One of the PAM advantages is its possibility to be generalized for a non-gausian shape of the signal. One more SEM procedure for sequential one-by-one signal elimination was developed, which features are promising for processing of multipeak clusters.

The developed procedures were then generalized on the case of correlated noise. In the most complicate cases of overlapping signals when they are undistinguishable producing an unimodal histogram the comparable analysis of three computational schemes: SEM-PAM, LM-Pam and MINUIT was fulfilled on simulated data. It shows that for the acceptable precision ( $\sim 0.1$ of timebin) LM-PAM procedure is the fastest (one order of magnitude faster then MINUITE) with the confident resolution of two signals, if they are not closer then $\sigma_{r}$ (otherwise one would has a dramatic lost of accuracy and increase of computing time).

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## Агакишиев Г. и др.

E10-97:105
Эффективные алгоритмы разрешения сигналов детекторов
с гауссоподобной формой отклика
Вследствие большой множественности вторичных частиц в центральных взаимодействиях релятивистских ядер с ядрами решающим моментом, для систем восстановления вершин является требование высокой точности и скорости аггоритмов, предназначенных для определения пространственных координат каждой заряженной частицы, регистрируемой этими детекторами.

Предложены эффективные алгоритмы разрешения сигналов в детекторах с гауссоподобной формой сигнала, например, в системе восстановления вершин, основанной на силиконовых дрейфовых камерах.

Изложены некоторые повые алгоритмы восстановления перекрываююцихяя сигналов, которые позволяют справиться с высокой плотностью вторичных частиц в ядерных столкновениях с учетом асимметрии сигнала и корреляции шума.

Работа выполнена в Лаборатории высоких энергий и Лаборатории вычислительной техники и автоматизации ОИЯИ.

Сообшение Объединенног института ядерных исследований Дубна, 1997

## Agakichiev G. et al.

E10-97-105
Effective Pulse Resolution Algorithms for Detectors
with Gaussian-Like Signal Shape
Due to an extremely high multiplicity of central ultrarelativistic nucleus-nucleus collisions the crucial point of vertex tracking systems is the demand for the highest accuracy and speed of any algorithm intended to determine the position of each charge particle detected by this systems.

We propose effective pulse resolution algorithms for the detectors with Gaussian-like signal shape, for example, a vertex tracking system based on silicon drift detectors (SiDD).

In this paper we present some new algorithms of overlapping peak resolution, which finally enables us to cope with the high charged multiplicity environment in nucleus collisions taking into account both such factors as SiDD signal asymmetry and the noise correlation.

The investigation has been performed at the Laboratory of High Energies and Laboratory of Computing Techniques and Automation, JINR.


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